CHAPTER 1

Basic Principles of Digital Systems

OUTLINE

1.1 Digital Versus Analog Electronics
1.2 Digital Logic Levels
1.3 The Binary Number System
1.4 Hexadecimal Numbers
1.5 Digital Waveforms

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

• Describe some differences between analog and digital electronics.
• Understand the concept of HIGH and LOW logic levels.
• Explain the basic principles of a positional notation number system.
• Translate logic HIGHs and LOWs into binary numbers.
• Count in binary, decimal, or hexadecimal.
• Convert a number in binary, decimal, or hexadecimal to any of the other number bases.
• Calculate the fractional binary equivalent of any decimal number.
• Distinguish between the most significant bit and least significant bit of a binary number.
• Describe the difference between periodic, aperiodic, and pulse waveforms.
• Calculate the frequency, period, and duty cycle of a periodic digital waveform.
• Calculate the pulse width, rise time, and fall time of a digital pulse.

Digital electronics is the branch of electronics based on the combination and switching of voltages called logic levels. Any quantity in the outside world, such as temperature, pressure, or voltage, can be symbolized in a digital circuit by a group of logic voltages that, taken together, represent a binary number.

Each logic level corresponds to a digit in the binary (base 2) number system. The binary digits, or bits, 0 and 1, are sufficient to write any number, given enough places. The hexadecimal (base 16) number system is also important in digital systems. Since every combination of four binary digits can be uniquely represented as a hexadecimal digit, this system is often used as a compact way of writing binary information.

Inputs and outputs in digital circuits are not always static. Often they vary with time. Time-varying digital waveforms can have three forms:

1. Periodic waveforms, which repeat a pattern of logic 1s and 0s
2. Aperiodic waveforms, which do not repeat
3. Pulse waveforms, which produce a momentary variation from a constant logic level
1.1 Digital Versus Analog Electronics

**KEY TERMS**

- **Continuous** Smoothly connected. An unbroken series of consecutive values with no instantaneous changes.
- **Discrete** Separated into distinct segments or pieces. A series of discontinuous values.
- **Analog** A way of representing some physical quantity, such as temperature or velocity, by a proportional continuous voltage or current. An analog voltage or current can have any value within a defined range.
- **Digital** A way of representing a physical quantity by a series of binary numbers. A digital representation can have only specific discrete values.

The study of electronics often is divided into two basic areas: analog and digital electronics. Analog electronics has a longer history and can be regarded as the “classical” branch of electronics. Digital electronics, although newer, has achieved greater prominence through the advent of the computer age. The modern revolution in microcomputer chips, as part of everything from personal computers to cars and coffee makers, is founded almost entirely on digital electronics.

The main difference between analog and digital electronics can be stated simply. Analog voltages or currents are continuously variable between defined values, and digital voltages or currents can vary only by distinct, or discrete, steps.

Some keywords highlight the differences between digital and analog electronics:

<table>
<thead>
<tr>
<th>Analog</th>
<th>Digital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuously variable</td>
<td>Discrete steps</td>
</tr>
<tr>
<td>Amplification</td>
<td>Switching</td>
</tr>
<tr>
<td>Voltages</td>
<td>Numbers</td>
</tr>
</tbody>
</table>

An example often used to illustrate the difference between analog and digital devices is the comparison between a light dimmer and a light switch. A light dimmer is an analog device, since it can make the light it controls vary in brightness anywhere within a defined range of values. The light can be fully on, fully off, or at some brightness level in between. A light switch is a digital device, since it can turn the light on or off, but there is no value in between those two states.

The light switch/light dimmer analogy, although easy to understand, does not show any particular advantage to the digital device. If anything, it makes the digital device seem limited.

One modern application in which a digital device is clearly superior to an analog one is digital audio reproduction. Compact disc players have achieved their high level of popularity because of the accurate and noise-free way in which they reproduce recorded music. This high quality of sound is possible because the music is stored, not as a magnetic copy of the sound vibrations, as in analog tapes, but as a series of numbers that represent amplitude steps in the sound waves.

Figure 1.1 shows a sound waveform and its representation in both analog and digital forms.

The analog voltage, shown in Figure 1.1b, is a copy of the original waveform and introduces distortion both in the storage and playback processes. (Think of how a photocopy deteriorates in quality if you make a copy of a copy, then a copy of the new copy, and so on. It doesn’t take long before you can’t read the fine print.)

A digital audio system doesn’t make a copy of the waveform, but rather stores a code (a series of amplitude numbers) that tells the compact disc player how to re-create the original sound every time a disc is played. During the recording process, the sound waveform...
is “sampled” at precise intervals. The recording transforms each sample into a digital number corresponding to the amplitude of the sound at that point. The “samples” (the voltages represented by the vertical bars) of the digitized audio waveform shown in Figure 1.1c are much more widely spaced than they would be in a real digital audio system. They are shown this way to give the general idea of a digitized waveform. In real digital audio systems, each amplitude value can be indicated by a number having as many as 16,000 to 65,000 possible values. Such a large number of possible values means the voltage difference between any two consecutive digital numbers is very small. The numbers can thus correspond extremely closely to the actual amplitude of the sound waveform. If the spacing between the samples is made small enough, the reproduced waveform is almost exactly the same as the original.

**SECTION 1.1 REVIEW PROBLEM**

1.1 What is the basic difference between analog and digital audio reproduction?

### 1.2 Digital Logic Levels

#### KEY TERMS

- **Logic level**: A voltage level that represents a defined digital state in an electronic circuit.
- **Logic HIGH** (or logic 1): The higher of two voltages in a digital system with two logic levels.
- **Logic LOW** (or logic 0): The lower of two voltages in a digital system with two logic levels.
- **Positive logic**: A system in which logic LOW represents binary digit 0 and logic HIGH represents binary digit 1.
- **Negative logic**: A system in which logic LOW represents binary digit 1 and logic HIGH represents binary digit 0.

Digitally represented quantities, such as the amplitude of an audio waveform, are usually represented by binary, or base 2, numbers. When we want to describe a digital quantity electronically, we need to have a system that uses voltages or currents to symbolize binary numbers.

The binary number system has only two digits, 0 and 1. Each of these digits can be denoted by a different voltage called a **logic level**. For a system having two logic levels, the
lower voltage (usually 0 volts) is called a logic LOW or logic 0 and represents the digit 0.
The higher voltage (traditionally 5 V, but in some systems a specific value such as 1.8 V,
2.5 V or 3.3 V) is called a logic HIGH or logic 1, which symbolizes the digit 1. Except for
some allowable tolerance, as shown in Figure 1.2, the range of voltages between HIGH and
LOW logic levels is undefined.

![Figure 1.2: Logic Levels Based on +5 V and 0 V](image)

The system assigning the digit 1 to a logic HIGH and digit 0 to logic LOW is called
positive logic. Throughout the remainder of this text, logic levels will be referred to as
HIGH/LOW or 1/0 interchangeably.

(A complementary system, called negative logic, also exists that makes the assign-
ment the other way around.)

### 1.3 The Binary Number System

<table>
<thead>
<tr>
<th><strong>KEY TERMS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Binary number system</strong></td>
</tr>
<tr>
<td><strong>Positional notation</strong></td>
</tr>
<tr>
<td><strong>Bit</strong></td>
</tr>
</tbody>
</table>

**Positional Notation**

The binary number system is based on the number 2. This means that we can write any
number using only two binary digits (or bits), 0 and 1. Compare this to the decimal system,
which is based on the number 10, where we can write any number with only ten decimal
digits, 0 to 9.

The binary and decimal systems are both positional notation systems; the value of a
digit in either system depends on its placement within a number. In the decimal number
845, the digit 4 really means 40, whereas in the number 9426, the digit 4 really means 400
\((845 = 800 + 40 + 5; 9426 = 9000 + 400 + 20 + 6)\). The value of the digit is determined by
*what* the digit is as well as *where* it is.

In the decimal system, a digit in the position immediately to the left of the decimal point
is multiplied by 1 \((10^0)\). A digit two positions to the left of the decimal point is mul-

NOTE

For the voltages in Figure 1.2:

\[ +5 \text{ V} = \text{Logic HIGH} = 1 \]
\[ 0 \text{ V} = \text{Logic LOW} = 0 \]
1.3 • The Binary Number System

A digit in the next position left is multiplied by 100 (10^2). The positional multipliers, as you move left from the decimal point, are ascending powers of 10.

The same idea applies in the binary system, except that the positional multipliers are powers of 2 (2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16, 2^5 = 32, . . . ). For example, the binary number 101 has the decimal equivalent:

\[(1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)\]
\[= (1 \times 4) + (0 \times 2) + (1 \times 1)\]
\[= 4 + 0 + 1\]
\[= 5\]

**EXAMPLE 1.1** Calculate the decimal equivalents of the binary numbers 1010, 111, and 10010.

**SOLUTIONS**

1010 = \((1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)\)
\[= (1 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1)\]
\[= 8 + 0 + 2 + 0\]
\[= 10\]

111 = \((1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)\)
\[= (1 \times 4) + (1 \times 2) + (1 \times 1)\]
\[= 4 + 2 + 1\]
\[= 7\]

10010 = \((1 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)\)
\[= (1 \times 16) + (0 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1)\]
\[= 16 + 0 + 2 + 0\]
\[= 18\]

**Binary Inputs**

**KEY TERMS**

**Most significant bit** The leftmost bit in a binary number. This bit has the number’s largest positional multiplier.

**Least significant bit** The rightmost bit of a binary number. This bit has the number’s smallest positional multiplier.

A major class of digital circuits, called combinational logic, operates by accepting logic levels at one or more input terminals and producing a logic level at an output. In the analysis and design of such circuits, it is frequently necessary to find the output logic level of a circuit for all possible combinations of input logic levels.

The digital circuit in the black box in Figure 1.3 has three inputs. Each input can have two possible states, LOW or HIGH, which can be represented by positive logic as 0 or 1. The number of possible input combinations is 2^3 = 8. (In general, a circuit with n binary inputs has 2^n input combinations, ranging from 0 to 2^n – 1.) Table 1.1 shows a list of these combinations, both as logic levels and binary numbers, and their decimal equivalents.
A list of output logic levels corresponding to all possible input combinations, applied in ascending binary order, is called a truth table. This is a standard form for showing the function of a digital circuit.

The input bits on each line of Table 1.1 can be read from left to right as a series of 3-bit binary numbers. The numerical values of these eight input combinations range from 0 to 7 \((2^3 = 8\) possible input combinations, having decimal equivalents ranging from 0 to \(2^3 - 1\)) in decimal.

Bit A is called the **most significant bit** (MSB), and bit C is called the **least significant bit** (LSB). As these terms imply, a change in bit A is more significant, since it has the greatest effect on the number of which it is part.

Table 1.2 shows the effect of changing each of these bits in a 3-bit binary number and compares the changed number to the original by showing the difference in magnitude. A change in the MSB of any 3-bit number results in a difference of 4. A change in the LSB of any binary number results in a difference of 1. (Try it with a few different numbers.)

<table>
<thead>
<tr>
<th>Logic Level</th>
<th>Binary Value</th>
<th>Decimal Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A B C</td>
<td>A B C</td>
<td></td>
</tr>
<tr>
<td>L L L</td>
<td>0 0 0</td>
<td>0</td>
</tr>
<tr>
<td>L L H</td>
<td>0 0 1</td>
<td>1</td>
</tr>
<tr>
<td>L H L</td>
<td>0 1 0</td>
<td>2</td>
</tr>
<tr>
<td>L H H</td>
<td>0 1 1</td>
<td>3</td>
</tr>
<tr>
<td>H L L</td>
<td>1 0 0</td>
<td>4</td>
</tr>
<tr>
<td>H L H</td>
<td>1 0 1</td>
<td>5</td>
</tr>
<tr>
<td>H H L</td>
<td>1 1 0</td>
<td>6</td>
</tr>
<tr>
<td>H H H</td>
<td>1 1 1</td>
<td>7</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>TABLE 1.2 Effect of Changing the LSB and MSB of a Binary Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>Original</td>
</tr>
<tr>
<td>Change MSB</td>
</tr>
<tr>
<td>Difference = 4</td>
</tr>
<tr>
<td>Change LSB</td>
</tr>
<tr>
<td>Difference = 1</td>
</tr>
</tbody>
</table>

**EXAMPLE 1.2**

Figure 1.4 shows a 4-input digital circuit. List all the possible binary input combinations to this circuit and their decimal equivalents. What is the value of the MSB?

**FIGURE 1.4**
Example 1.2: 4-Input Digital Circuit
1.3 • The Binary Number System

SOLUTION Since there are four inputs, there will be \(2^4 = 16\) possible input combinations, ranging from 0000 to 1111 (0 to 15 in decimal). Table 1.3 shows the list of all possible input combinations.

The MSB has a value of 8 (decimal).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>13</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>15</td>
</tr>
</tbody>
</table>

Knowing how to construct a binary sequence is a very important skill when working with digital logic systems. Two ways to do this are:

1. **Learn to count in binary.** You should know all the binary numbers from 0000 to 1111 and their decimal equivalents (0 to 15). *Make this your first goal in learning the basics of digital systems.*

   Each binary number is a unique representation of its decimal equivalent. You can work out the decimal value of a binary number by adding the weighted values of all the bits.

   For instance, the binary equivalent of the decimal sequence 0, 1, 2, 3 can be written using two bits: the 1’s bit and the 2’s bit. The binary count sequence is:

   
   \[\begin{align*}
   00 &= 0 + 0 \\
   01 &= 0 + 1 \\
   10 &= 2 + 0 \\
   11 &= 2 + 1 \\
   \end{align*}\]

   To count beyond this, you need another bit: the 4’s bit. The decimal sequence 4, 5, 6, 7 has the binary equivalents:

   \[\begin{align*}
   100 &= 4 + 0 + 0 \\
   101 &= 4 + 0 + 1 \\
   110 &= 4 + 2 + 0 \\
   111 &= 4 + 2 + 1 \\
   \end{align*}\]

   The two least significant bits of this sequence are the same as the bits in the 0 to 3 sequence; a repeating pattern has been generated.
The sequence from 8 to 15 requires yet another bit: the 8’s bit. The three LSBs of this sequence repeat the 0 to 7 sequence. The binary equivalents of 8 to 15 are:

- \(1000 = 8\)
- \(1001 = 9\)
- \(1010 = 10\)
- \(1011 = 11\)
- \(1100 = 12\)
- \(1101 = 13\)
- \(1110 = 14\)
- \(1111 = 15\)

Practice writing out the binary sequence until it becomes familiar. In the 0 to 15 sequence, it is standard practice to write each number as a 4-bit value, as in Example 1.2, so that all numbers have the same number of bits. Numbers up to 7 have leading zeros to pad them out to 4 bits.

This convention has developed because each bit has a physical location in a digital circuit; we know a particular bit is logic 0 because we can measure 0 V at a particular point in a circuit. A bit with a value of 0 doesn’t go away just because there is not a 1 at a more significant location.

While you are still learning to count in binary, you can use a second method.

2. **Follow a simple repetitive pattern.** Look at Tables 1.1 and 1.3 again. Notice that the least significant bit follows a pattern. The bits alternate with every line, producing the pattern 0, 1, 0, 1, . . . . The 2’s bit alternates every two lines: 0, 0, 1, 1, 0, 0, 1, 1, . . . . The 4’s bit alternates every four lines: 0, 0, 0, 0, 1, 1, 1, 1, . . . . This pattern can be expanded to cover any number of bits, with the number of lines between alternations doubling with each bit to the left.

### Decimal-to-Binary Conversion

There are two methods commonly used to convert decimal numbers to binary: sum of powers of 2 and repeated division by 2.

#### Sum of Powers of 2

You can convert a decimal number to binary by adding up powers of 2 by inspection, adding bits as you need them to fill up the total value of the number. For example, convert \(57_{10}\) to binary.

\[\begin{align*}
64_{10} &> 57_{10} > 32_{10} \\
\text{• We see that } 32 (=2^5) \text{ is the largest power of two that is smaller than 57. Set the 32’s bit to 1 and subtract 32 from the original number, as shown below.} \\
57 - 32 & = 25 \\
\text{• The largest power of two that is less than 25 is 16. Set the 16’s bit to 1 and subtract 16 from the accumulated total.} \\
25 - 16 & = 9 \\
\text{• 8 is the largest power of two that is less than 9. Set the 8’s bit to 1 and subtract 8 from the total.} \\
9 - 8 & = 1 \\
\text{• 4 is greater than the remaining total. Set the 4’s bit to 0.} \\
\text{• 2 is greater than the remaining total. Set the 2’s bit to 0.}
\end{align*}\]
1.3 • The Binary Number System

• 1 is left over. Set the 1’s bit to 1 and subtract 1.

\[ 1 - 1 = 0 \]

• Conversion is complete when there is nothing left to subtract. Any remaining bits should be set to 0.

\[
\begin{array}{cccccc}
32 & 16 & 8 & 4 & 2 & 1 \\
1 & & & & & 57 - 32 = 25 \\
32 & 16 & 8 & 4 & 2 & 1 \\
1 & 1 & & & & 57 - (32 + 16) = 9 \\
32 & 16 & 8 & 4 & 2 & 1 \\
1 & 1 & 1 & & & 57 - (32 + 16 + 8) = 1 \\
32 & 16 & 8 & 4 & 2 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 57 - (32 + 16 + 8 + 1) = 0 \\
\end{array}
\]

\[ 57_{10} = 111001_2 \]

EXAMPLE 1.3

Convert \( 92_{10} \) to binary using the sum-of-powers-of-2 method.

SOLUTION

\[
\begin{array}{cccccc}
32 & 16 & 8 & 4 & 2 & 1 \\
1 & & & & & 92 - 64 = 28 \\
32 & 16 & 8 & 4 & 2 & 1 \\
1 & 0 & 1 & & & 92 - (64 + 16) = 12 \\
32 & 16 & 8 & 4 & 2 & 1 \\
1 & 0 & 1 & 1 & & 92 - (64 + 16 + 8) = 4 \\
32 & 16 & 8 & 4 & 2 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 92 - (64 + 16 + 8 + 4) = 0 \\
\end{array}
\]

\[ 92_{10} = 1011100_2 \]

Repeated Division by 2

Any decimal number divided by 2 will leave a remainder of 0 or 1. Repeated division by 2 will leave a string of 0s and 1s that become the binary equivalent of the decimal number. Let us use this method to convert \( 46_{10} \) to binary.

1. Divide the decimal number by 2 and note the remainder.

\[ 46/2 = 23 + \text{remainder} \ 0 \ (\text{LSB}) \]

The remainder is the least significant bit of the binary equivalent of 46.

2. Divide the quotient from the previous division and note the remainder. The remainder is the second LSB.

\[ 23/2 = 11 + \text{remainder} \ 1 \]
3. Continue this process until the quotient is 0. The last remainder is the most significant bit of the binary number.

\[
\begin{align*}
11/2 & = 5 + \text{remainder } 1 \\
5/2 & = 2 + \text{remainder } 1 \\
2/2 & = 1 + \text{remainder } 0 \\
1/2 & = 0 + \text{remainder } 1 \quad (\text{MSB})
\end{align*}
\]

To write the binary equivalent of the decimal number, read the remainders from the bottom up.

\[46_{10} = 101110_2\]

**EXAMPLE 1.4**

Use repeated division by 2 to convert 115\textsubscript{10} to a binary number.

**SOLUTION**

\[
\begin{align*}
115/2 & = 57 + \text{remainder } 1 \; (\text{LSB}) \\
57/2 & = 28 + \text{remainder } 1 \\
28/2 & = 14 + \text{remainder } 0 \\
14/2 & = 7 + \text{remainder } 0 \\
7/2 & = 3 + \text{remainder } 1 \\
3/2 & = 1 + \text{remainder } 1 \\
1/2 & = 0 + \text{remainder } 1 \; (\text{MSB})
\end{align*}
\]

Read the remainders from bottom to top: 1110011.

\[115_{10} = 1110011_2\]

In any decimal-to-binary conversion, the number of bits in the binary number is the exponent of the smallest power of 2 that is larger than the decimal number.

For example, for the numbers 92\textsubscript{10} and 46\textsubscript{10},

\[
\begin{align*}
2^7 & = 128 > 92 \quad 7 \text{ bits: } 1011100 \\
2^6 & = 64 > 46 \quad 6 \text{ bits: } 101110
\end{align*}
\]

**Fractional Binary Numbers**

**KEY TERMS**

*Radix point*  The generalized form of a decimal point. In any positional number system, the radix point marks the dividing line between positional multipliers that are positive and negative powers of the system’s number base.

*Binary point*  A period (“.”) that marks the dividing line between positional multipliers that are positive and negative powers of 2 (e.g., first multiplier right of binary point = \(2^{-1}\); first multiplier left of binary point = \(2^0\)).

In the decimal system, fractional numbers use the same digits as whole numbers, but the digits are written to the right of the decimal point. The multipliers for these digits are negative powers of 10—\(10^{-1}\) (1/10), \(10^{-2}\) (1/100), \(10^{-3}\) (1/1000), and so on.

So it is in the binary system. Digits 0 and 1 are used to write fractional binary numbers, but the digits are to the right of the **binary point**—the binary equivalent of the decimal point. (The decimal point and binary point are special cases of the *radix point*, the general name for any such point in any number system.)
Each digit is multiplied by a positional factor that is a negative power of 2. The first four multipliers on either side of the binary point are:

\[
\begin{array}{cccccccc}
2^3 & 2^2 & 2^1 & 2^0 & \cdot & 2^{-1} & 2^{-2} & 2^{-3} \\
= 8 & = 4 & = 2 & = 1 & & = 1/2 & = 1/4 & = 1/8
\end{array}
\]

**EXAMPLE 1.5**

Write the binary fraction 0.101101 as a decimal fraction.

**SOLUTION**

\[
\begin{align*}
1 \times 1/2 &= 1/2 \\
0 \times 1/4 &= 0 \\
1 \times 1/8 &= 1/8 \\
1 \times 1/16 &= 1/16 \\
0 \times 1/32 &= 0 \\
1 \times 1/64 &= 1/64 \\
1/2 + 1/8 + 1/16 + 1/64 &= 32/64 + 8/64 + 4/64 + 1/64 \\
&= 45/64 \\
&= 0.703125_{10}
\end{align*}
\]

**Fractional-Decimal-to-Fractional-Binary Conversion**

Simple decimal fractions such as 0.5, 0.25, and 0.375 can be converted to binary fractions by a sum-of-powers method. The above decimal numbers can also be written 0.5 = 1/2, 0.25 = 1/4, and 0.375 = 3/8 = 1/4 + 1/8. These numbers can all be represented by negative powers of 2. Thus, in binary,

\[
\begin{align*}
0.5_{10} &= 0.12 \\
0.25_{10} &= 0.0112 \\
0.375_{10} &= 0.01112
\end{align*}
\]

The conversion process becomes more complicated if we try to convert decimal fractions that cannot be broken into powers of 2. For example, the number 1/5 = 0.2_{10} cannot be exactly represented by a sum of negative powers of 2. (Try it.) For this type of number, we must use the method of repeated multiplication by 2.

**Method:**

1. Multiply the decimal fraction by 2 and note the integer part. The integer part is either 0 or 1 for any number between 0 and 0.999. . . . The integer part of the product is the first digit to the left of the binary point.

\[
0.2 \times 2 = 0.4 \quad \text{Integer part: 0}
\]

2. Discard the integer part of the previous product. Multiply the fractional part of the previous product by 2. Repeat step 1 until the fraction repeats or terminates.

\[
\begin{align*}
0.4 \times 2 &= 0.8 \quad \text{Integer part: 0} \\
0.8 \times 2 &= 1.6 \quad \text{Integer part: 1} \\
0.6 \times 2 &= 1.2 \quad \text{Integer part: 1} \\
0.2 \times 2 &= 0.4 \quad \text{Integer part: 0}
\end{align*}
\]

(Fraction repeats; product is same as in step 1)
Read the above integer parts from top to bottom to obtain the fractional binary number. Thus, \(0.2_{10} = 0.00110011\ldots_2 = 0.0011_2\). The bar shows the portion of the digits that repeats.

**EXAMPLE 1.6** Convert \(0.95_{10}\) to its binary equivalent.

**SOLUTION**

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Binary Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.95 (\times 2 = 1.90)</td>
<td>Integer part: 1</td>
</tr>
<tr>
<td>0.90 (\times 2 = 1.80)</td>
<td>Integer part: 1</td>
</tr>
<tr>
<td>0.80 (\times 2 = 1.60)</td>
<td>Integer part: 1</td>
</tr>
<tr>
<td>0.60 (\times 2 = 1.20)</td>
<td>Integer part: 1</td>
</tr>
<tr>
<td>0.20 (\times 2 = 0.40)</td>
<td>Integer part: 0</td>
</tr>
<tr>
<td>0.40 (\times 2 = 0.80)</td>
<td>Integer part: 0</td>
</tr>
<tr>
<td>0.80 (\times 2 = 1.60)</td>
<td>Fraction repeats last four digits</td>
</tr>
</tbody>
</table>

\(0.95_{10} = 0.11100\ldots_2\)

**SECTION 1.3 REVIEW PROBLEMS**

1.2. How many different binary numbers can be written with 6 bits?
1.3. How many can be written with 7 bits?
1.4. Write the sequence of 7-bit numbers from 1010000 to 1010111.
1.5. Write the decimal equivalents of the numbers written for Problem 1.4.

### 1.4 Hexadecimal Numbers

After binary numbers, hexadecimal (base 16) numbers are the most important numbers in digital applications. Hexadecimal, or hex, numbers are primarily used as a shorthand form of binary notation. Since 16 is a power of 2 \(2^4 = 16\), each hexadecimal digit can be converted directly to four binary digits. Hex numbers can pack more digital information into fewer digits.

Hex numbers have become particularly popular with the advent of small computers, which use binary data having 8, 16, or 32 bits. Such data can be represented by 2, 4, or 8 hexadecimal digits, respectively.

**Counting in Hexadecimal**

The positional multipliers in the hex system are powers of sixteen: \(16^0 = 1, 16^1 = 16, 16^2 = 256, 16^3 = 4096\), and so on.

We need 16 digits to write hex numbers; the decimal digits 0 through 9 are not sufficient. The usual convention is to use the capital letters A through F, each letter representing a number from \(10_{10}\) through \(15_{10}\). Table 1.4 shows how hexadecimal digits relate to their decimal and binary equivalents.

**TABLE 1.4 Hex Digits and Their Binary and Decimal Equivalents**

<table>
<thead>
<tr>
<th>Hex</th>
<th>Decimal</th>
<th>Binary</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0000</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0001</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0010</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0011</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0100</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0101</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0110</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>0111</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>1000</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>1001</td>
</tr>
<tr>
<td>A</td>
<td>10</td>
<td>1010</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
<td>1011</td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td>1100</td>
</tr>
<tr>
<td>D</td>
<td>13</td>
<td>1101</td>
</tr>
<tr>
<td>E</td>
<td>14</td>
<td>1110</td>
</tr>
<tr>
<td>F</td>
<td>15</td>
<td>1111</td>
</tr>
</tbody>
</table>

**NOTE**

**Counting Rules for Hexadecimal Numbers:**

1. Count in sequence from 0 to F in the least significant digit.
2. Add 1 to the next digit to the left and start over.
3. Repeat in all other columns.

For instance, the hex numbers between 19 and 22 are 19, 1A, 1B, 1C, 1D, 1E, 1F, 20, 21, 22. (The decimal equivalents of these numbers are 25 through 34.)
EXAMPLE 1.7
What is the next hexadecimal number after 999? After 99F? After 9FF? After FFF?

SOLUTION  The hexadecimal number after 999 is 99A. The number after 99F is 9A0. The number after 9FF is A00. The number after FFF is 1000.

EXAMPLE 1.8
List the hexadecimal digits from 190\textsubscript{16} to 200\textsubscript{16}, inclusive.

SOLUTION  The numbers follow the counting rules: Use all the digits in one position, add 1 to the digit one position left, and start over.

For brevity, we will list only a few of the numbers in the sequence:

190, 191, 192, \ldots, 199, 19A, 19B, 19C, 19D, 19E, 19F,
1A0, 1A1, 1A2, \ldots, 1AF, 1B0, 1B1, 1B2, \ldots, 1BF, 1C0, \ldots, 1CF, 1D0, \ldots, 1DF, 1E0, \ldots, 1EF, 1F0, \ldots, 1FF, 200

SECTION 1.4A REVIEW PROBLEMS

1.6. List the hexadecimal numbers from FA9 to FB0, inclusive.
1.7. List the hexadecimal numbers from 1F9 to 200, inclusive.

Hexadecimal-to-Decimal Conversion

To convert a number from hex to decimal, multiply each digit by its power-of-16 positional multiplier and add the products. In the following examples, hexadecimal numbers are indicated by a final “H” (e.g., 1F7H), rather than a “16” subscript.

EXAMPLE 1.9
Convert 7C6H to decimal.

SOLUTION

\begin{align*}
7 & \times 16^2 = 7_{10} \times 256_{10} = 1792_{10} \\
C & \times 16^1 = 12_{10} \times 16_{10} = 192_{10} \\
6 & \times 16^0 = 6_{10} \times 1_{10} = 6_{10} \\
\end{align*}

\[1990_{10}\]

EXAMPLE 1.10
Convert 1FD5H to decimal.

SOLUTION

\begin{align*}
1 & \times 16^3 = 1_{10} \times 4096_{10} = 4096_{10} \\
F & \times 16^2 = 15_{10} \times 256_{10} = 3840_{10} \\
D & \times 16^1 = 13_{10} \times 16_{10} = 208_{10} \\
5 & \times 16^0 = 5_{10} \times 1_{10} = 5_{10} \\
\end{align*}

\[8149_{10}\]

SECTION 1.4B REVIEW PROBLEM

1.8  Convert the hexadecimal number A30F to its decimal equivalent.

Decimal-to-Hexadecimal Conversion

Decimal numbers can be converted to hex by the sum-of-weighted-hex-digits method or by repeated division by 16. The main difficulty we encounter in either method is
remembering to convert decimal numbers 10 through 15 into the equivalent hex digits, A through F.

**Sum of Weighted Hexadecimal Digits**
This method is useful for simple conversions (about three digits). For example, the decimal number 35 is easily converted to the hex value 23.

\[
35_{10} = 32_{10} + 3_{10} = (2 \times 16) + (3 \times 1) = 23_{16}
\]

---

**EXAMPLE 1.11**
Convert 175\(_{10}\) to hexadecimal.

**SOLUTION**

Since 256\(_{10}\) = 16\(^2\), the hexadecimal number will have two digits.

\[
(11 \times 16) > 175 > (10 \times 16)
\]

\[
\begin{array}{c|c}
16 & 1 \\
A & 175 - (A \times 16) = 175 - 160 = 15 \\
16 & 175 - ((A \times 16) + (F \times 1)) \\
A & 175 - (160 + 15) = 0
\end{array}
\]

**Repeated Division by 16**
Repeated division by 16 is a systematic decimal-to-hexadecimal conversion method that is not limited by the size of the number to be converted.

It is similar to the repeated-division-by-2 method used to convert decimal numbers to binary. Divide the decimal number by 16 and note the remainder, making sure to express it as a hex digit. Repeat the process until the quotient is zero. The last remainder is the most significant digit of the hex number.

**EXAMPLE 1.12**
Convert 31581\(_{10}\) to hexadecimal.

**SOLUTION**

\[
\begin{align*}
31581/16 &= 1973 + \text{remainder 13 (D) (LSD)} \\
1973/16 &= 123 + \text{remainder 5} \\
123/16 &= 7 + \text{remainder 11 (B)} \\
7/16 &= 0 + \text{remainder 7 (MSD)}
\end{align*}
\]

\[
31581_{10} = 7B5DH
\]

---

**SECTION 1.4C REVIEW PROBLEM**
1.9 Convert the decimal number 8137 to its hexadecimal equivalent.

**Conversions Between Hexadecimal and Binary**
Table 1.4 shows all 16 hexadecimal digits and their decimal and binary equivalents. Note that for every possible 4-bit binary number, there is a hexadecimal equivalent.

Binary-to-hex and hex-to-binary conversions simply consist of making a conversion between each hex digit and its binary equivalent.
EXAMPLE 1.13
Convert 7EF8H to its binary equivalent.

**SOLUTION** Convert each digit individually to its equivalent value:

- **7H** = 01112
- **EH** = 11102
- **FH** = 11112
- **8H** = 10002

The binary number is all the above binary numbers in sequence:

7EF8H = 111110111110002

The leading zero (the MSB of 0111) has been left out.

SECTION 1.4D REVIEW PROBLEMS

1.10 Convert the hexadecimal number 934B to binary.

1.11 Convert the binary number 11001000001101001001 to hexadecimal.

1.5 Digital Waveforms

**KEY TERM**

**Digital waveform** A series of logic 1s and 0s plotted as a function of time.

The inputs and outputs of digital circuits often are not fixed logic levels but **digital waveforms**, where the input and output logic levels vary with time. There are three possible types of digital waveform. **Periodic** waveforms repeat the same pattern of logic levels over a specified period of time. **Aperiodic** waveforms do not repeat. **Pulse** waveforms follow a HIGH-LOW-HIGH or LOW-HIGH-LOW pattern and may be periodic or aperiodic.

**Periodic Waveforms**

**KEY TERMS**

**Periodic waveform** A time-varying sequence of logic HIGHs and LOWs that repeats over a specified period of time.

**Period (T)** Time required for a periodic waveform to repeat. Unit: seconds (s).

**Frequency (f)** Number of times per second that a periodic waveform repeats. $f = 1/T$ Unit: Hertz (Hz).

**Time HIGH** ($t_h$) Time during one period that a waveform is in the HIGH state. Unit: seconds (s).

**Time LOW** ($t_l$) Time during one period that a waveform is in the LOW state. Unit: seconds (s).

**Duty cycle (DC)** Fraction of the total period that a digital waveform is in the HIGH state. $DC = t_h/T$ (often expressed as a percentage: $% DC = \frac{t_h}{T} \times 100\%$).

**Periodic waveforms** repeat the same pattern of HIGHs and LOWs over a specified period of time. The waveform may or may not be symmetrical; that is, it may or may not be HIGH and LOW for equal amounts of time.
EXAMPLE 1.14
Calculate the time LOW, time HIGH, period, frequency, and percent duty cycle for each of the periodic waveforms in Figure 1.5.

FIGURE 1.5
Example 1.14: Periodic Digital Waveforms

How are the waveforms similar? How do they differ?

SOLUTION

a. Time LOW: $t_l = 3$ ms
   Time HIGH: $t_h = 1$ ms
   Period: $T = t_l + t_h = 3$ ms + 1 ms = 4 ms
   Frequency: $f = 1/T = 1/(4$ ms) = 0.25 kHz = 250 Hz
   Duty cycle: $%DC = (t_h/T) \times 100\% = (1$ ms/4 ms) $\times 100\%$  
     = 25\%
     (1 ms = 1/1000 second; 1 kHz = 1000 Hz.)

b. Time LOW: $t_l = 2$ ms
   Time HIGH: $t_h = 2$ ms
   Period: $T = t_l + t_h = 2$ ms + 2 ms = 4 ms
   Frequency: $f = 1/T = 1/(4$ ms) = 0.25 kHz = 250 Hz
   Duty cycle: $%DC = (t_h/T) \times 100\% = (2$ ms/4 ms) $\times 100\%$  
     = 50\%

   The waveforms all have the same period but different duty cycles. A square waveform, shown in Figure 1.5b, has a duty cycle of 50%.

c. Time LOW: $t_l = 1$ ms
   Time HIGH: $t_h = 3$ ms
   Period: $T = t_l + t_h = 1$ ms + 3 ms = 4 ms
   Frequency: $f = 1/T = 1/(4$ ms) = 0.25 kHz and 250 Hz
   Duty cycle: $%DC = (t_h/T) \times 100\% = (3$ ms/4 ms) $\times 100\%$  
     = 75\%

Aperiodic Waveforms

A periodic waveform is a time-varying sequence of logic HIGHs and LOWs that does not repeat. An aperiodic waveform does not repeat a pattern of 0s and 1s. Thus, the parameters of time HIGH, time LOW, frequency, period, and duty cycle have no meaning for an aperiodic waveform. Most waveforms of this type are one-of-a-kind specimens. (It is also worth noting that most digital waveforms are aperiodic.)
Figure 1.6 shows some examples of aperiodic waveforms.

![Aperiodic Digital Waveforms](image)

**EXAMPLE 1.15**

A digital circuit generates the following strings of 0s and 1s:

a. 0011111011010100011000
b. 0011001100110011001100110011

c. 0000000111111000000001111

d. 1011101110111011101110111011

The time between two bits is always the same. Sketch the resulting digital waveform for each string of bits. Which waveforms are periodic and which are aperiodic?

**SOLUTION**  
Figure 1.7 shows the waveforms corresponding to the strings of bits above. The waveforms are easier to draw if you break up the bit strings into smaller groups of, say, 4 bits each. For instance:

a. 0011 1111 0110 1011 0100 0011 0000

All of the waveforms except Figure 1.7a are periodic.

![Waveforms](image)

**Pulse Waveforms**

**KEY TERMS**

- **Pulse**  A momentary variation of voltage from one logic level to the opposite level and back again.
- **Amplitude**  The instantaneous voltage of a waveform. Often used to mean maximum amplitude, or peak voltage, of a pulse.
- **Edge**  The part of the pulse that represents the transition from one logic level to the other.
- **Rising edge**  The part of a pulse where the logic level is in transition from a LOW to a HIGH.
Falling edge  The part of a pulse where the logic level is a transition from a HIGH to a LOW.

Leading edge  The edge of a pulse that occurs earliest in time.

Trailing edge  The edge of a pulse that occurs latest in time.

Pulse width \((t_w)\)  Elapsed time from the 50% point of the leading edge of a pulse to the 50% point of the trailing edge.

Rise time \((t_r)\)  Elapsed time from the 10% point to the 90% point of the rising edge of a pulse.

Fall time \((t_f)\)  Elapsed time from the 90% point to the 10% point of the falling edge of a pulse.

Figure 1.8 shows the forms of both an ideal and a nonideal pulse. The rising and falling edges of an ideal pulse are vertical. That is, the transitions between logic HIGH and LOW levels are instantaneous. There is no such thing as an ideal pulse in a real digital circuit. Circuit capacitance and other factors make the pulse more like the nonideal pulse in Figure 1.8b.

Pulses can be either positive-going or negative-going, as shown in Figure 1.9. In a positive-going pulse, the measured logic level is normally LOW, goes HIGH for the duration

---

**FIGURE 1.8**
Ideal and Nonideal Pulses

**FIGURE 1.9**
Pulse Edges
of the pulse, and returns to the LOW state. A negative-going pulse acts in the opposite direc-
tion.

Nonideal pulses are measured in terms of several timing parameters. Figure 1.10 shows the 10%, 50%, and 90% points on the rising and falling edges of a nonideal pulse. (100% is the maximum amplitude of the pulse.)

**FIGURE 1.10**
Pulse Width, Rise Time, Fall Time

The 50% points are used to measure pulse width because the edges of the pulse are not vertical. Without an agreed reference point, the pulse width is indeterminate. The 10% and 90% points are used as references for the rise and fall times, since the edges of a nonideal pulse are nonlinear. Most of the nonlinearity is below the 10% or above the 90% point.

**EXAMPLE 1.16**
Calculate the pulse width, rise time, and fall time of the pulse shown in Figure 1.11.

**FIGURE 1.11**
Example 1.16: Pulse

**SOLUTION** From the graph in Figure 1.11, read the times corresponding to the 10%, 50%, and 90% values of the pulse on both the leading and trailing edges.

<table>
<thead>
<tr>
<th>Leading edge:</th>
<th>Trailing edge:</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%: 2 µs</td>
<td>90%: 20 µs</td>
</tr>
<tr>
<td>50%: 5 µs</td>
<td>50%: 25 µs</td>
</tr>
<tr>
<td>90%: 8 µs</td>
<td>10%: 30 µs</td>
</tr>
</tbody>
</table>
SUMMARY

1. The two basic areas of electronics are analog and digital electronics. Analog electronics deals with continuously variable quantities; digital electronics represents the world in discrete steps.

2. Digital logic uses defined voltage levels, called logic levels, to represent binary numbers within an electronic system.

3. The higher voltage in a digital system represents the binary digit 1 and is called a logic HIGH or logic 1. The lower voltage in a system represents the binary digit 0 and is called a logic LOW or logic 0.

4. The logic levels of multiple locations in a digital circuit can be combined to represent a multibit binary number.

5. Binary is a positional number system (base 2) with two digits, 0 and 1, and positional multipliers that are powers of 2.

6. The bit with the largest positional weight in a binary number is called the most significant bit (MSB); the bit with the smallest positional weight is called the least significant bit (LSB). The MSB is also the leftmost bit in the number; the LSB is the rightmost bit.

7. A decimal number can be converted to binary by sum of powers of 2 (add place values to get a total) or repeated division by 2 (divide by 2 until quotient is 0; remainders are the binary value).

8. The hexadecimal number system is based on 16. It uses 16 digits, from 0–9 and A–F, with power-of-16 multipliers.

9. Each hexadecimal digit uniquely corresponds to a 4-bit binary value. Hex digits can thus be used as shorthand for binary.

10. A digital waveform is a sequence of bits over time. A waveform can be periodic (repetitive), aperiodic (nonrepetitive), or pulsed (a single variation and return between logic levels.)

11. Periodic waveforms are measured by period ($T$: time for one cycle), time HIGH ($t_{ih}$), time LOW ($t_{il}$), frequency ($f$: number of cycles per second), and duty cycle ($DC$ or $\%DC$: fraction of cycle in HIGH state).

12. Pulse waveforms are measured by pulse width ($t_w$: time from 50% of leading edge to 50% of trailing edge), rise time ($tr$: time from 10% to 90% of rising edge) and fall time ($tf$: time from 90% to 10% of falling edge).

GLOSSARY

Amplitude The instantaneous voltage of a waveform. Often used to mean maximum amplitude, or peak voltage, of a pulse.

Analog A way of representing some physical quantity, such as temperature or velocity, by a proportional continuous voltage or current. An analog voltage or current can have any value within a defined range.

Aperiodic waveform A time-varying sequence of logic HIGHs and LOWs that does not repeat.

Binary number system A number system used extensively in digital systems, based on the number 2. It uses two digits to write any number.

Bit Binary digit. A 0 or a 1.

Pulse width: 50% of leading edge to 50% of trailing edge.

$$t_w = 25 \ \mu s - 5 \ \mu s = 20 \ \mu s$$

Rise time: 10% of rising edge to 90% of rising edge.

$$t_r = 8 \ \mu s - 2 \ \mu s = 6 \ \mu s$$

Fall time: 90% of falling edge to 10% of falling edge.

$$t_f = 30 \ \mu s - 20 \ \mu s = 10 \ \mu s$$

SECTION 1.5 REVIEW PROBLEMS

A digital circuit produces a waveform that can be described by the following periodic bit pattern: 0011001100110011.

1.12 What is the duty cycle of the waveform?

1.13 Write the bit pattern of a waveform with the same duty cycle and twice the frequency of the original.

1.14 Write the bit pattern of a waveform having the same frequency as the original and a duty cycle of 75%.
Problems

Continuous Smoothly connected. An unbroken series of consecutive values with no instantaneous changes.

Digital A way of representing a physical quantity by a series of binary numbers. A digital representation can have only specific discrete values.

Digital waveform A series of logic 1s and 0s plotted as a function of time.

Discrete Separated into distinct segments or pieces. A series of discontinuous values.

Duty cycle (DC) Fraction of the total period that a digital waveform is in the HIGH state. DC = t_H / T (often expressed as a percentage: %DC = t_H / T × 100%).

Edge The part of the pulse that represents the transition from one logic level to the other.

Fall time (t_f) Elapsed time from the 90% point to the 10% point of the falling edge of a pulse.

Falling edge The part of a pulse where the logic level is in transition from a HIGH to a LOW.

Frequency (f) Number of times per second that a periodic waveform repeats. f = 1 / T Unit: Hertz (Hz).

Hexadecimal number system Base-16 number system. Hexadecimal numbers are written with sixteen digits, 0–9 and A–F, with power-of-16 positional multipliers.

Leading edge The edge of a pulse that occurs earliest in time.

Least significant bit (LSB) The rightmost bit of a binary number. This bit has the number’s smallest positional multiplier.

Logic HIGH The higher of two voltages in a digital system with two logic levels.

Logic level A voltage level that represents a defined digital state in an electronic circuit.

Logic LOW The lower of two voltages in a digital system with two logic levels.

Most significant bit (MSB) The leftmost bit in a binary number. This bit has the number’s largest positional multiplier.

Negative logic A system in which logic LOW represents binary digit 1 and logic HIGH represents binary digit 0.

Period (T) Time required for a period waveform to repeat. Unit: seconds (s).

Periodic waveform A time-varying sequence of logic HIGHs and LOWs that repeats over a specified period of time.

Positional notation A system of writing numbers in which the value of a digit depends not only on the digit, but also on its placement within a number.

Positive logic A system in which logic LOW represents binary digit 0 and logic HIGH represents binary digit 1.

Pulse A momentary variation of voltage from one logic level to the opposite level and back again.

Pulse width (t_w) Elapsed time from the 50% point of the leading edge of a pulse to the 50% point of the trailing edge.

Radix point The generalized form of a decimal point. In any positional number system, the radix point marks the dividing line between positional multipliers that are positive and negative powers of the system’s number base.

Rise time (t_r) Elapsed time from the 10% point to the 90% point of the rising edge of a pulse.

Rising edge The part of a pulse where the logic level is in transition from a LOW to a HIGH.

Time HIGH (t_H) Time during one period that a waveform is in the HIGH state. Unit: seconds (s).

Time LOW (t_L) Time during one period that a waveform is in the LOW state. Unit: seconds (s).

Trailing edge The edge of a pulse that occurs latest in time.

PROBLEMS

Problem numbers set in color indicate more difficult problems; those with underlines indicate most difficult problems.

Section 1.1 Digital Versus Analog Electronics

1.1 Which of the following quantities is analog in nature and which digital? Explain your answers.

- a. Water temperature at the beach
- b. Weight of a bucket of sand
- c. Grains of sand in a bucket
- d. Waves hitting the beach in one hour
- e. Height of a wave
- f. People in a square mile

Section 1.2 Digital Logic Levels

1.2 A digital logic system is defined by the voltages 3.3 volts and 0 volts. For a positive logic system, state which voltage corresponds to a logic 0 and which to a logic 1.

Section 1.3 The Binary Number System

1.3 Calculate the decimal values of each of the following binary numbers:

| a. 100  | f. 11101 |
| b. 1000 | g. 111011 |
| c. 11001 | h. 1011101 |
| d. 110  | i. 100001 |
| e. 10101 | j. 10111001 |

1.4 Translate each of the following combinations of HIGH (H) and LOW (L) logic levels to binary numbers using positive logic:

| a. H H L H | d. L L L H |
| b. L H L H | e. H L L |
| c. H L H L |
**CHAPTER 1 • Basic Principles of Digital Systems**

1.5 List the sequence of binary numbers from 101 to 1000.

1.6 List the sequence of binary numbers from 10000 to 11111.

1.7 What is the decimal value of the most significant bit for the numbers in Problem 1.6?

1.8 Convert the following decimal numbers to binary. Use the sum-of-powers-of-2 method for parts a, c, e, and g. Use the repeated-division-by-2 method for parts b, d, f, and h.

   a. \(75_{10}\)
   b. \(83_{10}\)
   c. \(237_{10}\)
   d. \(198_{10}\)
   e. \(63_{10}\)
   f. \(64_{10}\)
   g. \(4087_{10}\)
   h. \(8193_{10}\)

1.9 Convert the following fractional binary numbers to their decimal equivalents.

   a. \(0.101\)
   b. \(0.011\)
   c. \(0.1101\)

1.10 Convert the following fractional binary numbers to their decimal equivalents.

   a. \(0.01\)
   b. \(0.0101\)
   c. \(0.01010101\)

1.11 The numbers in Problem 1.10 are converging to a closer and closer binary approximation of a simple fraction that can be expressed by decimal integers \(a/b\). What is the fraction?

1.12 What is the simple decimal fraction \((a/b)\) represented by the repeating binary number \(0.101010\ldots\)?

1.13 Convert the following decimal numbers to their binary equivalents. If a number has an integer part larger than 0, calculate the integer and fractional parts separately.

   a. \(0.75_{10}\)
   b. \(0.625_{10}\)
   c. \(0.1875_{10}\)
   d. \(0.65_{10}\)
   e. \(1.75_{10}\)
   f. \(3.95_{10}\)
   g. \(67.84_{10}\)

**Section 1.4 Hexadecimal Numbers**

1.14 Write all the hexadecimal numbers in sequence from \(308_{16}\) to \(321_{16}\) inclusive.

1.15 Write all the hexadecimal numbers in sequence from \(9F7_{16}\) to \(A03_{16}\) inclusive.

1.16 Convert the following hexadecimal numbers to their decimal equivalents.

   a. \(1A0H\)
   b. \(10AH\)
   c. \(FFFH\)
   d. \(1000H\)
   e. \(F3C8H\)
   f. \(D3B4H\)
   g. \(C000H\)
   h. \(30BAFH\)

1.17 Convert the following decimal numbers to their hexadecimal equivalents.

   a. \(709_{10}\)
   b. \(1889_{10}\)
   c. \(4095_{10}\)
   d. \(4096_{10}\)
   e. \(10128_{10}\)
   f. \(32000_{10}\)
   g. \(32768_{10}\)

1.18 Convert the following hexadecimal numbers to their binary equivalents.

   a. \(F3C8H\)
   b. \(D3B4H\)
   c. \(8037H\)
   d. \(FABDH\)
   e. \(30ACH\)
   f. \(3E7B6H\)
   g. \(743DCFH\)

1.19 Convert the following binary numbers to their hexadecimal equivalents.

   a. \(10111101000110_{2}\)
   b. \(101101101010_{2}\)
   c. \(110001011011_{2}\)
   d. \(11010111000100_{2}\)
   e. \(101010111000101_{2}\)
   f. \(1100110001011011_{2}\)
   g. \(10100000000000000_{2}\)

**Section 1.5 Digital Waveforms**

1.20 Calculate the time LOW, time HIGH, period, frequency, and percent duty cycle for the waveforms shown in Figure 1.12. How are the waveforms similar? How do they differ?

1.21 Which of the waveforms in Figure 1.13 are periodic and which are aperiodic? Explain your answers.

1.22 Sketch the pulse waveforms represented by the following strings of 0s and 1s. State which waveforms are periodic and which are aperiodic.

   a. \(1100111011100111011000000110110101\)
   b. \(1110001110000111010010001100011011111\)
   c. \(1111111100000000011111111111111111\)
   d. \(0110011001100011100110100101101101\)
   e. \(01110110100110101010110101101110101\)

1.23 Calculate the pulse width, rise time, and fall time of the pulse shown in Figure 1.14.

1.24 Repeat Problem 1.23 for the pulse shown in Figure 1.15.
ANSWERS TO SECTION REVIEW PROBLEMS

Section 1.1
1.1 An analog audio system makes a direct copy of the recorded sound waves. A digital system stores the sound as a series of binary numbers.

Section 1.3
1.2 64; 1.3. 128; 1.4. 1010000, 1010001, 1010010, 1010011, 1010100, 1010101, 1010110, 1010111; 1.5. 80, 81, 82, 83, 84, 85, 86, 87.

Section 1.4a
1.6 FA9, FAA, FAB, FAC, FAD, FAE, FAF, FB0, 1.7 1F9, IFA, IFB, IFC, 1FD, IFE, IFF, 200.

Section 1.4b
1.8 4174310.

Section 1.4c
1.9 IFC9.

Section 1.4d
1.10 1001001101001011. 1.11 C8349.

Section 1.5
1.12 50%; 1.13 0101010101010101; 1.14 0111011011011011.
CHAPTER 2

LOGIC FUNCTIONS AND GATES

OUTLINE

2.1 Basic Logic Functions
2.2 Logic Switches and LED Indicators
2.3 Derived Logic Functions
2.4 DeMorgan’s Theorems and Gate Equivalence
2.5 Enable and Inhibit Properties of Logic Gates
2.6 Integrated Circuit Logic Gates

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

• Describe the basic logic functions: AND, OR, and NOT
• Draw simple switch circuits to represent AND, OR and Exclusive OR functions.
• Draw simple logic switch circuits for single-pole single-throw (SPST) and normally open and normally closed pushbutton switches.
• Describe the use of light-emitting diodes (LEDs) as indicators of logic HIGH and LOW states.
• Describe those logic functions derived from the basic ones: NAND, NOR, Exclusive OR, and Exclusive NOR.
• Explain the concept of active levels and identify active LOW and HIGH terminals of logic gates.
• Choose appropriate logic functions to solve simple design problems.
• Draw the truth table of any logic gate.
• Draw any logic gate, given its truth table.
• Draw the DeMorgan equivalent form of any logic gate.
• Determine when a logic gate will pass a digital waveform and when it will block the signal.
• Describe several types of integrated circuit packaging for digital logic gates.

All digital logic functions can be synthesized by various combinations of the three basic logic functions: AND, OR, and NOT. These so-called Boolean functions are the basis for all further study of combinational logic circuitry. (Combinational logic circuits are digital circuits whose outputs are functions of their inputs, regardless of the order the inputs are applied.) Standard circuits, called logic gates, have been developed for these and for more complex digital logic functions.

Logic gates can be represented in various forms. A standard set of distinctive-shape symbols has evolved as a universally understandable means of representing the various functions in a circuit. A useful pair of mathematical theorems, called DeMorgan’s theorems, enables us to draw these gate symbols in different ways to represent different aspects of the same function. A newer way of representing standard logic gates is outlined in IEEE/ANSI Standard 91-1984, a standard copublished by the Institute of Electrical and
Electronic Engineers and the American National Standards Institute. It uses a set of symbols called rectangular-outline symbols.

Logic gates can be used as electronic switches to block or allow passage of digital waveforms. Each logic gate has a different set of properties for enabling (passing) or inhibiting (blocking) digital waveforms.

2.1 Basic Logic Functions

<table>
<thead>
<tr>
<th>KEY TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Boolean variable</strong></td>
</tr>
<tr>
<td><strong>Boolean algebra</strong></td>
</tr>
<tr>
<td><strong>Boolean expression</strong></td>
</tr>
<tr>
<td><strong>Logic gate</strong></td>
</tr>
</tbody>
</table>

At its simplest level, a digital circuit works by accepting logic 1s and 0s at one or more inputs and producing 1s or 0s at one or more outputs. A branch of mathematics known as **Boolean algebra** (named after 19th-century mathematician George Boole) describes the relation between inputs and outputs of a digital circuit. We call these input and output values **Boolean variables** and the functions **Boolean expressions**, **logic functions**, or **Boolean functions**. The distinguishing characteristic of these functions is that they are made up of variables and constants that can have only two possible values: 0 or 1.

All possible operations in Boolean algebra can be created from three basic logic functions: AND, OR, and NOT.1 Electronic circuits that perform these logic functions are called **logic gates**. When we are analyzing or designing a digital circuit, we usually don’t concern ourselves with the actual circuitry of the logic gates, but treat them as black boxes that perform specified logic functions. We can think of each variable in a logic function as a circuit input and the whole function as a circuit output.

In addition to gates for the three basic functions, there are also gates for compound functions that are derived from the basic ones. NAND gates combine the NOT and AND functions in a single circuit. Similarly, NOR gates combine the NOT and OR functions. Gates for more complex functions, such as Exclusive OR and Exclusive NOR, also exist. We will examine all these devices later in the chapter.

NOT, AND, and OR Functions

<table>
<thead>
<tr>
<th>KEY TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Truth table</strong></td>
</tr>
<tr>
<td><strong>Inverter</strong></td>
</tr>
<tr>
<td><strong>Bubble</strong></td>
</tr>
</tbody>
</table>

---

1Words in uppercase letters represent either logic functions (AND, OR, NOT) or logic levels (HIGH, LOW). The same words in lowercase letters represent their conventional nontechnical meanings.
2.1 • Basic Logic Functions

Distinctive-shape symbols  Graphic symbols for logic circuits that show the function of each type of gate by a special shape.

IEEE/ANSI Standard 91-1984  A standard format for drawing logic circuit symbols as rectangles with logic functions shown by a standard notation inside the rectangle for each device.

Rectangular-outline symbols  Rectangular logic gate symbols that conform to IEEE/ANSI Standard 91-1984.

Qualifying symbol  A symbol in IEEE/ANSI logic circuit notation, placed in the top center of a rectangular symbol, that shows the function of a logic gate. Some of the qualifying symbols include: \(^1\) = “buffer”; \(\& = \text{“AND”}; \implies 1 = \text{“OR”}

Buffer  An amplifier that acts as a logic circuit. Its output can be inverting or non-inverting.

### NOT Function

The NOT function, the simplest logic function, has one input and one output. The input can be either HIGH or LOW (1 or 0), and the output is always the opposite logic level. We can show these values in a truth table, a list of all possible input values and the output resulting from each one. Table 2.1 shows a truth table for a NOT function, where \(A\) is the input variable and \(Y\) is the output.

The NOT function is represented algebraically by the Boolean expression:

\[ Y = \overline{A} \]

This is pronounced “\(Y\) equals NOT \(A\)” or “\(Y\) equals \(A\) bar.” We can also say “\(Y\) is the complement of \(A\).”

The circuit that produces the NOT function is called the NOT gate or, more usually, the inverter. Several possible symbols for the inverter, all performing the same logic function, are shown in Figure 2.1.

The symbols shown in Figure 2.1a are the standard distinctive-shape symbols for the inverter. The triangle represents an amplifier circuit, and the bubble (the small circle on the input or output) represents inversion. There are two symbols because sometimes it is convenient to show the inversion at the input and sometimes it is convenient to show it at the output.

Figure 2.1b shows the rectangular-outline inverter symbol specified by IEEE/ANSI Standard 91-1984. This standard is most useful for specifying the symbols for more complex digital devices. We will show the basic gates in both distinctive-shape and rectangular-outline symbols, although most examples will use the distinctive-shape symbols.

The “1” in the top center of the IEEE symbol is a qualifying symbol, indicating the logic gate function. In this case, it shows that the circuit is a buffer, an amplifying circuit used as a digital logic element. The arrows at the input and output of the two IEEE symbols show inversion, like the bubbles in the distinctive-shape symbols.

### AND Function

#### KEY TERMS

AND gate  A logic circuit whose output is HIGH when all inputs (e.g., \(A\) AND \(B\) AND \(C\)) are HIGH.

Logical product  AND function.

The AND function combines two or more input variables so that the output is HIGH only if all the inputs are HIGH. The truth table for a 2-input AND function is shown in Table 2.2.

<table>
<thead>
<tr>
<th>(A)</th>
<th>(B)</th>
<th>(Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>0</td>
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</table>
**OR Function**

A logic circuit whose output is HIGH when at least one input (e.g., $A$ OR $B$ OR $C$) is HIGH.

**Logical sum** OR function.

The OR function combines two or more input variables in such a way as to make the output variable HIGH if at least one input is HIGH. Table 2.4 gives the truth table for the 2-input OR function.

### Table 2.4 2-input OR Function Truth Table

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Y$</th>
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</thead>
<tbody>
<tr>
<td>0</td>
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<td>0</td>
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</tbody>
</table>

### Table 2.3 3-input AND Function Truth Table

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</table>
The algebraic expression for the OR function is:

\[ Y = A + B \]

which is pronounced “\( Y \) equals \( A \) OR \( B \).” This is similar to the arithmetic addition function, but it is not the same. The last line of the truth table tells us that \( 1 + 1 = 1 \) (pronounced “\( 1 \) OR \( 1 \) equals \( 1 \)”), which is not what we would expect in standard arithmetic. The similarity to the addition function leads to the name **logical sum.** (This is different from the “arithmetic sum,” where, of course, \( 1 + 1 \) does not equal \( 1 \).)

Figure 2.5 shows the logic circuit symbols for an **OR gate.** The qualifying symbol for the OR function in IEEE/ANSI notation is “\( \geq 1 \),” which tells us that one or more inputs must be HIGH to make the output HIGH.

The OR function can be represented by a set of switches connected in parallel, as in Figure 2.6. The lamp is on when either switch \( A \) or switch \( B \) is closed. (Note that the lamp is also on if both \( A \) and \( B \) are closed. This property distinguishes the OR function from the Exclusive OR function, which we will study later in this chapter.)

Like AND gates, OR gates can have several inputs, such as the 3-input OR gates shown in Figure 2.7. Table 2.5 shows the truth table for this gate. Again, three inputs can be combined in eight different ways. The output is HIGH when at least one input is HIGH.

### Table 2.5 3-input OR Function Truth Table

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( Y )</th>
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### EXAMPLE 2.1

State which logic function is most suitable for the following operations. Draw a set of switches to represent each function.

1. A manager and one other employee both need a key to open a safe.
2. A light comes on in a storeroom when either (or both) of two doors is open. (Assume the switch closes when the door opens.)
3. For safety, a punch press requires two-handed operation.

### SOLUTION

1. Both keys are required, so this is an AND function. Figure 2.8a shows a switch representation of the function.
2. One or more switches closed will turn on the lamp. This OR function is shown in Figure 2.8b.

3. Two switches are required to activate a punch press, as shown in Figure 2.8c. This is an AND function.

**Active Levels**

**Key Terms**

**Active level** A logic level defined as the “ON” state for a particular circuit input or output. The active level can be either HIGH or LOW.

**Active HIGH** An active-HIGH terminal is considered “ON” when it is in the logic HIGH state. Indicated by the absence of a bubble at the terminal in distinctive-shape symbols.

**Active LOW** An active-LOW terminal is considered “ON” when it is in the logic LOW state. Indicated by a bubble at the terminal in distinctive-shape symbols.

An active level of a gate input or output is the logic level, either HIGH or LOW, of the terminal when it is performing its designated function. An active LOW is shown by a bubble or an arrow symbol on the affected terminal. If there is no bubble or arrow, we assume the terminal is active HIGH.
The AND function has active-HIGH inputs and an active-HIGH output. To make the output HIGH, inputs A AND B must both be HIGH. The gate performs its designated function only when all inputs are HIGH.

The OR gate requires input A OR input B to be HIGH for its output to be HIGH. The HIGH active levels are shown by the absence of bubbles or arrows on the terminals.

SECTION REVIEW PROBLEM FOR SECTION 2.1

A 4-input gate has input variables A, B, C, and D and output Y. Write a descriptive sentence for the active output state(s) if the gate is
- 2.1 AND;
- 2.2 OR.

2.2 Logic Switches and LED Indicators

Before continuing on, we should examine a few simple circuits that can be used for input or output in a digital circuit. Single-pole single-throw (SPST) and pushbutton switches can be used, in combination with resistors, to generate logic voltages for circuit inputs. Light emitting diodes (LEDs) can be used to monitor outputs of circuits.

Logic Switches

**KEY TERMS**

- $V_{CC}$: The power supply voltage in a transistor-based electronic circuit. The term often refers to the power supply of digital circuits.
- **Pull-up resistor**: A resistor connected from a point in an electronic circuit to the power supply of that circuit.

Figure 2.9a shows a single-pole single-throw (SPST) switch connected as a logic switch. An important premise of this circuit is that the input of the digital circuit to which it is connected has a very high resistance to current. When the switch is open, the current flowing through the pull-up resistor from $V_{CC}$ to the digital circuit is very small. Since the current is small, Ohm’s law states that very little voltage drops across the pull-up resistor; the voltage is about the same at one end as at the other. Therefore, an open switch generates a logic HIGH at point X.

![Figure 2.9](image)

**FIGURE 2.9**

SPST Logic Switch

When the switch is closed, the majority of current flows to ground, limited only by the value of the pull-up resistor. (Since a pull-up resistor is typically between 1 kΩ and 10 kΩ, the LOW-state current in the resistor is about 0.5 mA to 5 mA.) Point X is approximately at ground potential, or logic LOW. Thus the switch generates a HIGH when open and a LOW when closed. The pull-up resistor provides a connection to $V_{CC}$ in the HIGH state.
and limits power supply current in the LOW state. Figure 2.9b shows the voltage levels when the switch is closed and when it is open.

Figure 2.10 shows how pushbuttons can be used as logic inputs. Figure 2.10a shows a normally open pushbutton and a pull-up resistor. The pushbutton has a spring-loaded plunger that makes a connection between two internal contacts when pressed. When released, the spring returns the plunger to the “normal” (open) state. The logic voltage at X is normally HIGH, but LOW when the button is pressed.

![a. Normally open pushbutton](image-a)

![b. Normally closed pushbutton](image-b)

![c. Two-pole pushbutton](image-c)

**FIGURE 2.10**
Pushbuttons as Logic Switches

Figure 2.10b shows a normally closed pushbutton. The internal spring holds the plunger so that the connection is normally made between the two contacts. When the button is pressed, the connection is broken and the resistor pulls up the voltage at X to a logic HIGH. At rest, X is grounded and the voltage at X is LOW.

It is sometimes desirable to have normally HIGH and normally LOW levels available from the same switch. The two-pole pushbutton in Figure 2.10c provides such a function. The switch has a normally open and a normally closed contact. One contact of each switch is connected to the other, in an internal COMMON connection, allowing the switch to have three terminals rather than four. The circuit has two pull-up resistors, one for X and one for Y. X is normally HIGH and goes LOW when the switch is pressed. Y is opposite.

**LED Indicators**

**KEY TERMS**

**LED**  Light-emitting diode. An electronic device that conducts current in one direction only and illuminates when it is conducting.
A device used to indicate the status of a digital output is the **light-emitting diode** or **LED**. This is sometimes pronounced as a word (“led”) and sometimes said as separate initials (“ell ee dee”). This device comes in a variety of shapes, sizes, and colors, some of which are shown in the photo of Figure 2.11. The circuit symbol, shown in Figure 2.12, has two terminals, called the anode (positive) and cathode (negative). The arrow coming from the symbol indicates emitted light.

**FIGURE 2.11**
LEDs

**FIGURE 2.12**
Light-Emitting Diode (LED)

The electrical requirements for the LED are simple: current flows through the LED if the anode is more positive than the cathode by more than a specified value (about 1.5 volts). If enough current flows, the LED illuminates. If more current flows, the illumination is brighter. (If too much flows, the LED burns out, so a series resistor is used to keep the current in the required range.) Figure 2.13 shows a circuit in which an LED illuminates when a switch is closed.

Figure 2.14 shows an AND gate driving an LED. In Figure 2.14a, the LED is on when \( Y \) is HIGH (5 volts), since the anode of the LED is more positive than the cathode.
In Figure 2.14b, the LED turns on when \( Y \) is LOW (0 volts), again since the anode is more positive than the cathode.

Figure 2.15 shows a circuit in which an LED indicates the status of a logic switch. When the switch is open, the 1 k\( \Omega \) pull-up applies a HIGH to the inverter input. The inverter output is LOW, turning on the LED (anode is more positive than cathode). When the switch is closed, the inverter input is LOW. The inverter output is HIGH (same value as \( V_{CC} \)), making anode and cathode voltages equal. No current flows through the LED, and it is therefore off. Thus, the LED is on for a HIGH state at the switch and off for a LOW. Note, however, that the LED is on when the inverter output is LOW.

SECTION 2.2 REVIEW PROBLEM

2.3 A single-pole single-throw switch is connected such that one end is grounded and one end is connected to a 1 k\( \Omega \) pull-up resistor. The other end of the resistor connects to the circuit power supply, \( V_{CC} \). What logic level does the switch provide when it is open? When it is closed?

2.3 Derived Logic Functions

**KEY TERMS**

- **NAND gate**: A logic circuit whose output is LOW when all inputs are HIGH.
- **NOR gate**: A logic circuit whose output is LOW when at least one input is HIGH.
- **Exclusive OR gate**: A 2-input logic circuit whose output is HIGH when one input (but not both) is HIGH.
- **Exclusive NOR gate**: A 2-input logic circuit whose output is the complement of an Exclusive OR gate.
- **Coincidence gate**: An Exclusive NOR gate.

The basic logic functions, AND, OR, and NOT, can be combined to make any other logic function. Special logic gates exist for several of the most common of these derived functions. In fact, for reasons we will discover later, two of these derived-function gates, NAND and NOR, are the most common of all gates, and each can be used to create any logic function.

**NAND and NOR Functions**

The names NAND and NOR are contractions of NOT AND and NOT OR, respectively. The NAND is generated by inverting the output of an AND function. The symbols for the **NAND gate** and its equivalent circuit are shown in Figure 2.16.

The algebraic expression for the NAND function is:

\[ Y = A \cdot B \]
2.3 • Derived Logic Functions

The entire function is inverted because the bubble is on the NAND gate output. Table 2.6 shows the NAND gate truth table. The output is LOW when \( A \) AND \( B \) are HIGH.

We can generate the NOR function by inverting the output of an OR gate. The NOR function truth table is shown in Table 2.7. The truth table tells us that the output is LOW when \( A \) OR \( B \) is HIGH.

Figure 2.17 shows the logic symbols for the NOR gate.

The algebraic expression for the NOR function is:

\[
Y = \overline{A + B}
\]

The entire function is inverted because the bubble is on the gate output.

We know that the outputs of both gates are active LOW because of the bubbles on the output terminals. The inputs are active HIGH because there are no bubbles on the input terminals.

Multiple-Input NAND and NOR Gates

Table 2.8 shows the truth tables of the 3-input NAND and NOR functions. The logic circuit symbols for these gates are shown in Figure 2.18.

The truth tables of these gates can be generated by understanding the active levels of the gate inputs and outputs. The NAND output is LOW when \( A \) AND \( B \) AND \( C \) are HIGH. This is shown in the last line of the NAND truth table. The NOR output is LOW if one or more of \( A \) OR \( B \) OR \( C \) is HIGH. This describes all lines of the NOR truth table except the first.
Table 2.9 shows the truth table for the XOR function. Another way of looking at the Exclusive OR gate is that its output is HIGH when one and only one of the inputs is HIGH. (Multiple-input XOR circuits do not expand as simply as other functions. As we will see in a later chapter, an XOR output is HIGH when an odd number of inputs is HIGH.)

Unlike the OR gate, which is sometimes called an Inclusive OR, a HIGH at both inputs makes the output LOW. (We could say that the case in which both inputs are HIGH is excluded.)

The gate symbol for the **Exclusive OR** gate is shown in Figure 2.19.

Table 2.9 shows the truth table for the XOR function.

Table 2.9: Exclusive OR Function Truth Table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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</tr>
<tr>
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</tbody>
</table>

Another way of looking at the Exclusive OR gate is that its output is HIGH when the inputs are different and LOW when they are the same. This is a useful property in some applications, such as error detection in digital communication systems. (Transmitted data can be compared with received data. If they are the same, no error has been detected.)

The XOR function is expressed algebraically as:

\[ Y = A \oplus B \]

The Exclusive NOR function is the complement of the Exclusive OR function and shares some of the same properties. The symbol, shown in Figure 2.20, is an XOR gate.
with a bubble on the output, implying that the entire function is inverted. Table 2.10 shows the Exclusive NOR truth table.

The algebraic expression for the Exclusive NOR function is:

$$Y = \bar{A} \oplus B$$

The output of the **Exclusive NOR gate** is HIGH when the inputs are the same and LOW when they are different. For this reason, the XNOR gate is also called a **coincidence gate**. This same/different property is similar to that of the Exclusive OR gate, only opposite in sense. Many of the applications that make use of this property can use either the XOR or the XNOR gate.

### SECTION 2.3 REVIEW PROBLEMS

The output of a logic gate turns on an LED when it is HIGH. The gate has two inputs, each of which is connected to a logic switch, as shown in Figure 2.21.

2.4 What type of gate will turn on the light when the switches are in opposite positions?

2.5 Which gate will turn off the light only when both switches are HIGH?

2.6 What type of gate turns on the light only when both switches are LOW?

2.7 Which gate turns on the light when the switches are in the same position?

## 2.4 DeMorgan’s Theorems and Gate Equivalence

### KEY TERMS

- **DeMorgan’s theorems**: Two theorems in Boolean algebra that allow us to transform any gate from an AND-shaped to an OR-shaped gate and vice versa.
- **DeMorgan equivalent forms**: Two gate symbols, one AND-shaped and one OR-shaped, that are equivalent according to DeMorgan’s theorems.

Recall the truth table (repeated in Table 2.11) and description of a 2-input NAND gate. “Output $Y$ is LOW if inputs $A$ AND $B$ are HIGH.” Or, “Output $Y$ is LOW if all inputs are HIGH.” The condition of this sentence is satisfied in the last line of Table 2.11.

We could also describe the gate function by saying, “Output $Y$ is HIGH if $A$ OR $B$ (OR both) are LOW,” or, “The output is HIGH if at least one input is LOW.” These conditions are satisfied by the first three lines of Table 2.11.

The gates in Figure 2.22 represent positive- and negative-logic forms of a NAND gate. Figure 2.23 shows the logic equivalents of these gates. In the first case, we combine the in-
puts in an AND function, then invert the result. In the second case, we invert the variables, then combine the inverted inputs in an OR function. The Boolean function for the AND-shaped gate is given by:

\[ Y = A \cdot B \]

The Boolean expression for the OR-shaped gate is:

\[ Y = A + B \]

The gates shown in Figure 2.22 are called DeMorgan equivalent forms. Both gates have the same truth table, but represent different aspects or ways of looking at the NAND function. We can extend this observation to state that any gate (except XOR and XNOR) has two equivalent forms, one AND, one OR.

A gate can be categorized by examining three attributes: shape, input, and output. A question arises from each attribute:

1. What is its shape (AND/OR)?
   - AND: all
   - OR: at least one
2. What active level is at the gate inputs (HIGH/LOW)?
3. What active level is at the gate output (HIGH/LOW)?

The answers to these questions characterize any gate and allow us to write a descriptive sentence and a truth table for that gate. The DeMorgan equivalent forms of the gate will yield opposite answers to each of the above questions.

Thus the gates in Figure 2.22 have the following complementary attributes:

<table>
<thead>
<tr>
<th>Boolean Expression</th>
<th>Basic Gate</th>
<th>DeMorgan Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \overline{A} \cdot \overline{B} ]</td>
<td>( \overline{A} + \overline{B} )</td>
<td></td>
</tr>
<tr>
<td>Shape</td>
<td>AND</td>
<td>OR</td>
</tr>
<tr>
<td>Input Active Level</td>
<td>HIGH</td>
<td>LOW</td>
</tr>
<tr>
<td>Output Active Level</td>
<td>LOW</td>
<td>HIGH</td>
</tr>
</tbody>
</table>

### EXAMPLE 2.2

Analyze the shape, input, and output of the gates shown in Figure 2.24 and write a Boolean expression, a descriptive sentence, and a truth table of each one. Write an asterisk beside the active output level on each truth table. Describe how these gates relate to each other.

![Figure 2.24](image)
2.4 • DeMorgan’s Theorems and Gate Equivalence

SOLUTION

a. Boolean expression: \( Y = \overline{A + B} \)
   Shape: OR (at least one)
   Input: HIGH
   Output: LOW
   Descriptive sentence: Output \( Y \) is LOW if \( A \) OR \( B \) is HIGH.
   Truth table:
   
<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0*</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0*</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0*</td>
</tr>
</tbody>
</table>

b. Boolean expression: \( Y = \overline{A \cdot B} \)
   Shape: AND (all)
   Input: LOW
   Output: HIGH
   Descriptive sentence: Output \( Y \) is HIGH if \( A \) AND \( B \) are LOW.
   Truth table:
   
<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1*</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Both gates in this example yield the same truth table. Therefore they are DeMorgan equivalents of one another (positive- and negative-NOR gates).

The gates in Figures 2.22 and 2.24 yield the following algebraic equivalencies:

\[
\begin{align*}
\overline{A \cdot B} &= \overline{A} + \overline{B} \\
\overline{A + B} &= \overline{A} \cdot \overline{B}
\end{align*}
\]

These equivalencies are known as DeMorgan’s theorems. (You can remember how to use DeMorgan’s theorems by a simple rhyme: “Break the line and change the sign.”)

It is tempting to compare the first gate in Figure 2.22 and the second in Figure 2.24 and declare them equivalent. Both gates are AND-shaped, both have inversions. However, the comparison is false. The gates have different truth tables, as we have found in Tables 2.11 and 2.13. Therefore they have different logic functions and are not equivalent. The same is true of the OR-shaped gates in Figures 2.22 and 2.24. The gates may look similar, but since they have different truth tables, they have different logic functions and are therefore not equivalent.

The confusion arises when, after changing the logic input and output levels, you forget to change the shape of the gate. This is a common, but serious, error. These inequalities can be expressed as follows:

\[
\begin{align*}
\overline{A \cdot B} &\neq \overline{A} \cdot \overline{B} \\
\overline{A + B} &\neq \overline{A} + \overline{B}
\end{align*}
\]
As previously stated, any AND- or OR-shaped gate can be represented in its DeMorgan equivalent form. All we need to do is analyze a gate for its shape, input, and output, then change everything.

### EXAMPLE 2.3

Analyze the gate in Figure 2.25 and write a Boolean expression, descriptive sentence, and truth table for the gate. Mark active output levels on the truth table with asterisks. Find the DeMorgan equivalent form of the gate and write its Boolean expression and description.

#### FIGURE 2.25
Example 2.3: Logic Gates

![Logic Gates](image)

#### SOLUTION

**Boolean expression:** \( Y = A + B + C \)

**Shape:** OR (at least one)

**Input:** LOW

**Output:** LOW

**Descriptive sentence:** Output Y is LOW if A OR B OR C is LOW.

**Truth table:**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0*</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0*</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0*</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0*</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0*</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0*</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0*</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 2.26 shows the DeMorgan equivalent form of the gate in Figure 2.25. To create this symbol, we change the shape from OR to AND and invert the logic levels at both input and output.

#### FIGURE 2.26
Example 2.3: DeMorgan Equivalent of Gate in Figure 2.25

![DeMorgan](image)

**Boolean expression:** \( Y = ABC \)

**Descriptive sentence:** Output Y is HIGH if A AND B AND C are HIGH.

---

### SECTION 2.4 REVIEW PROBLEM

2.8 The output of a gate is described by the following Boolean expression:

\[
Y = \overline{A} + \overline{B} + \overline{C} + \overline{D}
\]

Write the Boolean expression for the DeMorgan equivalent form of this gate.
Enable and Inhibit Properties of Logic Gates

**KEY TERMS**

- **Digital signal (or pulse waveform)**: A series of 0s and 1s plotted over time.
- **True form**: Not inverted.
- **Complement form**: Inverted.
- **Enable**: A logic gate is enabled if it allows a digital signal to pass from an input to the output in either true or complement form.
- **Inhibit (or disable)**: A logic gate is inhibited if it prevents a digital signal from passing from an input to the output.
- **In phase**: Two digital waveforms are in phase if they are always at the same logic level at the same time.
- **Out of phase**: Two digital waveforms are out of phase if they are always at opposite logic levels at any given time.

In Chapter 1, we saw that a digital signal is just a string of bits (0s and 1s) generated over time. A major task of digital circuitry is the direction and control of such signals. Logic gates can be used to **enable** (pass) or **inhibit** (block) these signals. (The word “gate” gives a clue to this function; the gate can “open” to allow a signal through or “close” to block its passage.)

**AND and OR Gates**

The simplest case of the enable and inhibit properties is that of an AND gate used to pass or block a logic signal. Figure 2.27 shows the output of an AND gate under different conditions of input A when a digital signal (an alternating string of 0s and 1s) is applied to input B.

Recall the properties of an AND gate: both inputs must be HIGH to make the output HIGH. Thus, if input A is LOW, the output must always be LOW, regardless of the state of input B. The digital signal applied to B has no effect on the output, and we say that the gate is inhibited or disabled. This is shown in the first half of the timing diagram in Figure 2.27.

If A AND B are HIGH, the output is HIGH. When A is HIGH and B is LOW, the output is LOW. Thus, output Y is the same as input B if input A is HIGH; that is, Y and B are in phase with each other. The input waveform is passed to the output in **true form**, and we say the gate is enabled. The last half of the timing diagram in Figure 2.27 shows this waveform.

It is convenient to define terms for the A and B inputs. Since we apply a digital signal to B, we will call it the Signal input. Since input A controls whether or not the signal
Each type of logic gate has a particular set of enable/inhibit properties that can be predicted by examining the truth table of the gate. Let us examine the truth table of the AND gate to see how the method works.

Divide the truth table in half, as shown in Table 2.15. Since we have designated $A$ as the Control input, the top half of the truth table shows the inhibit function ($A = 0$), and the bottom half shows the enable function ($A = 1$). To determine the gate properties, we compare input $B$ (the Signal input) to the output in each half of the table.

Inhibit mode: If $A = 0$ and $B$ is pulsing ($B$ is continuously going back and forth between the first and second lines of the truth table), output $Y$ is always 0. Since the Signal input has no effect on the output, we say that the gate is disabled or inhibited.

Enable mode: If $A = 1$ and $B$ is pulsing ($B$ is going continuously between the third and fourth lines of the truth table), the output is the same as the Signal input. Since the Signal input affects the output, we say that the gate is enabled.

Each type of logic gate has a particular set of enable/inhibit properties that can be predicted by examining the truth table of the gate. Let us examine the truth table of the AND gate to see how the method works.

Divide the truth table in half, as shown in Table 2.15. Since we have designated $A$ as the Control input, the top half of the truth table shows the inhibit function ($A = 0$), and the bottom half shows the enable function ($A = 1$). To determine the gate properties, we compare input $B$ (the Signal input) to the output in each half of the table.

Inhibit mode: If $A = 0$ and $B$ is pulsing ($B$ is continuously going back and forth between the first and second lines of the truth table), output $Y$ is always 0. Since the Signal input has no effect on the output, we say that the gate is disabled or inhibited.

Enable mode: If $A = 1$ and $B$ is pulsing ($B$ is going continuously between the third and fourth lines of the truth table), the output is the same as the Signal input. Since the Signal input affects the output, we say that the gate is enabled.

### Table 2.15 AND Truth Table Showing Enable/Inhibit Properties

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Y$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>($Y = 0$)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Inhibit</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>($Y = B$)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Enable</td>
</tr>
</tbody>
</table>

### Table 2.16 OR Truth Table Showing Enable/Inhibit Properties

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Y$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>($Y = B$)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Enable</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>($Y = 1$)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>Inhibit</td>
</tr>
</tbody>
</table>

Each type of logic gate has a particular set of enable/inhibit properties that can be predicted by examining the truth table of the gate. Let us examine the truth table of the AND gate to see how the method works.

Divide the truth table in half, as shown in Table 2.15. Since we have designated $A$ as the Control input, the top half of the truth table shows the inhibit function ($A = 0$), and the bottom half shows the enable function ($A = 1$). To determine the gate properties, we compare input $B$ (the Signal input) to the output in each half of the table.

Inhibit mode: If $A = 0$ and $B$ is pulsing ($B$ is continuously going back and forth between the first and second lines of the truth table), output $Y$ is always 0. Since the Signal input has no effect on the output, we say that the gate is disabled or inhibited.

Enable mode: If $A = 1$ and $B$ is pulsing ($B$ is going continuously between the third and fourth lines of the truth table), the output is the same as the Signal input. Since the Signal input affects the output, we say that the gate is enabled.

### Example 2.4

Use the method just described to draw the output waveform of an OR gate if the input waveforms of $A$ and $B$ are the same as in Figure 2.27. Indicate the enable and inhibit portions of the timing diagram.

### Solution

Divide the OR gate truth table in half. Designate input $A$ the Control input and input $B$ the Signal input.

As shown in Table 2.16, when $A = 0$ and $B$ is pulsing, the output is the same as $B$ and the gate is enabled. When $A = 1$, the output is always HIGH. (At least one input HIGH makes the output HIGH.) Since $B$ has no effect on the output, the gate is inhibited. This is shown in Figure 2.29 in graphical form.
Example 2.4 shows that a gate can be in the inhibit state even if its output is HIGH. It is natural to think of the HIGH state as "ON," but this is not always the case. Enable or inhibit states are determined by the effect the Signal input has on the gate's output. If an input signal does not affect the gate output, the gate is inhibited. If the Signal input does affect the output, the gate is enabled.

**NAND and NOR Gates**

When inverting gates, such as NAND and NOR, are enabled, they will invert an input signal before passing it to the gate output. In other words, they transmit the signal in \textit{complement form}. Figures 2.30 and 2.31 show the output waveforms of a NAND and a NOR gate when a square waveform is applied to input \( B \) and input \( A \) acts as a Control input.

**FIGURE 2.30**
Enable/Inhibit Properties of a NAND Gate

**FIGURE 2.31**
Enable/Inhibit Properties of a NOR Gate
The truth table for the XOR gate, showing the gate’s dynamic properties, is given in Table 2.19.

Table 2.19  XOR Truth Table
Showing Dynamic Properties

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Notice that when the Control is HIGH (1), the output is in phase with the input signal. Table 2.19 shows the enable/inhibit properties of the gate.

Neither the XOR nor the XNOR gate has an inhibit state. The Control input on both of these gates acts only to determine whether the output waveform will be in or out of phase with the input signal. Figure 2.32 shows the dynamic properties of an XOR gate.

Figure 2.32  Dynamic Properties of an Exclusive OR Gate

The truth table for the XOR gate, showing the gate’s dynamic properties, is given in Table 2.19.

Notice that when \( A = 0 \), the output is in phase with \( B \) and when \( A = 1 \), the output is out of phase with \( B \). A useful application of this property is to use an XOR gate as a programmable inverter. When \( A = 1 \), the gate is an inverter; when \( A = 0 \), it is a noninverting buffer.

The XNOR gate has properties similar to the XOR gate. That is, an XNOR has no inhibit state, and the Control input switches the output in and out of phase with the Signal waveform, although not the same way as an XOR gate does. You will derive these properties in one of the end-of-chapter problems.

Table 2.20 summarizes the enable/inhibit properties of the six gates examined above.

Table 2.20  Summary of Enable/Inhibit Properties

<table>
<thead>
<tr>
<th>Control</th>
<th>AND</th>
<th>OR</th>
<th>NAND</th>
<th>NOR</th>
<th>XOR</th>
<th>XNOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = 0 )</td>
<td>( Y = 0 )</td>
<td>( Y = B )</td>
<td>( Y = 1 )</td>
<td>( Y = \overline{B} )</td>
<td>( Y = B )</td>
<td>( Y = \overline{B} )</td>
</tr>
<tr>
<td>( A = 1 )</td>
<td>( Y = B )</td>
<td>( Y = 1 )</td>
<td>( Y = \overline{B} )</td>
<td>( Y = 0 )</td>
<td>( Y = B )</td>
<td>( Y = \overline{B} )</td>
</tr>
</tbody>
</table>

SECTION 2.5 REVIEW PROBLEM

2.9 Briefly explain why an AND gate is inhibited by a LOW Control input and an OR gate is inhibited by a HIGH Control input.
Tristate Buffers

**KEY TERMS**

- **Tristate buffer** A gate having three possible output states: logic HIGH, logic LOW, and high-impedance.
- **High-impedance state** The output state of a tristate buffer that is neither logic HIGH nor logic LOW, but is electrically equivalent to an open circuit.
- **Bus** A common wire or parallel group of wires connecting multiple circuits.

In the previous section, logic gates were used to enable or inhibit signals in digital circuits. In the AND, NAND, NOR, and OR gates, however, the inhibit state was always logic HIGH or LOW. In some cases, it is desirable to have an output state that is neither HIGH nor LOW, but acts to electrically disconnect the gate output from the circuit. This third state is called the **high-impedance state** and is one of three available states in a class of devices known as **tristate buffers**.

Figure 2.33 shows the logic symbols for two tristate buffers, one with a noninverting output and one with an inverting output. The third input, \( \bar{O}E \) (Output enable), is an active-LOW signal that enables or disables the buffer output.

When \( \bar{O}E = 0 \), as shown in Figure 2.34a, the noninverting buffer transfers the input value directly to the output as a logic HIGH or LOW. When \( \bar{O}E = 1 \), as in Figure 2.34b, the output is electrically disconnected from any circuit to which it is connected. (The open switch in Figure 2.34b does not literally exist. It is shown as a symbolic representation of the electrical disconnection of the output in the high-impedance state.)

This type of enable/disable function is particularly useful when digital data are transferred from more than one source to one or more destinations along a common wire (or **bus**), as shown in Figure 2.35. (This is the underlying principle in modern computer systems, where multiple components use the same bus to pass data back and forth.) The destination circuit in Figure 2.35 can receive data from source 1 or source 2. If the source circuits were directly connected to the bus, they could produce contradictory logic levels at the destination. To prevent this, only one source is enabled at a time, with control of this switching left to the two tristate buffers.
2.6 Integrated Circuit Logic Gates

**KEY TERMS**

**Integrated circuit (IC)** An electronic circuit having many components, such as transistors, diodes, resistors, and capacitors, in a single package.

**Small scale integration (SSI)** An integrated circuit having 12 or fewer gates in one package.

**Medium scale integration (MSI)** An integrated circuit having the equivalent of 12 to 100 gates in one package.

**Large scale integration (LSI)** An integrated circuit having from 100 to 10,000 equivalent gates.

**Very large scale integration (VLSI)** An integrated circuit having more than 10,000 equivalent gates.

**Transistor-transistor logic (TTL)** A family of digital logic devices whose basic element is the bipolar junction transistor.

**Complementary metal-oxide-semiconductor (CMOS)** A family of digital logic devices whose basic element is the metal-oxide-semiconductor field effect transistor (MOSFET).

**Chip** An integrated circuit. Specifically, a chip of silicon on which an integrated circuit is constructed.

**Dual in-line package (DIP)** A type of IC with two parallel rows of pins for the various circuit inputs and outputs.

**Printed circuit board (PCB)** A circuit board in which connections between components are made with lines of copper on the surfaces of the circuit board.

**Breadboard** A circuit board for wiring temporary circuits, usually used for prototypes or laboratory work.

**Wire-wrap** A circuit construction technique in which the connecting wires are wrapped around the posts of a special chip socket, usually used for prototyping or laboratory work.

**Through-hole** A means of mounting DIP ICs on a circuit board by inserting the IC leads through holes in the board and soldering them in place.

**Surface-mount technology (SMT)** A system of mounting and soldering integrated circuits on the surface of a circuit board, as opposed to inserting their leads through holes on the board.

**Small outline IC (SOIC)** An IC package similar to a DIP, but smaller, which is designed for automatic placement and soldering on the surface of a circuit board. Also called gull-wing, for the shape of the package leads.

**Thin shrink small outline package (TSSOP)** A thinner version of an SOIC package.

**Plastic leaded chip carrier (PLCC)** A square IC package with leads on all four sides designed for surface mounting on a circuit board. Also called J-lead, for the profile shape of the package leads.

**Quad flat pack (QFP)** A square surface-mount IC package with gull-wing leads.

**Ball grid array (BGA)** A square surface-mount IC package with rows and columns of spherical leads underneath the package.

**Data sheet** A printed specification giving details of the pin configuration, electrical properties, and mechanical profile of an electronic device.

**Data book** A bound collection of data sheets. A digital logic data book usually contains data sheets for a specific logic family or families.

**Portable document format (PDF)** A format for storing published documents in compressed form.
All the logic gates we have looked at so far are available in integrated circuit form. Most of these small scale integration (SSI) functions are available either in transistor-transistor logic (TTL) or complementary metal-oxide-semiconductor (CMOS) technologies. TTL and CMOS devices differ not in their logic functions, but in their construction and electrical characteristics.

TTL and CMOS chips are designated by an industry-standard numbering system. TTL devices and the more recent members of the CMOS family are numbered according to the general format 74XXXN, where XX is a family identifier and N/N identifies the specific logic function. For example, the number 74ALS00 represents a quadruple 2-input NAND device (indicated by 00) in the advanced low power Schottky (ALS) family of TTL. (Earlier versions of CMOS had a different set of unrelated numbers of the form 4NN/NB or 4NN/UB where NN was the logic function designator. The suffixes B and UB stand for buffered and unbuffered, respectively.)

Table 2.21 lists the quadruple 2-input NAND function as implemented in different logic families. These devices all have the same logic function, but different electrical characteristics.

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Logic Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>74LS00</td>
<td>Low-power Schottky TTL</td>
</tr>
<tr>
<td>74ALSO00</td>
<td>Advanced low-power Schottky TTL</td>
</tr>
<tr>
<td>74F00</td>
<td>FAST TTL</td>
</tr>
<tr>
<td>74HC00</td>
<td>High-speed CMOS</td>
</tr>
<tr>
<td>74HCT00</td>
<td>High-speed CMOS (TTL-compatible inputs)</td>
</tr>
<tr>
<td>74LVX00</td>
<td>Low-voltage CMOS</td>
</tr>
<tr>
<td>74ABT00</td>
<td>Advanced BiCMOS (TTL/CMOS hybrid)</td>
</tr>
</tbody>
</table>

Table 2.22 lists several logic functions available in the high-speed CMOS family. These devices all have the same electrical characteristics, but different logic functions.

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>74HC00</td>
<td>Quadruple 2-input NAND</td>
</tr>
<tr>
<td>74HC02</td>
<td>Quadruple 2-input NOR</td>
</tr>
<tr>
<td>74HC04</td>
<td>Hex inverter</td>
</tr>
<tr>
<td>74HC08</td>
<td>Quadruple 2-input AND</td>
</tr>
<tr>
<td>74HC32</td>
<td>Quadruple 2-input OR</td>
</tr>
<tr>
<td>74HC86</td>
<td>Quadruple 2-input XOR</td>
</tr>
</tbody>
</table>

Until recently, the most common way to package logic gates has been in a plastic or ceramic dual in-line package, or DIP, which has two parallel rows of pins. The standard spacing between pins in one row is 0.1” (or 100 mil). For packages having fewer than 28 pins, the spacing between rows is 0.3” (or 300 mil). For larger packages, the rows are spaced by 0.6” (600 mil).

This type of package is designed to be inserted in a printed circuit board in one of two says: (a) the pins are inserted through holes in the circuit board and soldered in place; or (b) a socket is soldered to the circuit board and the IC is placed in the socket. The latter method is more expensive, but makes chip replacement much easier. A socket can occasionally cause its own problems by making a poor connection to the pins of the IC.

The DIP is also convenient for laboratory and prototype work, since it can also be inserted easily into a breadboard, a special type of temporary circuit board with internal connections between holes of a standard spacing. It is also convenient for wire-wrapping, a technique in which a special tool is used to wrap wires around posts on the underside of special sockets.
The outline of a 14-pin DIP is shown in Figure 2.36. There is a notch on one end to show the orientation of the pins. When the IC is oriented as shown and viewed from above, pin 1 is at the top left corner and the pins number counterclockwise from that point.

Besides DIP packages, there are numerous other types of packages for digital ICs, including, among others, small outline IC (SOIC), thin shrink small outline package (TSSOP), plastic leaded chip carrier (PLCC), quad flat pack (QFP), and ball grid array (BGA) packages. They are used mostly in applications where circuit board space is at a premium and in manufacturing processes relying on surface-mount technology (SMT). In fact, these devices represent the majority of IC packages found in new designs. Some of these IC packaging options are shown in Figure 2.37.

SMT is a sophisticated technology which relies on automatic placement of chips and soldering of pins onto the surface of a circuit board, not through holes in the circuit board. This technique allows a manufacturer to mount components on both sides of a circuit board.
Primarily due to the great reduction in board space requirements, most new ICs are available only in the newer surface-mount packages and are not being offered at all in the DIP package. However, we will look at DIP offerings in logic gates because they are inexpensive and easy to use with laboratory breadboards and therefore useful as a learning tool.

Logic gates come in packages containing several gates. Common groupings available in DIP packages are six 1-input gates, four 2-input gates, three 3-input gates, or two 4-input gates, although other arrangements are available. The usual way of stating the number of logic gates in a package is to use the numerical prefixes hex (6), quad or quadruple (4), triple (3), or dual (2).

Some common gate packages are listed in Table 2.23.

### Table 2.23 Some Common Logic Gate ICs

<table>
<thead>
<tr>
<th>Gate</th>
<th>Family</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>74HC00A</td>
<td>High-speed CMOS</td>
<td>Quad 2-input NAND</td>
</tr>
<tr>
<td>74HC02</td>
<td>High-speed CMOS</td>
<td>Quad 2-input NOR</td>
</tr>
<tr>
<td>74ALS04</td>
<td>Advanced low-power Schottky TTL</td>
<td>Hex inverter</td>
</tr>
<tr>
<td>74LS11</td>
<td>Low-power Schottky TTL</td>
<td>Triple 3-input AND</td>
</tr>
<tr>
<td>74F20</td>
<td>FAST TTL</td>
<td>Dual 4-input NAND</td>
</tr>
<tr>
<td>74HC27</td>
<td>High-speed CMOS</td>
<td>Triple 3-input NOR</td>
</tr>
</tbody>
</table>

Information about pin configurations, electrical characteristics, and mechanical specifications of a part is available in a **data sheet** provided by the chip manufacturer. A collection of data sheets for a particular logic family is often bound together in a **data book**. More recently, device manufacturers have been making data sheets available on their corporate World Wide Web sites in **portable document format (PDF)**, readable by a special program such as Adobe Acrobat Reader. Links to some of these manufacturers can be found on the Online Companion Web site for this book. (http://www.electronictech.com)

Figure 2.38 shows the internal diagrams of gates listed in Table 2.23. Notice that the gates can be oriented inside a chip in a number of ways. That is why it is important to confirm pin connections with a data sheet.

In addition to the gate inputs and outputs there are two more connections to be made on every chip: the power \((V_{CC})\) and ground connections. In TTL, connect \(V_{CC}\) to +5 Volts and GND to ground. In CMOS, connect the \(V_{CC}\) pin to the supply voltage (+3 V to +6 V) and GND to ground. The gates won’t work without these connections.

**Every chip requires power and ground.** This might seem obvious, but it’s surprising how often it is forgotten, especially by students who are new to digital electronics. Probably this is because most digital circuit diagrams don’t show the power connections, but assume that you know enough to make them.

The only place a chip gets its required power is through the \(V_{CC}\) pin. Even if the power supply is connected to a logic input as a logic HIGH, you still need to connect it to the power supply pin.

Even more important is a good ground connection. A circuit with no power connection will not work at all. A circuit without a ground may appear to work, but it will often produce bizarre errors that are very difficult to detect and repair.

In later chapters, we will work primarily with complex ICs in PLCC packages. The power and ground connections are so important to these chips that they will not be left to chance; they are provided on a specially designed circuit board. Only input and output pins are accessible for connection by the user.

As digital designs become more complex, it is increasingly necessary to follow good practices in board layout and prototyping procedure to ensure even minimal functionality.
Thus, hardware platforms for prototype and laboratory work will need to be at least partially constructed by the board manufacturer in order to supply the requirements of a stable circuit configuration.

SECTION REVIEW PROBLEM FOR SECTION 2.6

2.10. How are the pins numbered in a dual in-line package?

SUMMARY

1. Digital systems can be analyzed and designed using Boolean algebra, a system of mathematics that operates on variables that have one of two possible values.
2. Any Boolean expression can be constructed from the three simplest logic functions: NOT, AND, and OR.
3. A NOT gate, or inverter, has an output state that is in the opposite logic state of the input.
4. The main 2-input logic functions are described as follows, for inputs A and B and output Y:
   - **AND:** \( Y \) is HIGH if A AND B are HIGH. (\( Y = A \cdot B \))
   - **OR:** \( Y \) is HIGH if A OR B is HIGH. (\( Y = A + B \))
   - **NAND:** \( Y \) is LOW if A AND B are HIGH. (\( Y = A \cdot \overline{B} \))
   - **NOR:** \( Y \) is LOW if A OR B is HIGH. (\( Y = A + B \))
   - **XOR:** \( Y \) is HIGH if A OR B is HIGH, but not if both are HIGH. (\( Y = A \oplus B \))
   - **XNOR:** \( Y \) is LOW if A OR B is HIGH, but not if both are HIGH. (\( Y = A \oplus B \))
5. The function of a logic gate can be represented by a truth table, a list of all possible inputs in binary order and the output corresponding to each input state.
6. DeMorgan’s theorems (\( A \cdot B = A + \overline{B} \) and \( A + B = A \cdot \overline{B} \)) allow us to represent any gate in an AND form and an OR form.
7. To change a gate into its DeMorgan equivalent form, change its shape from AND to OR or vice versa and change the active levels of inputs and output.
8. A logic switch can be created from a single-pole single-throw switch by grounding one end and tying the other end to $V_{CC}$ through a pull-up resistor. The logic level is available on the same side of the switch as the resistor. An open switch is HIGH and a closed switch is LOW. A similar circuit can be made with a pushbutton switch.

9. A light emitting diode (LED) can be used to indicate logic HIGH or LOW levels. To indicate a HIGH, ground the cathode through a series resistor (about 470 Ω for a 5-volt power supply) and apply the logic level to the anode. To indicate a LOW, tie the anode to $V_{CC}$ through a series resistor and apply the logic level to the cathode.

10. Logic gates can be used to pass or block digital signals. For example, an AND gate will pass a digital signal applied to input $B$ if the input $A$ is HIGH ($Y = B$). If input $A$ is LOW, the signal is blocked and the gate output is always LOW ($Y = 0$). Similar properties apply to other gates, as summarized in Table 2.20.

11. Tristate buffers have outputs that generate logic HIGH and LOW when enabled and a high-impedance state when disabled. The high-impedance state is electrically equivalent to an open circuit.

Glossary

Active HIGH An active-HIGH terminal is considered “ON” when it is in the logic HIGH state. Indicated by the absence of a bubble at the terminal in distinctive-shape symbols.

Active level A logic level defined as the “ON” state for a particular circuit input or output. The active level can be either HIGH or LOW.

Active LOW An active-LOW terminal is considered “ON” when it is in the logic LOW state. Indicated by a bubble at the terminal in distinctive-shape symbols.

AND gate A logic circuit whose output is HIGH when all inputs (e.g., $A \land B \land C$) are HIGH.

Ball grid array (BGA) A square surface-mount IC package with rows and columns of spherical leads underneath the package.

Boolean algebra A system of algebra that operates on Boolean variables. The binary (two-state) nature of Boolean algebra makes it useful for analysis, simplification, and design of combinational logic circuits.

Boolean expression An algebraic expression made up of Boolean variables and operators, such as $\land$ (AND), $\lor$ (OR), or $\neg$ (NOT). Also referred to as a Boolean function or a logic function.

Boolean variable A variable having only two possible values, such as HIGH/Low, 1/0, On/Off, or True/False.

Breadboard A circuit board for wiring temporary circuits, usually used for prototypes or laboratory work.

Bubble A small circle indicating logical inversion on a circuit symbol.

Buffer An amplifier that acts as a logic circuit. Its output can be inverting or noninverting.

Bus A common wire or parallel group of wires connecting multiple circuits.

Chip An integrated circuit. Specifically, a chip of silicon on which an integrated circuit is constructed.

Clock generator A circuit that generates a periodic digital waveform.

Coincidence gate An Exclusive NOR gate.

Complement form Inverted.

Complementary metal-oxide-semiconductor (CMOS) A family of digital logic devices whose basic element is the metal-oxide-semiconductor field effect transistor (MOSFET).

Data book A bound collection of data sheets. A digital logic data book usually contains data sheets for a specific logic family or families.

Data sheet A printed specification giving details of the pin configuration, electrical properties, and mechanical profile of an electronic device.

DeMorgan equivalent forms Two gate symbols, one AND-shaped and one OR-shaped, that are equivalent according to DeMorgan’s theorems.

DeMorgan’s theorems Two theorems in Boolean algebra that allow us to transform any gate from an AND-shaped to an OR-shaped gate and vice versa.

Digital signal (or pulse waveform) A series of 0s and 1s plotted over time.

Distinctive-shape symbols Graphic symbols for logic circuits that show the function of each type of gate by a special shape.

Dual in-line package (DIP) A type of IC with two parallel rows of pins for the various circuit inputs and outputs.

Enable A logic gate is enabled if it allows a digital signal to pass from an input to the output in either true or complement form.

Exclusive NOR gate A two-input logic circuit whose output is the complement of an Exclusive OR gate.
Plastic leaded chip carrier (PLCC) A square IC package with leads on all four sides designed for surface mounting on a circuit board. Also called J-lead, for the profile shape of the package leads.

Exclusive OR gate A two-input logic circuit whose output is HIGH when one input (but not both) is HIGH.

Floating An undefined logic state, neither HIGH nor LOW.

High-impedance state The output state of a tristate buffer that is neither logic HIGH nor logic LOW, but is electrically equivalent to an open circuit.

IEEE/ANSI Standard 91-1984 A standard format for drawing logic circuit symbols as rectangles with logic functions shown by a standard notation inside the rectangle for each device.

In phase Two digital waveforms are in phase if they are always at the same logic level at the same time.

Inhibit (or disable) A logic gate is inhibited if it prevents a digital signal from passing from an input to the output.

Integrated circuit (IC) An electronic circuit having many components, such as transistors, diodes, resistors, and capacitors, in a single package.

Inverter Also called a NOT gate or an inverting buffer. A logic gate that changes its input logic level to the opposite state.

Large scale integration (LSI) An integrated circuit having from 100 to 10,000 equivalent gates.

LED Light emitting diode. An electronic device that conducts current in one direction only and illuminates when it is conducting.

Logic function See Boolean expression.

Logic gate An electronic circuit that performs a Boolean algebraic function.

Logical product AND function.

Logical sum OR function.

Medium scale integration (MSI) An integrated circuit having the equivalent of 12 to 100 gates in one package.

NAND gate A logic circuit whose output is LOW when all inputs are HIGH.

NOR gate A logic circuit whose output is LOW when at least one input is HIGH.

OR gate A logic circuit whose output is HIGH when at least one input (e.g., A OR B OR C) is HIGH.

Out of phase Two digital waveforms are out of phase if they are always at opposite logic levels at any given time.

Plastic leaded chip carrier (PLCC) A square IC package with leads on all four sides designed for surface mounting on a circuit board. Also called J-lead, for the profile shape of the package leads.

Portable document format (PDF) A format for storing published documents in compressed form.

Printed circuit board (PCB) A circuit board in which connections between components are made with lines of copper on the surfaces of the circuit board.

Pull-up resistor A resistor connected from a point in an electronic circuit to the power supply of that circuit. In a digital circuit it supplies the required logic level in a HIGH state and limits current from the power supply in the LOW state.

Quad flat pack (QFP) A square surface-mount IC package with gull-wing leads.

Qualifying symbol A symbol in IEEE/ANSI logic circuit notation, placed in the top center of a rectangular symbol, that shows the function of a logic gate. Some qualifying symbols include: 1 = “buffer”; & = “AND”; ≥1 = “OR”

Rectangular-outline symbols Rectangular logic gate symbols that conform to IEEE/ANSI Standard 91-1984.

Small outline IC (SOIC) An IC package similar to a DIP, but smaller, which is designed for automatic placement and soldering on the surface of a circuit board. Also called gull-wing, for the shape of the package leads.

Small-scale integration (SSI) An integrated circuit having 12 or fewer gates in one package.

Surface-mount technology (SMT) A system of mounting and soldering integrated circuits on the surface of a circuit board, as opposed to inserting their leads through holes on the board.

Thin shrink small outline package (TSSOP) A thinner version of an SOIC package.

Through-hole A means of mounting DIP ICs on a circuit board by inserting the IC leads through holes in the board and soldering them in place.

Transistor-transistor logic (TTL) A family of digital logic devices whose basic element is the bipolar junction transistor.

Tristate buffer A gate having three possible output states: logic HIGH, logic LOW, and high-impedance.

True form Not inverted.

Truth table A list of all possible input values to a digital circuit, listed in ascending binary order, and the output response for each input combination.

Vcc The power supply voltage in a transistor-based electronic circuit. The term often refers to the power supply of digital circuits.

Very large scale integration (VLSI) An integrated circuit having more than 10,000 equivalent gates.

PROBLEMS

Problem numbers set in color indicate more difficult problems; those with underlines indicate most difficult problems.

Section 2.1 Basic Logic Functions

2.1 Draw the symbol for the NOT gate (inverter) in both rectangular-outline and distinctive-shape forms.
side the line(s) of the truth table indicating when the gate output is in its active state.

2.5 Write a sentence that describes the operation of a 4-input OR gate with inputs \( J, K, L, \) and \( M \) and output \( N \). Make the truth table of this gate and draw an asterisk beside the line(s) of the truth table indicating when the gate output is in its active state.

2.6 State how three switches must be connected to represent a 3-input AND function. Draw a circuit diagram showing how this function can control a lamp.

2.7 State how four switches must be connected to represent a 4-input OR function. Draw a circuit diagram showing how this function can control a lamp.

**Section 2.2 Logic Switches and LED Indicators**

2.8 Sketch the circuit of a single-pole single-throw (SPST) switch used as a logic switch. Briefly explain how it works.

2.9 Refer to Figure 2.10 (logic pushbuttons). Should the normally open pushbutton be considered an active HIGH or active LOW device? Briefly explain your choice.

2.10 Should the normally closed pushbutton be considered an active HIGH or active LOW device? Why?

2.11 Briefly state what is required for an LED to illuminate.

2.12 Briefly state the relationship between the brightness of an LED and the current flowing through it. Why is a series resistor required?

2.13 Draw a circuit showing how an OR-gate output will illuminate an LED when the gate output is LOW. Assume the required series resistor is 470 \( \Omega \).

**Section 2.3 Derived Logic Functions**

2.14 For a 4-input NAND gate with inputs \( A, B, C, \) and \( D \) and output \( Y \):
   a. Write the truth table and a descriptive sentence.
   b. Write the Boolean expression.
   c. Draw the logic circuit symbol in both distinctive-shape and rectangular-outline symbols.

2.15 Repeat Problem 2.14 for a 4-input NOR gate.

2.16 State the active levels of the inputs and outputs of a NAND gate and a NOR gate.

2.17 Write a descriptive sentence of the operation of a 5-input NAND gate with inputs \( A, B, C, D, \) and \( E \) and output \( Y \). How many lines would the truth table of this gate have?

2.18 Repeat Problem 2.17 for a 5-input NOR gate.

2.19 A pump motor in an industrial plant will start only if the temperature and pressure of liquid in a tank exceed a certain level. The temperature sensor and pressure sensor, shown in Figure 2.39 each produce a logic HIGH if the measured quantities exceed this value. The logic circuit interface produces a HIGH output to turn on the motor. Draw the symbol and truth table of the gate that corresponds to the action of the logic circuit.

2.20 Repeat Problem 2.19 for the case in which the motor is activated by a logic LOW.

**Section 2.4 DeMorgan’s Theorems and Gate Equivalence**

2.21 Figure 2.40 shows a circuit for a two-way switch for a stairwell. This is a common circuit that allows you to turn on a light from either the top or the bottom of the stairwell and off at the other end. The circuit also allows anyone coming along after you to do the same thing, no matter which direction they are coming from.

   The lamp is ON when the switches are in the same positions and OFF when they are in opposite positions. What logic function does this represent? Draw the truth table of the function and use it to explain your reasoning.

2.22 Find the truth table for the logic circuit shown in Figure 2.41.

2.23 Recall the description of a 2-input Exclusive OR gate: “Output is HIGH if one input is HIGH, but not both.” This is not the best statement of the operation of a multiple-input XOR gate. Look at the truth table derived in Problem 2.22 and write a more accurate description of \( n \)-input XOR operation.

**FIGURE 2.39** Problem 2.19: Temperature and Pressure Sensors

**FIGURE 2.40** Problem 2.21: Circuit for Two-Way Switch

**FIGURE 2.41** Problem 2.22: Logic Circuit

2.24 For each of the gates in Figure 2.42:
   a. Write the truth table.
   b. Indicate with an * which lines on the truth table show the gate output in its active state.
Section 2.5 Enable and Inhibit Properties of Logic Gates

2.26 Draw the output waveform of the Exclusive NOR gate when a square waveform is applied to one input and
a. The other input is held LOW
b. The other input is held HIGH

2.27 Sketch the input waveforms represented by the following 32-bit sequences (use 1/4-inch graph paper, 1 square per bit):
A: 0000 0000 0000 1111 1111 1111 1111 0000
B: 1010 0111 0010 1011 0101 0011 1001 1011

Assume that these waveforms represent inputs to a logic gate. (Spaces are provided for readability only.) Sketch the waveform for gate output \( Y \) if the gate function is:
a. AND
b. OR
c. NAND
d. NOR
e. XOR
f. XNOR

2.28 Repeat Problem 2.27 for the waveforms shown in Figure 2.44.

2.29 The A and B waveforms shown in Figure 2.45 are inputs to an OR gate. Complete the sketch by drawing the waveform for output \( Y \).

2.30 Repeat Problem 2.29 for a NOR gate.

2.31 Figure 2.46 shows a circuit that will make a lamp flash at 3 Hz when the gasoline level in a car’s gas tank drops below a certain point. A float switch in the tank monitors the level of gasoline. What logic level must the float switch produce to make the light flash when the tank is approaching empty? Why?

2.32 Repeat Problem 2.31 for the case where the AND gate is replaced by a NOR gate.

2.33 Will the circuit in Figure 2.46 work properly if the AND gate is replaced by an Exclusive OR gate? Why or why not?

2.34 Make a truth table for the tristate buffers shown in Figure 2.33. Indicate the high-impedence state by the notation C B A Y

How does this compare to the waveform that would appear at the output of an Exclusive OR gate under the same conditions?
ANSWERS TO SECTION REVIEW PROBLEMS

Section 2.1
2.1 AND: “A AND B AND C AND D must be HIGH to make Y HIGH.”
2.2 OR: “A OR B OR C OR D must be HIGH to make Y HIGH.”

Section 2.2
2.3 When the switch is open, it provides a logic HIGH because of the pull-up resistor. A closed switch is LOW, due to the connection to ground.

Section 2.3
2.4 XOR; 2.5 NAND; 2.6 NOR; 2.7 XNOR.

Section 2.4
2.8 \( Y = \overline{A B C D} \)

Section 2.5
2.9 An AND needs two HIGH inputs to make a HIGH output. If the Control input is LOW, the output can never be HIGH; the output remains LOW. An OR output is HIGH if one input is HIGH. If the Control input is HIGH, the output is always HIGH, regardless of the level at the Signal input. In both cases, the output is “stuck” at one level, signifying that the gate is inhibited.

Section 2.6
2.10 Viewed from above, with the notch in the package away from you, pin 1 is on the left side at the far end. The pins are numbered counterclockwise from that point.

2.35 Name two logic families used to implement digital logic functions. How do they differ?
2.36 List the industry-standard numbers for a quadruple 2-input NAND gate in low power Schottky TTL, CMOS, and high-speed CMOS technologies.
2.37 Repeat Problem 2.36 for a quadruple 2-input NOR gate. How does each numbering system differentiate between the NAND and NOR functions?
2.38 List six types of packaging that a logic gate could come in.

“Hi-Z” How do the enable properties of these gates differ from gates such as AND and NAND?
In Chapter 3, we will examine the rudiments of combinational logic. A combinational logic circuit is one in which two or more gates are connected together to combine several Boolean inputs. These circuits can be represented several ways, as a logic diagram, truth table, or Boolean expression.

A Boolean expression for a network of logic gates is often not in its simplest form. In such a case, we may be using more components than would be required for the job, so it is of benefit to us if we can simplify the Boolean expression. Several tools are available to us, such as Boolean algebra and a graphical technique known as Karnaugh mapping. We can also simplify the Boolean expression by taking care to draw the logic diagrams in such a way as to automatically eliminate inverting functions within the circuit.
CHAPTER 3 • Boolean Algebra and Combinational Logic

3.1 Boolean Expressions, Logic Diagrams and Truth Tables

**KEY TERMS**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Logic gate network</td>
<td>Two or more logic gates connected together.</td>
</tr>
<tr>
<td>Logic diagram</td>
<td>A diagram, similar to a schematic, showing the connection of logic gates.</td>
</tr>
<tr>
<td>Combinational logic</td>
<td>Digital circuitry in which an output is derived from the combination of inputs, independent of the order in which they are applied.</td>
</tr>
<tr>
<td>Combinatorial logic</td>
<td>Another name for combinational logic.</td>
</tr>
</tbody>
</table>

In Chapter 2, we examined the functions of single logic gates. However, most digital circuits require multiple gates. When two or more gates are connected together, they form a **logic gate network**. These networks can be described by a truth table, a **logic diagram** (i.e., a circuit diagram), or a Boolean expression. Any one of these can be derived from any other.

A digital circuit built from gates is called a **combinational** (or **combinatorial**) logic circuit. The output of a combinational circuit depends on the **combination** of inputs. The inputs can be applied in any sequence and still produce the same result. For example, an AND gate output will always be HIGH if all inputs are HIGH, regardless of the order in which they became HIGH. This is in contrast to **sequential logic**, in which sequence matters; a sequential logic output may have a different value with two identical sets of inputs if those inputs were applied in a different order. We will study sequential logic in a later chapter.

**Boolean Expressions from Logic Diagrams**

**KEY TERMS**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bubble-to-bubble convention</td>
<td>The practice of drawing gates in a logic diagram so that inverting outputs connect to inverting inputs and noninverting outputs connect to noninverting inputs.</td>
</tr>
<tr>
<td>Order of precedence</td>
<td>The sequence in which Boolean functions are performed, unless otherwise specified by parentheses.</td>
</tr>
</tbody>
</table>

Writing the Boolean expression of a logic gate network is similar to finding the expression for a single gate. The difference is that in a multiple gate network, the inputs will usually not consist of single variables, but compound expressions that represent outputs of previous gates.

These compound expressions are combined according to the same rules as single variables. In an OR gate, with inputs $x$ and $y$, the output will always be $x + y$ regardless of whether $x$ and $y$ are single variables (e.g., $x = A$, $y = B$, output $= A + B$) or compound expressions (e.g., $x = AB$, $y = AC$, output $= AB + AC$).

Figure 3.1 shows a simple logic gate network, consisting of a single AND and a single OR gate. The AND gate combines inputs $A$ and $B$ to give the output expression $AB$. The OR combines the AND function and input $C$ to yield the compound expression $AB + C$.

![Figure 3.1](image-url)  
**FIGURE 3.1**  
Boolean Expression from a Gate Network
EXAMPLE 3.1
Derive the Boolean expression of the logic gate network shown in Figure 3.2a.

**Solution**

Figure 3.2b shows the gate network with the output terms indicated for each gate. The AND and NAND functions are combined in an OR function to yield the output expression:

\[ Y = AB + \overline{C} \overline{D} \]

The Boolean expression in Example 3.1 includes a NAND function. It is possible to draw the NAND in its DeMorgan equivalent form. If we choose the gate symbols so that outputs with bubbles connect to inputs with bubbles, we will not have bars over groups of variables, except possibly one bar over the entire function. In a circuit with many inverting functions (NANDs and NORs), this results in a cleaner notation and often a clearer idea of the function of the circuit. We will follow this notation, which we will refer to as the bubble-to-bubble convention, as much as possible.

EXAMPLE 3.2
Redraw the circuit in Figure 3.2 to conform to the bubble-to-bubble convention. Write the Boolean expression of the new logic diagram.

**Solution**

Figure 3.3 shows the new circuit. The NAND has been converted to its DeMorgan equivalent so that its active-HIGH output drives an active-HIGH input on the OR gate. The new Boolean expression is

\[ Y = AB + \overline{C} \overline{D} \]

Boolean functions are governed by an order of precedence. Unless otherwise specified, AND functions are performed first, followed by ORs. This order results in a form similar to that of linear algebra, where multiplication is performed before addition, unless otherwise specified.
Figure 3.4 shows two logic diagrams, one whose Boolean expression requires parentheses and one that does not.

![Logic Diagrams](image)

**a. No parentheses required (AND, then OR)**

**b. Parentheses required (OR, then AND)**

**FIGURE 3.4**

Order of Precedence

The AND functions in Figure 3.4a are evaluated first, eliminating the need for parentheses in the output expression. The expression for Figure 3.4b requires parentheses since the ORs are evaluated first.

---

**EXAMPLE 3.3**

Write the Boolean expression for the logic diagrams in Figure 3.5.

![Logic Diagrams](image)

**Ordinary Precedence**

**a.**

**Solution**  Examine the output of each gate and combine the resultant terms as required.

**Figure 3.5a:** Gate 1: $\overline{A} \cdot \overline{B}$  
Gate 2: $B \cdot C$  
Gate 3: $Y = \text{Gate 1} + \text{Gate 2} = \overline{A} \cdot \overline{B} + B \cdot C$  

**Figure 3.5b:** Gate 1: $\overline{P + Q} = \overline{P} + \overline{Q}$  
Gate 2: $\overline{Q + R}$  
Gate 3: $S = \text{Gate 1} \cdot \text{Gate 2} = (\overline{P + Q})(\overline{Q + R}) = (\overline{P} + \overline{Q})(\overline{Q} + \overline{R})$
Note that when two bubbles touch, they cancel out, as in the doubly inverted $P$ input or the connection between the outputs of gates 1 and 2 and the inputs of gate 3. In the resultant Boolean expression, bars of the same length cancel; bars of unequal length do not.

**SECTION 3.1A REVIEW PROBLEM**

3.1 Write the Boolean expression for the logic diagrams in Figure 3.6, paying attention to the rules of order of precedence.

![Logic Diagram](image)

**FIGURE 3.6**

Section Review Problem 3.1

**Logic Diagrams from Boolean Expressions**

<table>
<thead>
<tr>
<th><strong>KEY TERMS</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Levels of gating</strong> The number of gates through which a signal must pass from input to output of a logic gate network.</td>
</tr>
<tr>
<td><strong>Double-rail inputs</strong> Boolean input variables that are available to a circuit in both true and complement form.</td>
</tr>
<tr>
<td><strong>Synthesis</strong> The process of creating a logic circuit from a description such as a Boolean equation or truth table.</td>
</tr>
</tbody>
</table>

We can derive a logic diagram from a Boolean expression by applying the order of precedence rules. We examine an expression to create the first level of gating from the circuit inputs, then combine the output functions of the first level in the second level gates, and so forth. Input inverters are often not counted as a gating level, as we usually assume that each variable is available in both true (noninverted) and complement (inverted) form. When input variables are available to a circuit in true and complement form, we refer to them as double-rail inputs.

The first level usually will be AND gates if no parentheses are present, OR gates if parentheses are used. (Not always, however; parentheses merely tell us which functions to synthesize first.) Although we will try to eliminate bars over groups of variables by use of DeMorgan's theorems and the bubble-to-bubble convention, we should recognize that a bar over a group of variables is the same as having those variables in parentheses.

Let us examine the Boolean expression $Y = AC + BD + AD$. Order of precedence tells us that we synthesize the AND functions first. This yields three 2-input AND gates, with outputs $AC$, $BD$, and $AD$, as shown in Figure 3.7a. In the next step, we combine these AND functions in a 3-input OR gate, as shown in Figure 3.7b.
When the expression has OR functions in parentheses, we synthesize the ORs first, as for the expression \( Y = (A + B)(A + C + D)(B + C) \). Figure 3.8 shows this process. In the first step, we synthesize three OR gates for the terms \((A + B), (A + C + D), \) and \((B + C)\). We then combine these terms in a 3-input AND gate.

**EXAMPLE 3.4**

Synthesize the logic diagrams for the following Boolean expressions:

1. \( P = QRS + \overline{ST} \)
2. \( X = (W + Z + \overline{Y})\overline{V} + (\overline{W} + V)\overline{Y} \)

**Solution**

1. Recall that a bar over two variables acts like parentheses. Thus the \(QRS\) term is synthesized from a NAND, then an AND, as shown in Figure 3.9a. Also shown is the second AND term, ST.
Figure 3.9b shows the terms combined in an OR gate.

2. Figure 3.10 shows the synthesis of the second logic diagram in three stages. Figure 3.10a shows how the circuit inputs are first combined in two OR gates. We do this first because the ORs are in parentheses. In Figure 3.10b, each of these functions is combined in an AND gate, according to the normal order of precedence. The AND outputs are combined in a final OR function, as shown in Figure 3.10c.
EXAMPLE 3.5

Use DeMorgan’s theorem to modify the Boolean equation in part 1 of Example 3.4 so that there is no bar over any group of variables. Redraw Figure 3.9b to reflect the change.

Solution

\[ P = QRS + ST = Q(R + S) + ST \]

Figure 3.11a shows the modified logic diagram. The levels of gating could be further reduced from three to two (not counting input inverters) by “multiplying through” the parentheses to yield the expression:

\[ P = Q\bar{R} + Q\bar{S} + \bar{S}T \]

Figure 3.11b shows the logic diagram for this form. We will examine this simplification procedure more formally in a later section of this chapter.

Truth Tables from Logic Diagrams or Boolean Expressions

There are two basic ways to find a truth table from a logic diagram. We can examine the output of each gate in the circuit and develop its truth table. We then use our knowledge of gate properties to combine these intermediate truth tables into the final output truth table. Alternatively, we can develop a Boolean expression for the logic diagram and by examining the expression fill in the truth table in a single step. The former method is more thorough and probably easier to understand when you are learning the technique. The latter method is more efficient, but requires some practice and experience. We will look at both.

Examine the logic diagram in Figure 3.12. Since there are three binary inputs, there will be eight ways those inputs can be combined. Thus, we start by making an 8-line truth table, as in Table 3.1.

FIGURE 3.12
Logic Diagram for \( AB + C \)
The OR gate output will describe the function of the whole circuit. In order to assess the OR function, we must first evaluate the AND output. We add a column to the truth table for the AND gate and look for the lines in the table where both A AND B equal logic 1 (in this case, the last two rows). For these lines, we write a 1 in the AB column. Next, we look at the values in column C and the AB column. If there is a 1 in either column, we write a 1 in the column for the final output.

### Table 3.1 Truth Table for Figure 3.12

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>AB</th>
<th>AB + C</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
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<tr>
<td>1</td>
<td>0</td>
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<td>1</td>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

### Example 3.6

Derive the truth table for the logic diagram shown in Figure 3.13.

**FIGURE 3.13**

Example 3.6
Logic Diagram

**Solution**

The Boolean equation for Figure 3.13 is \((\overline{A} + \overline{B})(A + C)\). We will create a column for each input variable and for each term in parentheses, as well as a column for the final output. Table 3.2 shows the result. For the lines where \(A \text{ OR } B\) is 0, we write a 1 in the \((\overline{A} + \overline{B})\) column. Where \(A \text{ OR } C\) is 1, we write a 1 in the \((A + C)\) column. For the lines where there is a 1 in both the \((\overline{A} + \overline{B})\) AND \((A + C)\) columns, we write a 1 in the final output column.

### Table 3.2 Truth Table for Figure 3.13

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>((\overline{A} + \overline{B}))</th>
<th>((A + C))</th>
<th>((\overline{A} + \overline{B})(A + C))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
Another approach to finding a truth table involves analysis of the Boolean expression of a logic diagram. The logic diagram in Figure 3.14 can be described by the Boolean expression \( Y = \overline{A} \overline{C} + \overline{A} \overline{C} + \overline{B} \overline{D} \).

We can examine the Boolean expression to determine that the final output of the circuit will be HIGH under one of the following conditions:

1. \( A = 0 \) AND \( B = 1 \) AND \( C = 1 \);
2. \( A = 0 \) AND \( C = 0 \);
3. \( B = 0 \) AND \( D = 0 \).

All we have to do is look for these conditions in the truth table and write a 1 in the output column whenever a condition is satisfied. Table 3.3 shows the result of this analysis with each line indicating which term, or terms, contribute to the HIGH output.

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
<th>( Y )</th>
<th>terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \overline{A} \overline{C}, \overline{B} \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( \overline{A} \overline{C} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( \overline{B} \overline{D} )</td>
</tr>
<tr>
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<td>0</td>
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<td>1</td>
<td>0</td>
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<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \overline{A} \overline{C} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>( \overline{A} \overline{C} )</td>
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<tr>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>( \overline{A} \overline{C} )</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \overline{B} \overline{D} )</td>
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<td>1</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

SECTION 3.16 REVIEW PROBLEM

3.2 Find the truth table for the logic diagram shown in Figure 3.15.
3.2 Sum-of-Products and Product-of-Sums Forms

**KEY TERMS**

- **Product term** A term in a Boolean expression where one or more true or complement variables are ANDed (e.g., \(A \overline{B} C\)).
- **Minterm** A product term in a Boolean expression where all possible variables appear once in true or complement form (e.g., \(A \overline{B} C; A B \overline{C}\)).
- **Sum term** A term in a Boolean expression where one or more true or complement variables are ORed (e.g., \(A \overline{B} + \overline{C}\)).
- **Maxterm** A sum term in a Boolean expression where all possible variables appear once, in true or complement form (e.g., \((A + \overline{B} + C); (A + \overline{B} + C)\)).
- **Sum-of-products (SOP)** A type of Boolean expression where several product terms are summed (ORed) together (e.g., \(A B C + \overline{A} B C + A \overline{B} C\)).
- **Product-of-sums (POS)** A type of Boolean expression where several sum terms are multiplied (ANDed) together (e.g., \((A + \overline{B} + C)(A + B + C)(A + \overline{B} + C)\)).
- **Bus form** A way of drawing a logic diagram so that each true and complement input variable is available along a continuous conductor called a bus.

Suppose we have an unknown digital circuit, represented by the block in Figure 3.16. All we know is which terminals are inputs, which are outputs, and how to connect the power supply. Given only that information, we can find the Boolean expression of the output.

The first thing to do is find the truth table by applying all possible input combinations in binary order and reading the output for each one. Suppose the unknown circuit in Figure 3.16 yields the truth table shown in Table 3.4.

The truth table output is HIGH for three conditions:

1. When \(A\) AND \(B\) AND \(C\) are all LOW, OR
2. When \(A\) is LOW AND \(B\) AND \(C\) are HIGH, OR
3. When \(A\) is HIGH AND \(B\) AND \(C\) are LOW.

![Figure 3.16](image)

Each of those conditions represents a **minterm** in the output Boolean expression. (A minterm is a **product term** (AND term) that includes all variables \((A, B, C)\) in true or complement form.) The minterms are:

1. \(A \overline{B} \overline{C}\)
2. \(\overline{A} B C\)
3. \(A \overline{B} C\)

| Table 3.4 Truth Table for Figure 3.19 |
| --- | --- | --- |
| \(A\) | \(B\) | \(C\) | \(Y\) |
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 |
Since condition 1 OR condition 2 OR condition 3 produces a HIGH output from the circuit, the Boolean function \( Y \) consists of all three minterms summed (ORed) together, as follows:

\[
Y = \overline{A} \overline{B} C + \overline{A} B C + A \overline{B} C
\]

This expression is in a standard form called **sum-of-products** (SOP) form. Figure 3.17 shows the equivalent logic circuit.

The inputs \( A, B, \) and \( C \) and their complements are shown in **bus form**. Each variable is available, in true or complement form, at any point along a conductor. This is a useful, uncluttered notation for circuits that require several of the input variables more than once.

**NOTE**

We can derive an SOP expression from a truth table as follows:

1. Every line on the truth table that has a HIGH output corresponds to a minterm in the truth table’s Boolean expression.
2. Write all truth table variables for every minterm in true or complement form. If a variable is 0, write it in complement form (with a bar over it); if it is 1, write it in true form (no bar).
3. Combine all minterms in an OR function.

### EXAMPLE 3.7

Tables 3.5 and 3.6 show the truth tables for the Exclusive OR and the Exclusive NOR functions. Derive the sum-of-products expression for each of these functions and draw the logic diagram for each one.

#### Table 3.5 XOR

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( A \oplus B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

#### Table 3.6 XNOR

<table>
<thead>
<tr>
<th>( A )</th>
<th>( B )</th>
<th>( A \odot B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Solution

**XOR:** The truth table yields two product terms: \(A \overline{B}\) and \(A \overline{B}\). Thus, the SOP form of the XOR function is \(A \oplus B = AB + A\overline{B}\). Figure 3.18 shows the logic diagram for this equation.

![FIGURE 3.18](image)

**FIGURE 3.18**
Example 3.7
SOP Form of XOR Function

**XNOR:** The product terms for this function are: \(A \overline{B}\) and \(AB\). The SOP form of the XNOR function is \(A \oplus B = \overline{A} \overline{B} + AB\). The logic diagram in Figure 3.19 represents the XNOR function.

![FIGURE 3.19](image)

**FIGURE 3.19**
Example 3.7
SOP Form of XNOR Function

We can also find the Boolean function of a truth table in **product-of-sums** (POS) form. The product-of-sums form of a Boolean expression consists of a number of **max-terms** (i.e., **sum terms** (OR terms) containing all variables in true or complement form) that are ANDed together. To find the POS form of \(Y\), we will find the SOP expression for \(\overline{Y}\) and apply DeMorgan’s theorems.

Recall DeMorgan’s theorems:

\[
\begin{align*}
\overline{x + y + z} &= \overline{x} \overline{y} \overline{z} \\
\overline{xyz} &= \overline{x} + \overline{y} + \overline{z}
\end{align*}
\]

When the theorems were introduced, they were presented as two-variable theorems, but in fact they are valid for any number of variables.
Let’s reexamine Table 3.4. To find the sum-of-products expression for \( Y \), we wrote a minterm for each line where \( Y = 1 \). To find the SOP expression for \( Y \), we must write a minterm for each line where \( Y = 0 \). Variables \( A, B, \) and \( C \) must appear in each minterm, in true or complement form. A variable is in complement form (with a bar over the top) if its value is \( 0 \) in that minterm, and it is in true form (no bar) if its value is \( 1 \).

We get the following minterms for \( Y \):

\[
\begin{align*}
\bar{A} & \quad \bar{B} & \quad \bar{C} \\
\bar{A} & \quad \bar{B} & \quad C \\
A & \quad \bar{B} & \quad C \\
A & \quad B & \quad \bar{C} \\
A & \quad B & \quad C \\
\end{align*}
\]

Thus, the SOP form of \( \bar{Y} \) is

\[
\bar{Y} = \bar{A} \bar{B} C + A \bar{B} \bar{C} + \bar{A} B C + A B \bar{C} + A B C
\]

To get \( Y \) in POS form, we must invert both sides of the above expression and apply De-Morgan’s theorems to the righthand side.

\[
Y = \bar{Y} = \bar{A} \bar{B} C + A \bar{B} \bar{C} + \bar{A} B C + A B \bar{C} + A B C
\]

This Boolean expression can be implemented by the logic circuit in Figure 3.20.

We don’t have to go through the whole process outlined above every time we want to find the POS form of a function. We can find it directly from the truth table, following the procedure summarized below. Use this procedure to find the POS form of the expression given by Table 3.4. The terms in this expression are the same as those derived by DeMorgan’s theorem.
3.2 • Sum-of-Products and Product-of-Sums Forms

Table 3.7 Truth Table for Example 3.8 (with minterms and maxterms)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Y</th>
<th>Minterms</th>
<th>Maxterms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \overline{A} \overline{B} \overline{C} \overline{D} )</td>
<td>( A + B + \overline{C} + \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( \overline{A} \overline{B} \overline{C} )</td>
<td>( A + B + C + \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( A \overline{B} \overline{C} \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( \overline{A} \overline{B} \overline{C} \overline{D} )</td>
<td>( A \overline{B} + \overline{C} + \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \overline{A} \overline{B} \overline{C} \overline{D} )</td>
<td>( A + \overline{B} + C + \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( A \overline{B} \overline{C} \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( A \overline{B} \overline{C} \overline{D} )</td>
<td>( A \overline{B} + \overline{C} + \overline{D} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<td>( A \overline{B} \overline{C} \overline{D} )</td>
<td>( \overline{A} + B + C + \overline{D} )</td>
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<td>0</td>
<td>( \overline{A} + B + C + \overline{D} )</td>
<td>( \overline{A} + B + C + \overline{D} )</td>
</tr>
</tbody>
</table>

Solution All minterms (for SOP form) and maxterms (for POS form) are shown in the last two columns of Table 3.5.

Boolean Expressions:

**SOP form:**
\[
Y = \overline{A} \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C} D + \overline{A} B \overline{C} \overline{D} + A \overline{B} \overline{C} \overline{D} + A \overline{B} C \overline{D} + A B \overline{C} \overline{D} + A B C \overline{D} + A B C D
\]

**POS form:**
\[
Y = (A + B + \overline{C} + \overline{D})(A + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + \overline{C} + D)
\]
\[
\overline{(A + B + C + D)}
\]

The logic circuits are shown in Figures 3.21 and 3.22.
FIGURE 3.21
Example 3.8
SOP Form

FIGURE 3.22
Example 3.8
POS Form
3.2 Review Problem

3.3 Find the SOP and POS forms of the Boolean functions represented by the following truth tables.

<table>
<thead>
<tr>
<th>a.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
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<tr>
<td>0</td>
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<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b.</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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</tr>
</tbody>
</table>

3.3 Theorems of Boolean Algebra

The main reason to learn Boolean algebra is to learn how to minimize the number of logic gates in a network. Boolean expressions with many terms, such as those represented by the logic diagrams in Figures 3.21 and 3.22, are seldom in their simplest form. It is often possible to apply some techniques of Boolean algebra to derive a simpler form of expression that requires fewer gates to implement.

For example, the logic circuit in Figure 3.21 requires eight 4-input AND gates and an 8-input OR gate. Using Boolean algebra, we can reduce its Boolean expression to

\[ Y = AD + AB + AC + AD + ABC. \]

This form can be implemented with 4 AND gates and a 4-input OR. You will use a simplification technique for this example in an end-of-chapter problem. In the meantime, let us examine some basic rules of Boolean algebra.

**Commutative, Associative, and Distributive Properties**

**Commutative property**  A mathematical operation is commutative if it can be applied to its operands in any order without affecting the result. For example, addition is commutative \((a + b = b + a)\), but subtraction is not \((a - b \neq b - a)\).

**Associative property**  A mathematical function is associative if its operands can be grouped in any order without affecting the result. For example, addition is associative \(((a + b) + c = a + (b + c))\), but subtraction is not \(((a - b) - c \neq a - (b - c))\).

**Distributive property**  Full name: distributive property of multiplication over addition. The property that allows us to distribute ("multiply through") an AND across several OR functions. For example, \(a(b + c) = ab + ac\).

AND and OR functions are both **commutative** and **associative**. The commutative property states that AND and OR operations are independent of input order. For inputs \(x\) and \(y\),

**Theorem 1:** \(xy = yx\)

and

**Theorem 2:** \(x + y = y + x\)

The associative property allows us to perform several two-input AND or OR functions in any order. In other words,

**Theorem 3:** \((xy)z = x(yz) = (xz)y\)

and

**Theorem 4:** \((x + y) + z = x + (y + z) = (x + z) + y\)
The **distributive** property allows us to “multiply through” an AND function across several OR functions. For example,

**Theorem 5:** \( x(y + z) = xy + xz \)

and

**Theorem 6:** \((x + y)(w + z) = xw + xz + yw + yz\)

Figure 3.23 shows the logic gate equivalents of these theorems.

---

**FIGURE 3.23**
Distributive Properties

---

**EXAMPLE 3.9**
Find the Boolean expression of the POS circuit in Figure 3.24a. Apply the distributive property to transform the circuit to an SOP form.

---

**FIGURE 3.24**
Example 3.9
Distributive Property

---

a. POS form

b. SOP form
Solution  The Boolean expression for Figure 3.24a is $Y = (A + B)(C + D)$. Using the distributive property, we get the expression $Y = AC + BC + AD + BD$. The logic diagram for this expression is shown in Figure 3.24b.

In Example 3.9, we see that the distributive property can be used to convert a POS circuit to SOP or vice versa. In this case, the circuit was not simplified, just transformed.

EXAMPLE 3.10

Write the Boolean expression for the circuit in Figure 3.25a. Use the distributive property to convert this to an SOP circuit.

Solution  The Boolean expression for Figure 3.25a is $AB(C + D)(E + F)$. The distributive property can be applied in two stages:

$$Y = (ABC + ABD)(E + F)$$
$$= ABCE + ABCF + ABDE + ABDF$$

The logic diagram for this equation is shown in Figure 3.25b. This results in a network that is “wider” (more gates on one level), but also “flatter” (fewer levels). The advantage of the second circuit is that signals would pass through the network faster, since it has fewer levels of gating.

Single-Variable Theorems

There are thirteen theorems that can be used to manipulate a single variable in a Boolean expression. An easy way to remember these theorems is to divide them into three groups:

1. Six theorems: $x$ AND/OR/XOR 0/1
2. Six theorems: $x$ AND/OR/XOR $x/\bar{x}$
3. One theorem: Double Inversion
The theorems in the first group can be generated by asking what happens when \( x \), a Boolean variable or expression, is at one input of an AND, an OR, or an XOR gate and a 0 or a 1 is at the other.

Examine the truth table of the gate in question. Hold one input of the gate constant and find the effect of the other on the output. This is the same procedure we used in Chapter 2 to examine the enable/inhibit properties of logic gates.

Each of these six theorems can be represented by a logic gate, as shown in Figure 3.26.

**FIGURE 3.26**
X AND/OR/XOR 0/1

<table>
<thead>
<tr>
<th></th>
<th>( x )</th>
<th>( y )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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</table>

**Theorem 7:** \( x \cdot 0 = 0 \)

For any Boolean variable, if \( x = 0 \), the output \( Y \) is also 0.

**Theorem 8:** \( x + 0 = x \)

For any Boolean variable, if \( x = 1 \), the output \( Y \) is also 1.
3.3 • Theorems of Boolean Algebra

If \( x = 0, Y = 0 \)
If \( x = 1, Y = 1 \)
(XOR acts as a noninverting buffer.)

**Theorem 9:** \( x \oplus 0 = x \)

\( x \cdot 1: \)

\[
\begin{array}{c|cc}
A & x & Y \\
\hline
0 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

If \( x = 0, Y = 0 \)
If \( x = 1, Y = 1 \)
(HIGH input enables AND gate.)

**Theorem 10:** \( x \cdot 1 = x \)

\( x + 1: \)

\[
\begin{array}{c|cc}
A & x & Y \\
\hline
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

If \( x = 0, Y = 1 \)
If \( x = 1, Y = 1 \)
(One input always HIGH, therefore output is always HIGH.)

**Theorem 11:** \( x + 1 = 1 \)

\( x \oplus 1: \)

\[
\begin{array}{c|cc}
A & x & Y \\
\hline
0 & 0 & 0 \\
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
\end{array}
\]

If \( x = 0, Y = 1 \)
If \( x = 1, Y = 0 \)
(XOR acts as an inverting buffer.)

**Theorem 12** \( x \oplus 1 = \overline{x} \)

\( x \text{ AND/OR/XOR } x/\overline{x} \)

Six theorems are generated by combining a Boolean variable or expression, \( x \), with itself or its complement in an AND, an OR, or an XOR function.

Again, we can use the AND, OR, and XOR truth tables. For the first three theorems, we look only at the lines where both inputs are the same. For the other three, we use the lines where the inputs are different.
Figure 3.27 shows the logic gates that represent these theorems.

**Theorem 13:** \( x \cdot x = x \)

**Theorem 14:** \( x + x = x \)

**Theorem 15:** \( x \oplus x = 0 \)
\[ x \cdot \overline{x} = 0 \]

\[ x + x = 1 \]

\[ x \oplus \overline{x} = 1 \]

**Theorem 16:** \( x \cdot \overline{x} = 0 \)

**Theorem 17:** \( x + \overline{x} = 1 \)

**Theorem 18:** \( x \oplus \overline{x} = 1 \)

**Double Inversion**

The final single-variable theorem is just common sense. It states that a variable or expression inverted twice is the same as the original variable or expression. It is given by:

**Theorem 19:** \( \overline{\overline{x}} = x \)

This theorem is illustrated by the two inverters in Figure 3.28.

**Multivariable Theorems**

There are numerous multivariable theorems we could learn, but we will look only at five of the most useful.
DeMorgan’s Theorems
We have already seen DeMorgan’s theorems. We will list them again, but will not comment further on them at this time.

**Theorem 20:** \( xy = x + y \)

**Theorem 21:** \( x + y = xy \)

**Other Multivariable Theorems**

**Theorem 22:** \( x + xy = x \)

**Proof:**

\[
x + xy = x (1 + y) \quad \text{(Distributive property)} \\
= x \cdot 1 \quad \text{(1 + y = 1; Theorem)} \\
= x
\]

Figure 3.29 illustrates the circuit in this theorem. Note that the equivalent is not a circuit at all, but a single, unmodified variable. Thus, the circuit shown need never be built.

**FIGURE 3.29**

Theorem 22

---

**EXAMPLE 3.11**

Simplify the following Boolean expressions, using Theorem 22 and other rules of Boolean algebra. Draw the logic circuits of the unsimplified and simplified expressions.

a. \( H = KL + K \)

b. \( Y = (A + B)CD + (A + B) \)

c. \( W = (PQR + \overline{P} \overline{Q})(S + T) + (P + \overline{Q})(S + T) + (S + T) \)

**Solution** Figure 3.30 shows the logic circuits for the unsimplified and simplified versions of the above expressions.

a. Let \( x = K \), let \( y = L \):

\[
H = x + xy = K + KL
\]

Theorem 22 states \( x + xy = x \). Therefore \( K + KL = K \).

b. Let \( x = (A + B) \), let \( y = CD \):

\[
Y = x + xy = x = A + B
\]

c. Let \( x = S + T \), let \( y = (\overline{P} + \overline{Q}) \):

\[
W = (PQR + \overline{P} \overline{Q})(S + T) + (S + T) = (S + T)
\]

Since \( x + xy = x \), \( (\overline{P} + \overline{Q})(S + T) + (S + T) = (S + T) \).

\[
W = (PQR + \overline{P} \overline{Q})(S + T) + (S + T)
\]

Let \( x = S + T \), let \( y = (PQR + \overline{P} \overline{Q}) \):

\[
W = x + xy = x = S + T
\]

**Alternate method:**

\[
W = (PQR + \overline{P} \overline{Q})(S + T) + (\overline{P} + \overline{Q})(S + T) + (S + T)
\]

By the distributive property:

\[
W = ((PQR + \overline{P} \overline{Q}) + (\overline{P} + \overline{Q}))(S + T) + (S + T)
\]

Let \( x = S + T \), let \( y = ((PQR + \overline{P} \overline{Q}) + (\overline{P} + \overline{Q})) \):

\[
W = x + xy = x = S + T
\]
Example 3.11
Logic Circuits for Un simplified and Simplified Expressions
Theorem 23: \( (x + y)(x + z) = x + yz \)

Proof: \( (x + y)(x + z) = xx + xz + xy + yz \) (Distributive property)
\[ = x + xy + xz + yz \] (Associative property)
\[ = x + xz + yz \] (\( xx = x \) (Theorem 22))
\[ = (x + xz) + yz \] (Associative property)
\[ = x + yz \] (Theorem 22)

Figure 3.31 shows the logic circuits for the left and right sides of the equation for Theorem 23. This theorem is a special case of one of the distributive properties, Theorem 6, where \( w = x \).

**EXAMPLE 3.12**

Simplify the following Boolean expressions, using Theorem 23 and other rules of Boolean algebra. Draw the logic circuits of the unsimplified and simplified expressions.

a. \( L = (M + \bar{N})(M + \bar{P}) \)
b. \( Y = (A + B + AB)(A + B + C) \)

**Solution**

Figure 3.32 shows the logic circuits for the unsimplified and simplified versions of the above expressions.

**FIGURE 3.31**
Theorem 23

**FIGURE 3.32**
Example 3.12
Logic Circuits for Unsimplified and Simplified Expressions
3.3 • Theorems of Boolean Algebra

Theorem 23: \((x + y)(x + z) = x + yz\)

a. Let \(x = M\), let \(y = \overline{N}\), let \(z = \overline{P}\):
\[
L = (x + y)(x + z) = x + yz = M + \overline{N}\overline{P}
\]
b. Let \(x = A + B\), let \(y = AB\), let \(z = C\):
\[
Y = (x + y)(x + z) = x + yz = \overline{A + B} + ABC = \overline{A}\overline{B} + ABC
\]

Theorem 24: \(x + \overline{xy} = x + y\)

Proof: Since \((x + y)(x + z) = x + yz\), then for \(y = \overline{x}\):
\[
x + \overline{xy} = (x + \overline{x})(x + y) = 1 \cdot (x + y) = x + y
\]

Figure 3.33 illustrates Theorem 24 with a logic circuit.

\[\text{FIGURE 3.33} \]

Theorem 24

\[\text{NOTE}\]

Here is another way to remember Theorem 24:
If a variable \((x)\) is ORed with a term consisting of a different variable \((y)\) AND the first variable’s complement \((\overline{x})\), the complement disappears.
\[
x + \overline{xy} = x + y
\]

**EXAMPLE 3.13**

Simplify the following Boolean expressions, using Theorem 24 and other rules of Boolean algebra. Draw the logic circuits of the unsimplified and simplified forms of the expressions.

a. \(W = \overline{U} + U\overline{V}\)

b. \(P = \overline{QR}S + (\overline{Q} + R + \overline{S})T\)
\[
J = KM(\overline{K} + L + M) + KM
\]

**Solution**

Figure 3.34 shows the circuits for the unsimplified and simplified expressions.

Theorem 24: \(x + \overline{xy} = x + y\)

a. Let \(x = \overline{U}\), let \(y = \overline{V}\):
\[
W = x + \overline{xy} = x + y = \overline{U} + \overline{V}
\]
b. \(P = \overline{QRS} + (\overline{Q} + R + \overline{S})T\)
\[
P = \overline{QRS} + \overline{Q}RS T \quad \text{(DeMorgan’s theorem)}
\]
Let \(x = \overline{QRS}\), let \(y = T\):
\[
P = x + \overline{xy} = x + y = \overline{QRS} + T
\]
c. Let \( x = KM \), let \( y = (\overline{K} + L + M) \):

\[
J = x + \overline{xy} = x + y = KM + \overline{K} + L + M
\]

\[
= \overline{K} + L + (M + KM) \quad \text{(Associative property)}
\]

\[
= \overline{K} + L + M \quad \text{(Theorem 22)}
\]

The rules of Boolean algebra are summarized in Table 3.8. Don’t try to memorize all these rules. The commutative, associative, and distributive properties are the same...
as their counterparts in ordinary algebra. The single-variable theorems can be reasoned out by your knowledge of logic gate operation. That leaves only five multivariable theorems.

<table>
<thead>
<tr>
<th>Table 3.8</th>
<th>Theorems of Boolean Algebra</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commutative Properties</strong></td>
<td></td>
</tr>
<tr>
<td>1. ( x + y = y + x )</td>
<td></td>
</tr>
<tr>
<td>2. ( x \cdot y = y \cdot x )</td>
<td></td>
</tr>
<tr>
<td><strong>Associative Properties</strong></td>
<td></td>
</tr>
<tr>
<td>3. ( x + (y + z) = (x + y) + z )</td>
<td></td>
</tr>
<tr>
<td>4. ( x(yz) = (xy)z )</td>
<td></td>
</tr>
<tr>
<td><strong>Distributive Properties</strong></td>
<td></td>
</tr>
<tr>
<td>5. ( x(y + z) = xy + xz )</td>
<td></td>
</tr>
<tr>
<td>6. ( (x + y)(w + z) = xw + xz + yw + yz )</td>
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<tr>
<td><strong>x AND/OR/XOR 0/1</strong></td>
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<tr>
<td>7. ( x \cdot 0 = 0 )</td>
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<tr>
<td>8. ( x + 0 = x )</td>
<td></td>
</tr>
<tr>
<td>9. ( x \oplus 0 = x )</td>
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<tr>
<td>10. ( x \cdot 1 = x )</td>
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<tr>
<td>11. ( x + 1 = 1 )</td>
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<tr>
<td>12. ( x \oplus 1 = \overline{x} )</td>
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<tr>
<td><strong>x AND/OR/XOR x/\overline{x}</strong></td>
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</tr>
<tr>
<td>13. ( x \cdot x = x )</td>
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<tr>
<td>14. ( x + x = x )</td>
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<tr>
<td>15. ( x \oplus x = 0 )</td>
<td></td>
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<tr>
<td>16. ( x \cdot \overline{x} = 0 )</td>
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<tr>
<td>17. ( x + \overline{x} = 1 )</td>
<td></td>
</tr>
<tr>
<td>18. ( x \oplus \overline{x} = 1 )</td>
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</tr>
<tr>
<td><strong>Double Inversion</strong></td>
<td></td>
</tr>
<tr>
<td>19. ( \overline{\overline{x}} = x )</td>
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</tr>
<tr>
<td><strong>DeMorgan’s Theorems</strong></td>
<td></td>
</tr>
<tr>
<td>20. ( \overline{xy} = \overline{x} + \overline{y} )</td>
<td></td>
</tr>
<tr>
<td>21. ( x + y = \overline{x\overline{y}} )</td>
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</tr>
<tr>
<td><strong>Other Multivariable Theorems</strong></td>
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<tr>
<td>22. ( x + xy = x )</td>
<td></td>
</tr>
<tr>
<td>23. ( (x + y)(z + x) = x + yz )</td>
<td></td>
</tr>
<tr>
<td>24. ( x + xy = x + y )</td>
<td></td>
</tr>
</tbody>
</table>

### SECTION 3.3 REVIEW PROBLEMS

3.4 Use theorems of Boolean algebra to simplify the following Boolean expressions.

a. \( Y = \overline{AC} + (\overline{A} + \overline{C})D \)

b. \( Y = \overline{A} + \overline{C} + ACD \)

c. \( Y = (\overline{AB} + \overline{BC})(\overline{AB} + \overline{C}) \)
3.4 Simplifying SOP and POS Expressions

**KEY TERMS**

**Maximum SOP simplification**  The form of an SOP Boolean expression that cannot be further simplified by canceling variables in the product terms. It may be possible to get a POS form of the expression with fewer terms or variables.

**Maximum POS simplification**  The form of a POS Boolean expression that cannot be further simplified by canceling variables in the sum terms. It may be possible to get an SOP form of the expression with fewer terms or variables.

Earlier in this chapter, we discovered that we can generate a Boolean equation from a truth table and express it in sum-of-products (SOP) or product-of-sums (POS) form. From this equation, we can develop a logic circuit diagram. The next step in the design or analysis of a circuit is to simplify its Boolean expression as much as possible, with the ultimate aim of producing a circuit that has fewer physical components than the unsimplified circuit.

In Section 3.2, we found the SOP and POS forms of the Boolean expression represented by Table 3.9. These forms yield the logic diagrams shown in Figures 3.17 and 3.20. For convenience, the circuits are illustrated again in Figure 3.35. The corresponding algebraic expressions can be simplified by the rules of Boolean algebra to give us a simpler circuit in each case.

**Table 3.9**  Truth Table for the SOP and POS Networks in Figure 3.35

<table>
<thead>
<tr>
<th>A</th>
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</table>

**FIGURE 3.35**  Unsimplified SOP and POS Networks

The sum-of-products and product-of-sums expressions represented by Table 3.9 are:

\[ Y = \overline{A} \overline{B} \overline{C} + \overline{A} B C + \overline{A} B \overline{C} \quad \text{(SOP)} \]

and

\[ Y = (A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + C)(\overline{A} + B + C)(\overline{A} + B + C) \quad \text{(POS)} \]
3.4 • Simplifying SOP and POS Expressions

The SOP form is fairly easy to simplify:

\[ Y = \overline{A} \overline{B} \overline{C} + \overline{A} B C + A \overline{B} C \]

\[ = (A + A) \overline{B} \overline{C} + A B C \quad \text{(Distributive property)} \]

\[ = 1 \cdot \overline{B} \overline{C} + A B C \quad (x + \overline{x} = 1) \]

\[ = \overline{B} \overline{C} + A B C \quad (x \cdot 1 = x) \]

Since we cannot cancel any more SOP terms, we can call this final form the maximum SOP simplification. The logic diagram for the simplified expression is shown in Figure 3.36.

**FIGURE 3.36**
Simplified SOP Circuit

Two terms in an SOP expression can be reduced to one if they are identical except for one variable that is in true form in one term and complement form in the other. Such a grouping of a variable and its complement always cancels.

\[ x\overline{y} \overline{z} + xy z = xy(\overline{z} + z) = xy \]

There is a similar procedure for the POS form. Examine the following expression:

\[ Y = (A + B + \overline{C})(A + B + C) \]

Recall Theorem 23: \((x + y)(x + z) = x + yz\).

Let \( x = A + B \), let \( y = C \), let \( z = C \).

\[ Y = (A + B) + \overline{C} C \quad \text{(Theorem 23)} \]

\[ = (A + B) + 0 \quad (x\overline{x} = 0) \]

\[ = (A + B) \quad (x + 0 = x) \]

**NOTE**
A POS expression can be simplified by grouping two terms that are identical except for one variable that is in true form in one term and complement form in the other.

\[ (x + y + \overline{z})(x + y + z) = (x + y) + \overline{z} z = x + y \]

Let us use this procedure to simplify the POS form of the previous Boolean expression, shown again below with the terms numbered for our reference. The numbered value of each term corresponds to the binary value of the line in the truth table from which it is derived.

\[
\begin{align*}
(1) & \quad (2) & \quad (5) & \quad (6) & \quad (7) \\
Y &= (A + B + \overline{C})(A + \overline{B} + C)(A + B + \overline{C})(A + B + C)(A + \overline{B} + \overline{C})
\end{align*}
\]

There can be more than one way to simplify an expression. The following grouping of the numbered POS terms is one possibility.
Combining the above terms, we get the expression:

\[ Y = (B + C)(B + C)(A + B) \]

Figure 3.37 shows the logic diagram for this expression. Compare this logic diagram and that of Figure 3.36 with the unsimplified circuits of Figure 3.35. Since there are no more cancellations of POS terms possible, we can call this the maximum POS simplification. We can, however, apply other rules of Boolean algebra and simplify further.

\[ Y = (B + C)(B + C)(A + B) \]

Let’s simplify it further:

\[ Y = (B + C)(B + C)(A + B) \]

\[ = (B + C)(B + C)(A + B) \]

\[ = B + C + A \]

\[ (x \cdot x = 0) \]

This is the same result we got when we simplified the SOP form of the expression.

To be sure you are getting the maximum SOP or POS simplification, you should be aware of the following guidelines:

1. Each term must be grouped with another, if possible.
2. When attempting to group all terms, it is permissible to group a term more than once, such as term (6) above. The theorems \( x \cdot x = x \) (POS forms) and \( x + x = x \) (SOP forms) imply that using a term more than once does not change the Boolean expression.
3. Each pair of terms should have at least one term that appears only in that pair. Otherwise, you will have redundant terms that will need to be canceled later. For example, another possible group in the POS simplification above is terms (5) and (7). But since both these terms are in other groups, this pair is unnecessary and would yield a term you would have to cancel.

EXAMPLE 3.14 Find the maximum SOP simplification for the Boolean function represented by Table 3.10. Draw the logic diagram for the simplified expression.

Solution SOP form:

\[ Y = A \overline{B} \overline{C} D + A \overline{B} \overline{C} D + A \overline{B} \overline{C} D + A \overline{B} \overline{C} D + A \overline{B} \overline{C} D + A \overline{B} \overline{C} D \]
Group the terms as follows:

(8) + (9): \( A \overline{B} \overline{C} \overline{D} + A \overline{B} \overline{C} D = A \overline{B} \overline{C} \)

(10) + (11): \( A \overline{B} \overline{C} \overline{D} + A \overline{B} C D = A \overline{B} C \)

(12) + (14): \( A \overline{B} \overline{C} \overline{D} + A B C \overline{D} = A B \overline{D} \)

Combine the simplified groups and apply techniques of Boolean algebra to simplify further:

\[
Y = A \overline{B} \overline{C} + A \overline{B} C + A B \overline{D} \\
= A \overline{B}(\overline{C} + C) + A B \overline{D} \\
= A \overline{B} + A B \overline{D} \\
= A(\overline{B} + B \overline{D}) \quad \text{Distributive property} \\
= A(\overline{B} + \overline{D}) \quad \text{Theorem 24: } x + \overline{x}y = x + y \\
= A \overline{B} + A \overline{D} 
\]

Figure 3.38 Shows the logic diagram of the simplified expression.
3.5 Simplification by the Karnaugh Map Method

**KEY TERMS**

- **Karnaugh map** A graphical tool for finding the maximum SOP or POS simplification of a Boolean expression. A Karnaugh map works by arranging the terms of an expression in such a way that variables can be canceled by grouping minterms or maxterms.

- **Cell** The smallest unit of a Karnaugh map, corresponding to one line of a truth table. The input variables are the cell’s coordinates, and the output variable is the cell’s contents.

- **Adjacent cell** Two cells are adjacent if there is only one variable that is different between the coordinates of the two cells. For example, the cells for minterms $ABC$ and $AB'C$ are adjacent.

- **Pair** A group of two adjacent cells in a Karnaugh map. A pair cancels one variable in a K-map simplification.

- **Quad** A group of four adjacent cells in a Karnaugh map. A quad cancels two variables in a K-map simplification.

- **Octet** A group of eight adjacent cells in a Karnaugh map. An octet cancels three variables in a K-map simplification.

In Example 3.14, we derived a sum-of-products Boolean expression from a truth table and simplified the expression by grouping minterms that differed by one variable. We made this task easier by breaking up the truth table into groups of four lines. (It is difficult for the eye to grasp an overall pattern in a group of 16 lines.) We chose groups of four because variables $A$ and $B$ are the same in any one group and variables $C$ and $D$ repeat the same binary sequence in each group. This allows us to see more easily when we have terms differing by only one variable.

The Karnaugh map, or K-map, is a graphical tool for simplifying Boolean expressions that uses a similar idea. A K-map is a square or rectangle divided into smaller squares called cells, each of which represents a line in the truth table of the Boolean expression to be mapped. Thus, the number of cells in a K-map is always a power of 2, usually 4, 8, or 16. The coordinates of each cell are the input variables of the truth table. The cell content is the value of the output variable on that line of the truth table. Figure 3.39 shows the formats of Karnaugh maps for Boolean expressions having two, three, and four variables, respectively.

There are two equivalent ways of labeling the cell coordinates: numerically or by true and complement variables. We will use the numerical labeling since it is always the same, regardless of the chosen variables.

The cells in the Karnaugh maps are set up so that the coordinates of any two adjacent cells differ by only one variable. By grouping adjacent cells according to specified rules, we can simplify a Boolean expression by canceling variables in their true and complement forms, much as we did algebraically in the previous section.

**Two-Variable Map**

Table 3.12 shows the truth table of a two-variable Boolean expression.

The Karnaugh map shown in Figure 3.40 is another way of showing the same information as the truth table. Every line in the truth table corresponds to a cell, or square, in the Karnaugh map.

The coordinates of each cell correspond to a unique combination of input variables $(A, B)$. The content of the cell is the output value for that input combination. If the truth table output is 1 for a particular line, the content of the corresponding cell is also 1. If the output is 0, the cell content is 0.
The SOP expression of the truth table is

\[ Y = A \bar{B} + \bar{A}B \]

which can be simplified as follows:

\[ Y = \bar{A} (\bar{B} + B) \]

\[ = \bar{A} \]
We can perform the same simplification by grouping the adjacent pair of 1s in the Karnaugh map, as shown in Figure 3.41.

**NOTE**

When we circle a pair of 1s in a K-map, we are grouping the common variable in two minterms, then factoring out and canceling the complements.

To find the simplified form of the Boolean expression represented in the K-map, we examine the coordinates of all the cells in the circled group. We retain coordinate variables that are the same in all cells and eliminate coordinate variables that are different in different cells.

In this case:
- \( A \) is a coordinate of both cells of the circled pair. (Keep \( A \).)
- \( B \) is a coordinate of one cell of the circled pair, and \( B \) is a coordinate of the other. (Discard \( B \).)

\[ Y = A \]

### Three- and Four-Variable Maps

Refer to the forms of three- and four-variable Karnaugh maps shown in Figure 3.39. Each cell is specified by a unique combination of binary variables. This implies that the three-variable map has 8 cells (since \( 2^3 = 8 \)) and the four-variable map has 16 cells (since \( 2^4 = 16 \)).

The variables specifying the row (both maps) or the column (the four-variable map) do not progress in binary order; they advance such that there is only one change of variable per row or column. For example, the numbering of the rows is 00, 01, 11, 10, rather than the binary order 00, 01, 10, 11. If we were to use binary order, adjacent cells in rows 2 and 3 or 3 and 4 would differ by two variables, meaning we could not factor out and cancel a pair of complements by grouping these cells. For instance, we cannot cancel complementary variables from the pair \( A \overline{B} \overline{C} + \overline{A} B \overline{C} \), which differs by two variables.

**NOTE**

The number of cells in a group must be a power of 2, such as 1, 2, 4, 8, or 16.

A group of four adjacent cells is called a quad. Figure 3.42 shows a Karnaugh map for a Boolean function whose terms can be grouped in a quad. The Boolean expression displayed in the K-map is:

\[ Y = \overline{A \overline{B} C} + \overline{A} B \overline{C} + A B C + A \overline{B} \overline{C} \]

\( A \) and \( B \) are both part of the quad coordinates in true and complement form. (Discard \( A \) and \( B \)).

\( C \) is a coordinate of each cell in the quad. (Keep \( C \)).

\[ Y = C \]

Grouping cells in a quad is equivalent to factoring two complementary pairs of variables and canceling them.

\[ Y = (A + \overline{A})(B + \overline{B})C = C \]

You can verify that this is the same as the original expression by multiplying out the terms.

An octet is a group of eight adjacent cells. Figure 3.43 shows the Karnaugh map for the following Boolean expression:
Simplification by the Karnaugh Map Method

Variables A, C, and D are all coordinates of the octet cells in true and complement form. (Discard A, C, and D.)

B is a coordinate of each cell. (Keep B.)

\[ Y = B \]

The algebraic equivalent of this octet is an expression where three complementary variables are factored out and canceled.

\[ Y = (A + \overline{A})B(C + \overline{C})(D + \overline{D}) = B \]

### NOTE

A Karnaugh map completely filled with 1s implies that all input conditions yield an output of 1. For a Boolean expression \( Y, Y = 1 \).

#### Grouping Cells Along Outside Edges

The cells along an outside edge of a three- or four-variable map are adjacent to cells along the opposite edge (only one change of variable). Thus we can group cells “around the outside” of the map to cancel variables. In the case of the four-variable map, we can also group the four corner cells as a quad, since they are all adjacent to one another.

### EXAMPLE 3.15

Use Karnaugh maps to simplify the following Boolean expressions:

a. \( Y = \overline{A} B \overline{C} \overline{D} + \overline{A} B C \overline{D} + \overline{A} B \overline{C} D + \overline{A} B C D \)

b. \( Y = A B \overline{C} \overline{D} + A B C \overline{D} + A B \overline{C} D + A B C D \)

### Solutions

Figure 3.44 shows the Karnaugh maps for the Boolean expressions labeled a and b. Cells in each map are grouped in a quad.

#### FIGURE 3.44

Example 3.15

K-Maps

\[ a. \quad A \text{ and } C \text{ are both coordinates of two cells in true form and two cells in complement form. (Discard } A \text{ and } C.\]

\[ B \text{ is a coordinate of each cell. (Keep } B.\]

\[ Y = \overline{B} \]

\[ b. \quad A \text{ and } C \text{ are both coordinates of two cells in true form and two cells in complement form. (Discard } A \text{ and } C.\]

\[ \overline{B} \text{ and } D \text{ are coordinates of each cell. (Keep } \overline{B} \text{ and } \overline{D}.\]

\[ Y = \overline{B} \overline{D} \]
Loading a K-Map From a Truth Table

NOTE
We don’t need a Boolean expression to fill a Karnaugh map if we have the function’s truth table.

Figures 3.45 and 3.46 show truth table and Karnaugh map forms for three- and four-variable Boolean expressions. The numbers in parentheses show the order of terms in binary sequence for both forms.

The Karnaugh map is not laid out in the same order as the truth table. That is, it is not laid out in a binary sequence. This is due to the criterion for cell adjacency: no more than one variable change between rows or columns is permitted.

**FIGURE 3.45**
Order of Terms (Three-Variable Function)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
</table>
| 0 | 0 | 0 | (0)
| 0 | 0 | 1 | (1)
| 0 | 1 | 0 | (2)
| 0 | 1 | 1 | (3)
| 1 | 0 | 0 | (4)
| 1 | 0 | 1 | (5)
| 1 | 1 | 0 | (6)
| 1 | 1 | 1 | (7)

**FIGURE 3.46**
Order of Terms (Four-Variable Function)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Y</th>
</tr>
</thead>
</table>
| 0 | 0 | 0 | 0 | (0)
| 0 | 0 | 0 | 1 | (1)
| 0 | 0 | 1 | 0 | (2)
| 0 | 0 | 1 | 1 | (3)
| 0 | 1 | 0 | 0 | (4)
| 0 | 1 | 0 | 1 | (5)
| 0 | 1 | 1 | 0 | (6)
| 0 | 1 | 1 | 1 | (7)

Filling in a Karnaugh map from a truth table is easy when you understand a system for doing it quickly. For the three-variable map, fill row 1, then row 2, skip to row 4, then go back to row 3. By doing this, you trace through the cells in binary order. Use the mnemonic phrase “1, 2, skip, back” to help you remember this.

The system for the four-variable map is similar but must account for the columns as well. The rows get filled in the same order as the three-variable map, but within each row, fill column 1, then column 2, skip to column 4, then go back to column 3. Again, “1, 2, skip, back.”
The four-variable map is easier to fill from the truth table if we break up the truth table into groups of four lines, as we have done in Figure 3.46. Each group is one row in the Karnaugh map. Following this system will quickly fill the cells in binary order.

Go back and follow the order of terms on the four-variable map in Figure 3.46, using this system. (Remember, for both rows and columns, “1, 2, skip, back.”)

**Multiple Groups**

If there is more than one group of 1s in a K-map simplification, each group is a term in the maximum SOP simplification of the mapped Boolean expression. The resulting terms are ORed together.

---

**EXAMPLE 3.17**

Use the Karnaugh map method to simplify the Boolean function represented by Table 3.13.

### Table 3.13  Truth Table for Example 4.3

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Y</th>
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<tbody>
<tr>
<td>0</td>
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</table>

### Solution

Figure 3.47 shows the Karnaugh map for the truth table in Table 3.14. There are two groups of 1s—a pair and a quad.

**Pair:**

- Variables $\overline{A}$, $\overline{B}$, and $\overline{D}$ are coordinates of both cells. (Keep $\overline{A} \overline{B} \overline{D}$.) $C$ is a coordinate of one cell and $\overline{C}$ is a coordinate of the other. (Discard $C$.)

**Term:** $\overline{A} \overline{B} \overline{D}$

**Quad:**

- Both $A$ and $C$ are coordinates of two cells in true form and two cells in complement form. (Discard $A$ and $C$.)
- $B$ and $D$ are coordinates of all four cells. (Keep $B \overline{D}$.)

**Term:** $B \overline{D}$

Combine the terms in an OR function:

$$Y = \overline{A} \overline{B} \overline{D} + B \overline{D}$$
Overlapping Groups

**NOTE**

A cell may be grouped more than once. The only condition is that every group must have at least one cell that does not belong to any other group. Otherwise, redundant terms will result.

---

### Example 3.18

Simplify the function represented by Table 3.14.

**Solution**  The Karnaugh map for the function in Table 3.14 is shown in Figure 3.48, with two different groupings of terms.

**Figure 3.48**

Example 3.18

K-Maps

**Table 3.14 Truth Table for Example 3.18**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

a. The simplified Boolean expression drawn from the first map has three terms.

\[
Y = \overline{A} \overline{B} + A \overline{B} + B \overline{C}
\]

b. The second map yields an expression with four terms.

\[
Y = \overline{A} \overline{B} + A \overline{B} + B \overline{C} + A \overline{C}
\]

One of the last two terms is redundant, since neither of the pairs corresponding to these terms has a cell belonging only to that pair. We could retain either pair of cells and its corresponding term, but not both.

We can show algebraically that the last term is redundant and thus make the expression the same as that in part a.

\[
Y = \overline{A} \overline{B} + A \overline{B} + B \overline{C} + A \overline{C}
\]

\[
\begin{align*}
&= \overline{A} \overline{B} + A \overline{B} + B \overline{C} + A (B + \overline{B}) C \\
&= \overline{A} \overline{B} + A \overline{B} + B \overline{C} + \overline{A} B C + \overline{A} \overline{B} C \\
&= \overline{A} \overline{B} (1 + C) + A \overline{B} + B \overline{C} (1 + \overline{A}) \\
&= \overline{A} \overline{B} + A \overline{B} + B \overline{C}
\end{align*}
\]
Conditions for Maximum Simplification

**NOTE**
The maximum simplification of a Boolean expression is achieved only if the circled groups of cells in its K-map are as large as possible and there are as few groups as possible.

**EXAMPLE 3.19**
Find the maximum SOP simplification of the Boolean function represented by Table 3.15.

**Solution** The values of Table 3.15 are loaded into the three K-maps shown in Figure 3.49. Three different ways of grouping adjacent cells are shown. One results in maximum simplification; the other two do not.

![K-Maps](image)

**Table 3.15** Truth Table for Example 3.19

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>Y</th>
</tr>
</thead>
<tbody>
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We get the maximum SOP simplification by grouping the two octets shown in Figure 3.49a. The resulting expression is

a. \( Y = \overline{B} + D \)

Figures 3.49b and c show two simplifications that are less than the maximum because the chosen cell groups are smaller than they could be. The resulting expressions are:

b. \( Y = \overline{AB} + A \overline{B} + D \)

c. \( Y = B + D \)

Neither of these expressions is the simplest possible, since both can be reduced by Boolean algebra to the form in Figure 3.49a.
Using K-Maps for Partially Simplified Circuits

Figure 3.50 shows a logic diagram that can be further simplified. If we want to use a Karnaugh map for this process, we must do one of two things:

1. Fill in the K-map from the existing product terms. Each product term that is not a minterm will represent more than one cell in the Karnaugh map. When the map is filled, regroup the cells for maximum simplification.

2. Expand the sum-of-products expression of the circuit to get a sum-of-minterms form. Each minterm represents one cell in the K-map. Group the cells for maximum simplification.

Figure 3.51 shows the K-map derived from the existing circuit and the regrouped cells that yield the maximum simplification.

The algebraic method requires us to expand the existing Boolean expression to get a sum of minterms. The original expression is:

$$Y = \overline{A} \overline{B} \overline{C} \overline{D} + A \overline{B} \overline{D} + \overline{A} C$$

The theorem $(x + \overline{x}) = 1$ implies that we can AND a variable with a term in true and complement form without changing the term. The expanded expression is:

$$Y = \overline{A} B \overline{C} \overline{D} + A B (C + \overline{C}) \overline{D} + \overline{A} (B + \overline{B}) C (D + \overline{D})$$

$$= \overline{A} B \overline{C} \overline{D} + A B C \overline{D} + A B \overline{C} \overline{D}$$

$$+ A B C D + \overline{A} B C \overline{D} + A B C D + \overline{A} B C \overline{D}$$

The terms of this expression can be loaded into a K-map and simplified, as shown in Figure 3.51b. Figure 3.52 shows the logic diagram for the simplified expression.
3.5 • Simplification by the Karnaugh Map Method

FIGURE 3.52
Simplified Circuit

EXAMPLE 3.20
Use a Karnaugh map to find the maximum SOP simplification of the circuit shown in Figure 3.53.

FIGURE 3.53
Example 3.20
Circuit to Be Simplified

Solution Figure 3.54a shows the Karnaugh map of Figure 3.53 with terms grouped as shown in the original circuit. Figure 3.54b shows the terms regrouped for the maximum simplification, which is given by:

\[ Y = \overline{A} \overline{D} + \overline{B} \overline{D} + \overline{A} B \overline{C} \]

Alternate method: The Boolean expression for the circuit in Figure 3.53 is:

\[ Y = \overline{A} B \overline{C} + \overline{A} C D + \overline{B} \overline{C} D + A \overline{B} C D \]

FIGURE 3.54
Example 3.20
Maximum Simplification of Figure 3.53

a. K-map from Figure 4.14
b. Maximum simplification
This expands to the following expression:

\[
Y = \overline{A} \overline{B} \overline{C} (D + \overline{D}) + \overline{A} (B + \overline{B}) C D + (A + \overline{A}) \overline{B} \overline{C} D + A \overline{B} C D
\]

\[
= \overline{A} \overline{B} C D + \overline{A} B \overline{C} D + A \overline{B} C D + \overline{A} B C D + A \overline{B} C D
\]

This expression can be loaded directly into the K-map and simplified, as shown in Figure 3.54b. The logic diagram for the simplified expression is shown in Figure 3.55.

**Don’t Care States**

**KEY TERMS**

**Don’t care state**  
An output state that can be regarded as either HIGH or LOW, as is most convenient. A don’t care state is the output state of a circuit for a combination of inputs that will never occur.

Sometimes a digital circuit will be intended to work only for certain combinations of inputs; any other input values will never be applied to the circuit.

In such a case, it may be to our advantage to use so-called don’t care states to simplify the circuit. A don’t care state is shown in a K-map cell as an “X” and can be either a 0 or a 1, depending on which case will yield the maximum simplification.

A common application of the don’t care state is a digital circuit designed for binary-coded decimal (BCD) inputs. In BCD, a decimal digit (0–9) is encoded as a 4-bit binary number (0000–1001). This leaves six binary states that are never used (1010, 1011, 1100, 1101, 1110, 1111). In any circuit designed for BCD inputs, these states are don’t care states.

All cells containing 1s must be grouped if we are looking for a maximum SOP simplification. (If necessary, a group can contain one cell.) The don’t care states can be used to maximize the size of these groups. We need not group all don’t care states, only those that actually contribute to a maximum simplification.

**EXAMPLE 3.21**

The circuit in Figure 3.56 is designed to accept binary-coded decimal inputs. The output is HIGH when the input is the BCD equivalent of 5, 7, or 9. If the BCD equivalent of the input is not 5, 7 or 9, the output is LOW. The output is not defined for input values greater than 9.

Find the maximum SOP simplification of the circuit.
Solution The Karnaugh map for the circuit is shown in Figure 3.57a.

We can designate three of the don’t care cells as 1s—those corresponding to input states 1011, 1101, and 1111. This allows us to group the 1s into two overlapping quads, which yield the following simplification.

\[ Y = D_4 D_1 + D_3 D_1 \]

The ungrouped don’t care states are treated as 0s. The corresponding circuit is shown in Figure 3.57b.

---

**EXAMPLE 3.22**

Applications

One type of decimal code is called 2421 code, so called because of the positional weights of its bits. (For example, 1011 in 2421 code is equivalent to \(2 + 2 + 1 = 5\) in decimal. 1100 is equivalent to decimal \(2 + 4 = 6\).) Table 3.16 shows how this code compares to its equivalent decimal digits and to the BCD code used in Example 3.21.

2421 code is sometimes used because it is “self-complementing,” a property that BCD code does not have, but that is useful in digital decimal arithmetic circuits.

The bits of the BCD code are designated \(D_4 D_3 D_2 D_1\). The bits of the 2421 code are designated \(Y_4 Y_3 Y_2 Y_1\).

Use the Karnaugh map method to design a logic circuit that accepts any BCD input and generates an output in 2421 code, as specified by Table 3.16.

**Solution** The required circuit is called a code converter. Each 4-bit BCD input corresponds to a 4-bit 2421 output. Thus, we must find four Boolean expressions, one for each

**Table 3.16** BCD and 2421 Code

<table>
<thead>
<tr>
<th>Decimal Equivalent</th>
<th>BCD Code (D_4 D_3 D_2 D_1)</th>
<th>2421 Code (Y_4 Y_3 Y_2 Y_1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0</td>
<td>0 0 0 0</td>
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<tr>
<td>1</td>
<td>0 0 0 1</td>
<td>0 0 0 1</td>
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<td>0 0 1 0</td>
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<td>3</td>
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<td>4</td>
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<td>5</td>
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<td>1 0 1 1</td>
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<td>6</td>
<td>0 1 1 0</td>
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<td>7</td>
<td>0 1 1 1</td>
<td>1 1 0 1</td>
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<td>8</td>
<td>1 0 0 0</td>
<td>1 1 1 0</td>
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<tr>
<td>9</td>
<td>1 0 0 1</td>
<td>1 1 1 1</td>
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</table>
FIGURE 3.58
Example 3.22
K-Maps: BCD to 2421

\[
\begin{array}{c|cccc}
D_3 & D_2 & D_1 & D_0 & \text{Output} \\
\hline
00 & 0 & 0 & 0 & 0 \\
01 & 1 & 0 & 1 & 1 \\
11 & 1 & 1 & 0 & 0 \\
10 & 1 & 1 & 1 & 1 \\
\end{array}
\]

\[Y_2 = D_3 + D_2D_1 + D_0D_1\]
\[Y_3 = D_3 + D_2D_1 + D_0D_1\]
\[Y_4 = D_3 + D_2D_1 + D_0D_1D_1\]

FIGURE 3.59
Example 3.22
BCD-to-2421 Code Converter
3.5 • Simplification by the Karnaugh Map Method

bit of the 2421 code. We can derive each Boolean expression from a truth table represented by the corresponding output column in Table 3.16.

We can load the 2421 values into four different Karnaugh maps, as shown in Figure 3.58. The cells corresponding to the unused input BCD codes 1010, 1011, 1100, 1101, 1110, and 1111 are don’t care states in each map.

The K-maps yield the following simplifications:

\[
Y_4 = D_4 + D_3 D_2 + D_3 D_1
\]
\[
Y_3 = D_4 + D_3 D_2 + D_3 D_1
\]
\[
Y_2 = D_4 + D_3 D_2 + D_3 D_2 D_1
\]
\[
Y_1 = D_1
\]

Figure 3.59 shows the logic diagram for these equations.

POS Simplification

Until now, we have looked only at obtaining the maximum SOP simplification from a Karnaugh map. It is also possible to find the maximum POS simplification from the same map.

Figure 3.60 shows a Karnaugh map with the cells grouped for an SOP simplification and a POS simplification. The SOP simplification is shown in Figure 3.60a and the POS simplification in Figure 3.60b.

![FIGURE 3.60](Image)

**FIGURE 3.60**

SOP and POS Forms on a K-Map

When we derive the POS form of an expression from a truth table, we use the lines where the output is 0 and we use the complements of the input variables on these lines as the elements of the selected maxterms. The same principle applies here.

The maxterms are:

\[(A + B + C)\] Top left cell
\[(\bar{A} + B + C)\] Bottom left cell
\[(\bar{A} + B + \bar{C})\] Bottom right cell

The variables are canceled in much the same way as in the SOP form. Remember, however, that the POS variables are the complements of the variables written beside the Karnaugh map.

If there is more than one simplified term, the terms are ANDed together, as in a full POS form.

Cancellations:

Outside pair: A is present in both true and complement form in the pair. (Discard A.)

B and C are present in both cells of the pair. (Keep B and C.)

Term: \(B + C\)
Bottom pair: \( \overline{A} \) and \( B \) are present in both cells of the pair. (Keep \( \overline{A} \) and \( B \).)

\( C \) is present in both true and complement form in the pair. (Discard \( C \).)

Term: \( \overline{A} + B \)

Maximum POS simplification:

\[ Y = (\overline{A} + B)(B + C) \]

Compare this with the maximum SOP simplification:

\[ Y = \overline{A} C + B \]

By the Boolean theorem \((x + y)(x + z) = x + yz\), we see that the SOP and POS forms are equivalent.

**EXAMPLE 3.23**

Find the maximum POS simplification of the logic function represented by Table 3.17.

**Solution**  Figure 3.61 shows the Karnaugh map from the truth table in Table 3.17. The cells containing 0s are grouped in two quads and there is a single 0 cell left over.

**Simplification:**

- **Corner quad:** \((B + D)\)
- **Horizontal quad:** \((A + B)\)
- **Single cell:** \((\overline{A} + B + C + \overline{D})\)

\[ Y = (A + B)(B + D)(\overline{A} + B + C + \overline{D}) \]

**Table 3.17  Truth Table for Example 3.23**

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**FIGURE 3.61  Example 3.23**

**POS Simplification of Table 3.17**

**SUMMARY**

1. Two or more gates connected together form a logic gate network or combinational logic circuit, which can be described by a truth table, a logic diagram, or a Boolean expression.
2. The output of a combinational logic circuit is always the same with the same combination of inputs, regardless of the order in which they are applied.
3. The order of precedence in a logic gate network is AND, then OR, unless otherwise indicated by parentheses.
4. DeMorgan’s theorems: \( \overline{x + y} = \overline{x} \cdot \overline{y} \)
   \( \overline{x \cdot y} = \overline{x} + \overline{y} \)
5. Inequalities: \( \overline{x \cdot y} \neq \overline{x} \cdot \overline{y} \)
6. A logic gate network can be drawn to simplify its Boolean expression by ensuring that bubbled (active-LOW) outputs drive bubbled inputs and outputs with no bubble (active-HIGH) drive inputs with no bubble. Some gates might need to be drawn in their DeMorgan equivalent form to achieve this.
7. In Boolean expressions, logic inversion bars of equal lengths
associative, property: A mathematical function is associative if its operands can be grouped in any order without affecting the result. For example, addition is associative \((a + b) + c = a + (b + c)\), but subtraction is not \((a - b) - c \neq a - (b - c)\).

bubble-to-bubble convention: The practice of drawing gates in a logic diagram so that inverting outputs connect to inverting inputs and noninverting outputs connect to noninverting inputs.

bus form: A way of drawing a logic diagram so that each true and complement input variable is available along a conductor called a bus.

cell: The smallest unit of a Karnaugh map, corresponding to one line of a truth table. The input variables are the cell’s coordinates and the output variable is the cell’s contents.

combinational logic: Digital circuitry in which an output is derived from the combination of inputs, independent of the order in which they are applied.

combinatorial logic: Another name for combinational logic.

commutative property: A mathematical operation is commutative if it can be applied to its operands in any order without affecting the result. For example, addition is commutative \((a + b = b + a)\), but subtraction is not \((a - b \neq b - a)\).

distributive property: Full name: distributive property of multiplication over addition. The property that allows us to distribute

Glossary

adjacent cell: Two cells are adjacent if there is only one variable that is different between the coordinates of the two cells.

don’t care state: An output state that can be regarded either as HIGH or LOW, as is most convenient. A don’t care state is the output state of a circuit for a combination of inputs that will never occur.

Karnaugh map: A graphical tool for finding the maximum SOP or POS simplification of a Boolean expression. A Karnaugh map works by arranging the terms of an expression in such a way that variables can be cancelled by grouping minterms or maxterms.

levels of gating: The number of gates through which a signal must pass from input to output of a logic gate network.

logic diagram: A diagram, similar to a schematic, showing the connection of logic gates.

logic gate network: Two or more logic gates connected together.

maximum POS simplification: The form of a POS Boolean expression which cannot be further simplified by cancelling variables in the sum terms. It may be possible to get an SOP form with fewer terms or variables.
Maximum SOP simplification  The form of an SOP Boolean expression which cannot be further simplified by cancelling variables in the product terms. It may be possible to get a POS form with fewer terms or variables.

Maxterm  A sum term in a Boolean expression where all possible variables appear once in true or complement form.

Minterm  A product term in a Boolean expression where all possible variables appear once in true or complement form.

Octet  A group of eight cells in a Karnaugh map. An octet cancels three variables in a K-map simplification.

Order of precedence  The sequence in which Boolean functions are performed, unless otherwise specified by parentheses.

Pair  A group of two cells in a Karnaugh map. A pair cancels one variable in a K-map simplification.

Product term  A term in a Boolean expression where one or more true or complement variables are ANDed.

Product-of-sums (POS)  A type of Boolean expression where several sum terms are multiplied (ANDed) together.

Quad  A group of four cells in a Karnaugh map. A quad cancels two variables in a K-map simplification.

Sum term  A term in a Boolean expression where one or more true or complement variables are ORed.

Sum-of-products (SOP)  A type of Boolean expression where several product terms are summed (ORed) together.

Synthesis  The process of creating a logic circuit from a description such as a Boolean equation or truth table.

PROBLEMS
Problem numbers set in color indicate more difficult problems: those with underlines indicate most difficult problems.

Section 3.1  Boolean Expressions, Logic Diagrams, and

Truth Tables
3.1  Write the unsimplified Boolean expression for each of the logic gate networks shown in Figure 3.62.
3.2  Write the unsimplified Boolean expression for each of the logic gate networks shown in Figure 3.63.
3.3  Redraw the logic diagrams of the gate networks shown in Figure 3.63 a, e, f, h, i, and j so that they conform to the

FIGURE 3.62
Problem 3.1
Logic Circuits
bubble-to-bubble convention. Rewrite the Boolean expression of each of the redrawn circuits.

3.4 The circuit in Figure 3.64 is called a majority vote circuit. It will turn on an active-HIGH indicator lamp only if a majority of inputs (two out of three) are HIGH. Write the Boolean expression for the circuit.

FIGURE 3.63
Problem 3.2
Logic Circuits
3.6 Draw the logic circuit for each of the following Boolean expressions:

a. \( Y = AB + BC \)

b. \( Y = ACD + BCD \)

c. \( Y = (A + B)(C + D) \)

d. \( Y = A + BC + D \)

e. \( Y = AC + B + C \)

f. \( Y = \overline{AC} + B + C \)

g. \( Y = \overline{ABD} + \overline{BC} + A + C \)

h. \( Y = \overline{AB} + \overline{AC} + \overline{BC} \)

i. \( Y = \overline{AB} + \overline{AC} + \overline{BC} \)

3.7 Use DeMorgan’s theorems to modify the Boolean equations in Problem 3.6, parts e, f, g, h, and i so that there is no bar over any group of variables. Redraw the logic diagrams of the circuits to reflect the changes. (The final circuit versions should conform to the bubble-to-bubble convention.)

3.8 Write the truth tables for the logic diagrams in Figure 3.62, parts b, c, e, f, and g.

3.9 Write the truth tables for the logic diagrams in Figure 3.63, parts a, h, i, and j.

3.10 Write the truth tables for the Boolean expression in Problem 3.6, parts c, d, e, f, h, and i.

Section 3.2 Sum-of-Products (SOP) and Product-of-Sums (POS) Forms

3.11 Find the Boolean expression, in both sum-of-products (SOP) and product-of-sums (POS) forms, for the logic function represented by the following truth table. Draw the logic diagram for each form.

3.12 Find the Boolean expression, in both sum-of-products (SOP) and product-of-sums (POS) forms, for the logic function represented by the following truth table. Draw the logic diagram for the SOP form only.

3.13 Find the Boolean expression, in both sum-of-products (SOP) and product-of-sums (POS) forms, for the logic function represented by the following truth table. Draw the logic diagram for the POS form only.

3.14 Find the Boolean expression, in both sum-of-products (SOP) and product-of-sums (POS) forms, for the logic function represented by the following truth table. Draw the logic diagram for the SOP form only.
3.15 Write the POS form of the 2-input XOR function. Draw the logic diagram of the POS form of the XOR function.

3.16 Write the POS form of the 2-input XNOR function. Draw the logic diagram of the POS form of the XNOR function.

3.17 Write the Boolean expression for the circuit shown in Figure 3.65. Use the distributive property to transform the circuit into a sum-of-products (SOP) circuit.

3.18 Write the Boolean expression for the circuit shown in Figure 3.66. Use the distributive property to transform the circuit into a sum-of-products (SOP) circuit.

3.19 Use the rules of Boolean algebra to simplify the following expressions as much as possible.

\[ Y = A A B + C \]

3.20 Use the rules of Boolean algebra to simplify the following expressions as much as possible.

\[ a. \ Y = A \bar{A} B + C \]
\[ b. \ J = K + L \bar{L} \]
\[ c. \ S = (T + U) V \bar{V} \]
\[ d. \ S = T + V \bar{V} \]
\[ e. \ S = (A \bar{B} + \bar{C})(\bar{B} \bar{D} + F) \]

3.21 Use the rules of Boolean algebra to simplify the following expressions as much as possible.

\[ a. \ Y = A B C D + (\bar{A} + B) \bar{C} \bar{D} + \bar{A} + \bar{B} \]
\[ b. \ Y = A \bar{B} C D + (A + \bar{B}) \bar{C} + \bar{D} + \bar{A} \]
\[ c. \ K = (\bar{L} \bar{M} + \bar{L} \bar{M}) (M \bar{N} + L M N) + M (\bar{N} \bar{L}) \]

3.22 Use the rules of Boolean algebra to find the maximum SOP and POS simplifications of the function represented by the following truth table.

3.23 Use the rules of Boolean algebra to find the maximum SOP and POS simplifications of the function represented by the following truth table.

3.24 Use the rules of Boolean algebra to find the maximum SOP and POS simplifications of the function represented by the following truth table.
### 3.25 Use the rules of Boolean algebra to find the maximum SOP simplification of the function represented by the following truth table.

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### 3.26 Use the rules of Boolean algebra to find the maximum SOP simplification of the function represented by the following truth table.

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### 3.27 Use the rules of Boolean algebra to find the maximum SOP simplification of the function represented by the following truth table.

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### 3.28 Use the rules of Boolean algebra to find the maximum SOP simplification of the function represented by the following truth table.

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### 3.29 Use the rules of Boolean algebra to find the maximum SOP simplification of the function represented by the following truth table.

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3.30 Use the rules of Boolean algebra to find the maximum SOP simplification of the function represented by the following truth table.

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Section 3.4 Simplification by the Karnaugh Map Method

3.31 Use the Karnaugh map method to find the maximum SOP simplification of the logic diagram in Figure 3.21.

3.32 Use the Karnaugh map method to reduce the following Boolean expressions to their maximum SOP simplifications:

- **a.** \(Y = \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} C + A \overline{B} C\)

- **b.** \(Y = \overline{A} \overline{B} C + \overline{A} B C + A \overline{B} C + A \overline{B} C\)

- **c.** \(Y = \overline{A} \overline{B} C + \overline{A} B C + A \overline{B} C + A \overline{B} C\)

- **d.** \(Y = \overline{A} \overline{B} C D + \overline{A} B C D + \overline{A} B C D + \overline{A} B C D + A \overline{B} C D + A B C D + A B C D + A B C D\)

3.33 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

3.34 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

3.35 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

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3.36 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

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### 3.37 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

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</table>

### 3.40 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

<table>
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<tr>
<th>A</th>
<th>B</th>
<th>C</th>
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<th>Y</th>
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<tbody>
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</table>

### 3.38 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

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<tr>
<th>A</th>
<th>B</th>
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### 3.41 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
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3.42 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

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<th>A</th>
<th>B</th>
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3.43 Use the Karnaugh map method to reduce the Boolean expression represented by the following truth table to simplest SOP form.

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<th>C</th>
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</table>

3.44 The circuit in Figure 3.67 represents the maximum SOP simplification of a Boolean function.

Use a Karnaugh map to derive the circuit for the maximum POS simplification.

3.45 Repeat Problem 3.44 for the circuit in Figure 3.68.

3.46 Refer to the BCD-to-2421 code converter developed in Example 3.22. Use a similar design procedure to develop the circuit of a 2421-to-BCD code converter.

3.47 Excess-3 code is a decimal code that is generated by adding 0011 (= 310) to a BCD code. Table 3.18 shows
ANSWERS TO SECTION REVIEW

3.48 Repeat Problem 3.47 for an Excess-3-to-BCD code converter.

Table 3.18 BCD and Excess-3 Code

<table>
<thead>
<tr>
<th>Decimal Equivalent</th>
<th>BCD Code</th>
<th>Excess-3 Code</th>
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</thead>
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<td></td>
<td>D4 D3 D2 D1</td>
<td>E4 E3 E2 E1</td>
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</tr>
<tr>
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<td>1 1 0 0</td>
</tr>
</tbody>
</table>

PROBLEMS

Section 3.1
3.1a \( Y = ABC + D \)
   \( b \) OUT = \((\overline{W} + X + Y)\overline{Z}\)

Section 3.2

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
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Section 3.3
3.3a SOP: \( Y = A \overline{B} \overline{C} + A \overline{B} \overline{C} \)
   POS: \( Y = (A + B + C)(A + B + \overline{C})(A + \overline{B} + C) \)
   \( (A + B + C)(A + \overline{B} + \overline{C})(A + \overline{B} + C) \)
   \( (A + \overline{B} + C) \)
   \( b \) SOP: \( Y = \overline{A} \overline{B} \overline{C} + A \overline{B} \overline{C} + A \overline{B} \overline{C} + A \overline{B} \overline{C} \)
   \( (A + \overline{B} + C)(A + \overline{B} + \overline{C})(A + \overline{B} + \overline{C}) \)
   \( (A + \overline{B} + C) \)

Section 3.4
3.4a \( Y = \overline{AC} \) or \( Y = \overline{A} + \overline{C} \)
   3.4b \( Y = \overline{AC} + D \) or \( Y = \overline{A} + \overline{C} + D \)

3.4c \( Y = A\overline{B} \)

Section 3.5
3.5 SOP: \( Y = \overline{A} \overline{B} \overline{C} \)  POS: \( Y = (\overline{A} + \overline{C})(B + C) \)
CHAPTER 4

Introduction to PLDs and MAX+PLUS II

O U T L I N E

4.1 What is a PLD?
4.2 Programming PLDs using MAX+PLUS II
4.3 Graphic Design File
4.4 Compiling MAX+PLUS II Files
4.5 Hierarchical Design
4.6 Text Design File (VHDL)
4.7 Creating a Physical Design

C H A P T E R  O B J E C T I V E S

Upon successful completion of this chapter you will be able to:

• Describe some advantages of programmable logic over fixed-function logic.

• Name some types of programmable logic devices (PLDs).

• Use Altera’s MAX+PLUS II PLD Design Software to enter simple combinational circuits using schematic capture.

• Use VHDL entity declarations, architecture bodies, and concurrent signal assignments to enter simple combinational circuits.

• Create circuit symbols from schematic or VHDL designs and use them in hierarchical designs for PLDs.

• Assign device and pin numbers to schematic or VHDL designs and compile them for programming Altera MAX7000S or FLEX10K20 devices.

• Program Altera PLDs via a JTAG interface and a ByteBlaster Parallel Port Download Cable.

In the first three chapters of this book, we examined logic gates and Boolean algebra. These basic foundations of combinational circuitry, as well as the sequential logic circuits we will study in a later chapter, form the fundamental building blocks of many digital integrated circuits (ICs).

In the past, such digital ICs were fixed in their logic functions; it was not possible to change designs without changing the chips in a circuit. Programmable logic offers the digital circuit designer the possibility of changing design function even after it has been built. A programmable logic device (PLD) can be programmed, erased, and reprogrammed many times, allowing easier prototyping and design modification. (The industry marketing buzz often refers to “rapid prototyping” and “reduced time to market.”) The number of IC packages required to implement a design with one or more PLDs is often reduced, compared to a design fabricated using standard fixed-function ICs.

PLDs can be programmed from a personal computer (PC) or workstation running special software. This software is often associated with a set of programs that allow us to design circuits for various PLDs. MAX+PLUS II, owned by Altera Corporation, is such a software package. MAX+PLUS II allows us to enter PLD designs, either as schematics or in several hardware description languages (specialized computer languages for modeling and synthesizing digital hardware). A design can contain components that are in themselves complete digital circuits. MAX+PLUS II converts the design information
into a binary form that can be transferred into a PLD via a special interface connected to the parallel port of a PC.

**4.1 What Is a PLD?**

**KEY TERMS**

**Programmable logic device (PLD)**  A digital integrated circuit that can be programmed by the user to implement any digital logic function.

**Complex PLD (CPLD)**  A digital device consisting of several programmable sections with internal interconnections between the sections.

**MAX+PLUS II**  CPLD design and programming software owned by Altera Corporation.

**Schematic capture**  A technique of entering CPLD design information by using a CAD (computer aided design) tool to draw a logic circuit as a schematic. The schematic can then be interpreted by design software to generate programming information for the CPLD.

**Compile**  The process used by CPLD design software to interpret design information (such as a drawing or text file) and create required programming information for a CPLD.

One of the most far-reaching developments in digital electronics has been the introduction of programmable logic devices (PLDs). Prior to the development of PLDs, digital circuits were constructed in various scales of integrated circuit logic, such as small scale integration (SSI) and medium scale integration (MSI) devices. These devices contained logic gates and other digital circuits. The functions were determined at the time of manufacture and could not be changed. This necessitated the manufacture of a large number of device types, requiring shelves full of data books just to describe them. Also, if a designer wanted a device with a particular function that was not in a manufacturer’s list of offerings, he or she was forced to make a circuit that used multiple devices, some of which might contain functions neither wanted nor needed, thus wasting circuit board space and design time.

Programmable logic provides a solution to these problems. A PLD is supplied to the user with no logic function programmed in at all. It is up to the designer to make the PLD perform in whatever way a design requires; only those functions required by the design need be programmed. Since several functions can usually be combined in the design and programmed onto a single chip, the package count and required board space can be reduced as well. Also, if a design needs to be changed, a PLD can be reprogrammed with the new design information, often without removing it from the circuit.

PLD is a generic term. There is a wide variety of PLD types, including PAL (programmable array logic), GAL (generic array logic), EPLD (erasable PLD), CPLD (complex PLD), FPGA (field-programmable gate array), as well as several others. We will be focussing on CPLDs as a representative type of PLD. Although terminology varies somewhat throughout the industry, we will use the term CPLD to mean a device with several programmable sections that are connected internally. In effect, a CPLD is several interconnected PLDs on a single chip. This structure is not apparent to the user and doesn’t really concern us at this time, except as background information. We will look at the structure of PALs, GALs, and CPLDs in Chapter 8. We will use the term “PLD” when we are referring to a generic device and “CPLD” as a more specific type of PLD.

A complication in the use of programmable logic is that we must use specialized computer software to design and program our circuit. Initially, this might seem as though we are adding another level of work to the design, but when these computer techniques are mastered, it shortens the design process greatly and yields a level of flexibility not otherwise available.
Let’s look at two examples, comparing the use of SSI logic versus programmable logic.

**EXAMPLE 4.1**

Figure 4.1 shows a majority vote circuit, as described in Problem 3.4 of Chapter 3. This circuit will produce a HIGH output when two out of three inputs are HIGH. Write the Boolean equation for the circuit and state the minimum number and type of 74HC devices required to build the circuit. How many packages would be required to build two such circuits?

![Majority Vote Circuit](image)

**Solution**

Boolean equation: \( Y = AB + BC + AC \)

Figure 4.2 shows the 74HC devices required to build the majority vote circuit: one 74HC08A quad 2-input AND gate and one 74HC4075 triple 3-input OR gate. Figure 4.2 also shows connections between the devices. Note that unused gate inputs are grounded and unused outputs are left open.

![74HC Devices Required to Build a Majority Vote Circuit](image)

Two majority vote circuits would require 6 ANDs and two ORs. This requires one more 74HC08A package.

**EXAMPLE 4.2**

Show how a CPLD can be programmed with a majority vote function, using a schematic capture tool. State how many CPLDs would be required to build two majority vote circuits.

**Solution**

A CPLD can be programmed by entering the schematic directly, using PLD programming software, such as Altera Corporation’s MAX+PLUS II. Figure 4.3 shows the circuit as entered in a MAX+PLUS II Graphic Design File.
FIGURE 4.3
MAX+PLUS II Graphic Design File of a Majority Vote Circuit

The design can be compiled by MAX+PLUS II to create the information required to program the CPLD with the majority vote circuit. If a second copy of the circuit is required, the first circuit can easily be duplicated by a Copy and Paste procedure. The two circuits can then be compiled together and used to program a single CPLD.

4.2 Programming PLDs using MAX+PLUS II

**KEY TERMS**
- **Design entry**  The process of using software tools to describe the design requirements of a PLD. Design entry can be done by entering a schematic or a text file that describes the required digital function.
- **Fitting**  Assigning internal PLD circuitry, as well as input and output pins, for a PLD design.
- **Simulation**  Verifying design function by specifying a set of inputs and observing the resultant outputs. Simulation is generally shown as a series of input and output waveforms.
- **Programming**  Transferring design information from the computer running PLD design software to the actual PLD chip.
- **Download**  Program a PLD from a computer running PLD design and programming software.
- **Software tools**  Specialized computer programs used to perform specific functions such as design entry, compiling, fitting, and so on. (Sometimes just called “tools.”)
- **Suite (of software tools)**  A related collection of tools for performing specific tasks. MAX+PLUS II is a suite of tools for designing and programming digital functions in a PLD.
- **Target device**  The specific PLD for which a digital design is intended.
- **Altera UP-1 board**  A circuit board, part of Altera’s University Program Design Laboratory Package, containing two CPLDs and a number of input and output devices.

In order to take a digital design from the idea stage to the programmed silicon chip, we must go through a series of steps known as the PLD Design Cycle. These include design entry, simulation, compiling, fitting, and programming. All steps require the use of PLD software, such as Altera’s MAX+PLUS II, a suite of software tools, to perform the various tasks of the design cycle. Some tasks, such as design entry, require a great deal of attention; others, such as fitting a design to a specified CPLD, are done automatically during the compiling process.

We will be using MAX+PLUS II as a vehicle for learning the concepts that relate to PLD design and programming. The target devices for our designs will be two Altera CPLDs, both installed on a circuit board available from Altera called the University Pro-
gram Design Laboratory Package. We will generally refer to this board, shown in Figure 4.4, as the Altera UP-1 board.

![Altera UP-1 Board](image)

**FIGURE 4.4**
Altera UP-1 Board

Figure 4.5 shows photos of the two CPLDs used in the Altera UP-1 Board. Figure 4.5a shows the CPLD from the MAX7000S family, part number EPM7128SLC84-7. Figure 4.5b shows the CPLD from Altera’s FLEX10K series, part number EPF10K20RC240-4. These part numbers are meaningful and will be discussed in detail in Chapter 8.

![CPLDs](image)

**FIGURE 4.5**
Altera MAX7000S and FLEX10K CPLDs

In the remaining part of this chapter, we will learn how to enter a design in MAX+PLUS II in both graphical and text format, how to compile the design, and how to download it into either one of the CPLDs on the Altera UP-1 circuit board.

Treat this design example as a tutorial in MAX+PLUS II. Follow along with all the steps on your own computer to get the maximum benefit from the chapter. If you do not have access to the Altera UP-1 board or an equivalent, you can still follow through most of the steps.
Although the examples in this book are created with the Altera UP-1 board in mind, they will easily adapt to other circuit boards carrying an Altera EPM7128S or other similar CPLD. One such board is available from Intectra Inc. For further information, contact Intectra at:

Intectra, Inc
2629 Terminal Blvd
Mountain View, CA 94043 U.S.A.
Ph 650-967-8818 Fx 650-967-8836
intectra@best.com
www.intectra.com (Web site in Spanish only)

### 4.3 Graphic Design File

**KEY TERMS**

- **Graphic Design File (gdf)**: A PLD design file in which the digital design is entered as a schematic.
- **Project**: A set of MAX+PLUS II files associated with a particular PLD design.

One way of entering PLD designs is to create a Graphic Design File. This type of file contains a representation of a digital circuit, such as in Figure 4.3, showing components and their interconnections, as well as specifying the inputs and output names of the circuit.

MAX+PLUS II automatically generates a number of other files to keep track of the PLD programming information represented by the Graphic Design File. These files, taken together, represent a project in MAX+PLUS II. All operations required to create a programming file for a CPLD are performed on a project, not a file. Thus, it is important during the design process to keep track of what the current project is. The MAX+PLUS II toolbar, shown in Figure 4.6, makes this fairly easy.

**FIGURE 4.6**
MAX+PLUS II Toolbar

The toolbar has a number of buttons that pertain to the current project of a PLD design. The operations performed by these buttons can all be done through the regular menus of MAX+PLUS II, but the toolbar offers a quick way to access many available functions. Not all buttons on the toolbar in Figure 4.6 are labeled, just the ones that you will find particularly convenient at this time. You can find out the function of any button by placing the cursor on the button and reading a description at the bottom of the window.
In particular, notice the buttons that create, open, and save files (standard Windows icons) and the button that sets the project to the current file. When creating a new file, make it standard practice to first Save the file, then Set Project to Current File. If you do this as a habit, you (and MAX+PLUS II) will always know what the current project is. If you don’t, you will find that you are saving or compiling some other project and wondering why your last set of changes didn’t work.

Another good practice is to create a new Windows folder for each new design that you enter. Since MAX+PLUS II creates many files in the design process, the folders would become unmanageable if designs were not kept in separate folders.

MAX+PLUS II installs a folder for working with design files called max2work. The examples in this text will be created in a subfolder of max2work. If you are working in a situation where many people share a computer and you have access to a network drive of your own, you may wish to keep your working files in a max2work folder on the network drive. Avoid storing your working files on a local hard drive unless you are the only one with regular access to the computer. Examples in this book will not specify a drive letter, but will indicate drive:\max2work\folder.

Most of these examples are also available on the accompanying CD in the folder called Student Files. A special icon, shown in the margin, will indicate the example filename.

In the following sections, we will go through the process of creating a file in detail, using the majority vote circuit of Figure 4.3 as an example. The example assumes that MAX+PLUS II is properly installed on your computer and running. For installation instructions, see the file SE_READ on the accompanying CD or the MAX+PLUS II Installation section of MAX+PLUS II Getting Started, available from Altera.

**Entering Components**

**KEY TERMS**

Primitives  Basic functional blocks, such as logic gates, used in PLD design files.

Instance  A single copy of a component in a PLD design file.

To create a Graphic Design File, click the New File icon on the tool bar or choose New on the MAX+PLUS II File menu. The dialog box, shown in Figure 4.7 appears. Select Graphic Editor file and choose OK.

Maximize the window and click the Save icon or choose Save As or Save from the File menu. In the dialog box shown in Figure 4.8, save the file in a new folder (e.g., drive:\max2work\maj_vote\maj_vote.gdf) and choose OK. (If you have not created the new folder, just type the complete path name in the File Name box. MAX+PLUS II will
create a new folder.) Click the icon to Set Project to Current File or choose this action from the File, Project menu.

The first design step is to lay out and align the required components. We require three 2-input AND gates, a 3-input OR gate, three input pins, and one output pin. These basic components are referred to as primitives. Let us start by entering three copies of the AND gate primitive, called and2.

Click the left mouse button to place the cursor (a flashing square) somewhere in the middle of the active window. Right-click to get a pop-up menu, shown in Figure 4.9, and choose Enter Symbol. The dialog box in Figure 4.10 appears. Type and2 in the Symbol Name box and choose OK. A copy or instance of the and2 primitive appears in the active window.

You can repeat the above procedure to get two more instances of the and2 primitive, or you can use the Copy and Paste commands. These are the same icons and File commands as for other Windows programs. Highlight the and2 symbol by clicking it. Right-click the symbol to get the pop-up menu shown in Figure 4.11 and choose Copy. You can also click the Copy icon on the toolbar or use the Copy command in the File menu.
FIGURE 4.10
Enter Symbol Dialog Box

FIGURE 4.11
Copying a Component
Paste an instance of the primitive by clicking to place the cursor, then right-clicking to bring up the menu shown in Figure 4.12. Choose Paste. The component will appear at the cursor location, marked in Figure 4.12 by the square at the top left corner of the pop-up menu.

Enter the remaining components by following the Enter Symbol procedure outlined above. The primitives are called or3, input, and output. When all components are entered we can align them, as in Figure 4.13 by highlighting, then dragging each one to a desired location.

Connecting Components

To connect components, click over one end of one component and drag a line to one end of a second component. When you drag the line, a horizontal and a vertical broken line mark the cursor position, as shown in Figure 4.14. These lines help you align connections properly.
FIGURE 4.14
Dragging a Line to Connect Components

A line will automatically make a connection to a perpendicular line, as shown in Figure 4.15.

A line can have one 90-degree bend, as at the inputs of the AND gates. If a line requires two bends, such as shown at the AND outputs in Figure 4.16, you must draw two separate lines.

Assigning Pin Names

Before a design can be compiled, its inputs and outputs must be assigned names. We could also specify pin numbers, if we wished to make the design conform to a particular CPLD, but it is not necessary to do so at this stage. It may not even be desirable to assign pin numbers, since the design we enter can be used as a component or subdesign of a larger circuit. We may also wish MAX+PLUS II to assign pins to make the best use of the CPLD’s internal resources. At any rate, we will leave this step out for now.
Figure 4.17 shows the naming procedure. Pins A and B have already been assigned names. Highlight a pin by clicking on it. Right-click the highlighted pin and choose Edit Pin Name from the pop-up menu. You could also double-click the pin name to highlight it. Type in the new name.

If there are several pins that are spaced one above the other, you can highlight the top pin name, as described above, then highlight successive pin names by using the Enter key.
4.4 Compiling MAX+PLUS II Files

**KEY TERMS**

- **Programmer Object File (pof)**  Binary file used to program a PLD of the Altera MAX series.
- **SRAM Object File (sof)**  Binary file used to configure a PLD of the Altera FLEX series.
- **Volatile**  A device is volatile if it does not retain its stored information after the power to the device is removed.
- **Nonvolatile**  Able to retain stored information after power is removed.

The MAX+PLUS II compiler converts design entry information into binary files that can be used to program a PLD. Before compiling, we should assign a target device to the design.

From the **Assign** menu, shown in Figure 4.18, select **Device**. From the dialog box in Figure 4.19, select the target device. For the Altera UP-1 board, this would be either the EPM7128SLC84-7 (shown) or the FLEX10K20RC240-4. The device family for the EPM7128S device is MAX7000S.

![Assign Menu](image)

**FIGURE 4.18**
Assign Menu

**NOTE**
To see the EPM7128SLC84-7 device, the box that says **Show Only Fastest Speed Grades** must be unchecked.

The compiler has a number of settings that can be chosen prior to the actual compile process. Figure 4.20 shows some of the settings that should be selected from the **Processing** menu of the **Compiler** window. You can open the **Compiler** window from the **MAX+PLUS II** menu or by clicking the **Compiler** button on the toolbar at the top of the screen.
Design Doctor is a utility that checks for adherence to good design practice and will warn you of any bad design choices. (Design Doctor will not stop the design from compiling, but will suggest potential problems that could result from a particular design.) The Timing SNF Extractor creates a Simulation Netlist File, which is required to perform a timing simulation of the design. We will perform this step in later MAX+PLUS II designs. (If you are not able to select the Timing SNF Extractor, then uncheck the Functional SNF Extractor option.) Smart Recompile allows the compiler to use previously compiled portions of the design to which no changes have been made. This allows the compiler to avoid having to compile the entire design each time a change is made to one part of the design.

To start the compile process, click Start in the Compiler window. While in progress, the window will look something like Figure 4.21. Message of three types may appear during the compile process. Info messages (green text) are for information only. Warning messages (blue text) tell you of potential, but nonfatal, problems with the design. Error messages (red text) inform you of design flaws that render the design unusable. A PLD can still be programmed if the compiler generates info or warning messages, but not if it generates an error.

Depending on the device chosen, the compiler generates either a Programmer Object File (pof) or SRAM Object File (sof). The pof is used to program a MAX-series PLD. The sof is used to configure a FLEX-series PLD. The difference is that the MAX de-
vice is **nonvolatile**, that is, it retains its programming information after the power has been removed. The FLEX-series device is **volatile**, meaning that its programming information must be loaded each time the device powers up.

### 4.5 Hierarchical Design

#### KEY TERMS

**Hierarchical design** A PLD design that is ordered in layers or levels. The highest level of design contains components that are themselves complete designs. These components may, in turn, have lower level designs embedded within them.

A MAX+PLUS II Graphical Design File can be used as part of a **hierarchical design**. That is, it can be represented as a component in a higher-level design. Figure 4.22 shows a gdf that is constructed as a hierarchical design. It contains two majority vote circuits whose

![Diagram of hierarchical design](image)
outputs are combined in an AND gate. Thus, the output would be HIGH if two out of three inputs were HIGH on both blocks labeled maj_vote. These blocks are complete designs in their own right, and thus form a lower level of the design hierarchy.

**Default Symbols and User Libraries**

**KEY TERMS**

- **Default symbol**: A graphical symbol that represents a PLD design as a block, showing only the design’s inputs and outputs. The symbol can be used as a component in any Graphic Design File.
- **User library**: A folder containing symbols that can be used in a gdf file.
- **Top level (of a hierarchy)**: The file in a hierarchy that contains components specified in other design files and is not itself a component of a higher-level file.

We can create a default symbol for the majority vote circuit of Figure 4.3 from the MAX+PLUS II File menu, as shown in Figure 4.23. This action will create a symbol file with the same name as the Graphic Design File and the extension sym. Before creating the symbol, make sure that the gdf is saved and that the project is set to the current file.

![Figure 4.23](image)

Creating a Default Symbol

The symbol can be embedded into a gdf, as in Figure 4.22.

Before we can use the new symbol, we must make sure that MAX+PLUS II knows where to find it. MAX+PLUS II looks for a component first in the present working directory, then in the user library folders in the order of priority listed in the User Libraries dialog box.

To create a path to a user library, select User Libraries from the Options menu (Figure 4.24) in MAX+PLUS II. In the resultant dialog box, shown in Figure 4.25, select the appropriate drive and directories by double-clicking on the name in the Directories box. When the desired directory appears in the Directory Name box, click...
Add, then OK.
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If you are using MAX+PLUS II on a shared computer (e.g., in a computer lab), you should be aware that a library path that points to another user’s directory can cause MAX+PLUS II to look there before (or instead of) looking in your directory, resulting in the apparent inability of MAX+PLUS II to find your file.

For example, suppose you have a file called \texttt{g:\max2work\my\_file.gdf}, where \texttt{g:\} is a network drive mapped exclusively to your user account. (i.e., everyone has a \texttt{g:\} drive mapping, unique to their user account.) Further suppose that another user, against standard lab protocol, has created a file with the same name on the local hard drive: \texttt{c:\max2work\my\_file.gdf}. (Don’t think this doesn’t happen. It does.)

At compile time, MAX+PLUS II will look for \texttt{my\_file.gdf} first in the directory where the active project resides, then in the folders specified in the user library paths. If the user library path \texttt{c:\max2work\} has a higher priority than \texttt{g:\max2work\}, it will compile the version of \texttt{my\_file.gdf} found on the \texttt{c:\} drive.

When you make changes to the copy on the \texttt{g:\} drive, they will not take effect because the file on \texttt{g:\} is not being compiled.

To remedy this, delete the user libraries that point to local drives, such as \texttt{a:\} or \texttt{c:\}. If you have no assigned network drive on your system, delete all user libraries except for your own. Since a user library is just the name of a folder where MAX+PLUS II should look for files, this won’t do any great harm.

Creating a Design Hierarchy

The circuit in Figure 4.22 is saved as \texttt{2votes.gdf}. If we double-click on either symbol labeled \texttt{maj\_vote}, the MAX+PLUS II Graphic Editor will bring the file \texttt{maj\_vote.gdf} to the foreground. Thus, we say that \texttt{2votes.gdf} is at the top level of the current hierarchy.

We can extend the hierarchy further by making a symbol for \texttt{2votes.gdf} and embedding it in a higher-level file called \texttt{4votes.gdf}, shown in Figure 4.26. This circuit generates a high output if (two out of three of (A11, B11, C11) are high) AND (two out of three of A21, B21, C21) are high.

\textbf{FIGURE 4.26}

Further Levels of Hierarchy (4votes.gdf)
(A21, B21, C21) are HIGH) OR the same is true for (A12, B12, C12) AND (A22, B22, C22). If we double-click on either symbol for 2votes, the Graphic Editor will bring the file 2votes.gdf to the foreground.

MAX+PLUS II can display the hierarchy of a design. To see the hierarchy structure, click the Hierarchy icon on the MAX+PLUS II toolbar (the yellow pyramid) or choose Hierarchy Display from the MAX+PLUS II menu. Figure 4.27 shows the hierarchy for the project 4votes. Note that the highest level has two subdesigns, each of which breaks down further into two subdesigns. Thus, using hierarchical design and symbols for gdf or other design files allows us to create multiple instances of a basic design (maj_vote.gdf) and use it in many places.

NOTE
In order to correctly show the hierarchy display, the top-level file of the project (in this case 4votes.gdf) must be compiled first.

4.6 Text Design File (VHDL)

An alternative to schematic entry, and ultimately a more powerful PLD design technique is the use of a text-based design tool, or hardware description language, such as Altera’s AHDL (Altera Hardware Description Language) or the industry-standard VHDL (VHSIC Hardware Description Language). A designer creates a text file, framed within a certain set of rules known as the syntax of the language and uses a compiler to create pro-
gramming data much as he or she would with a Graphic Design File. Hardware description languages can be used to generate hardware for hierarchical designs, either as components in graphic or text files or as higher level design entities containing other designs.

AHDL, while very easy to use, has a much narrower application than VHDL because it is one of many proprietary tools on the market aimed at the programming requirements of a particular manufacturer’s line of CPLDs. Since VHDL is an industry-standard language and the MAX+PLUS II compiler supports both languages, we will concentrate on VHDL.

VHDL was originally developed by defense contractors in the U.S. and is now the required standard for all ASICs (application specific integrated circuits) designed for the U.S. military. It has been standardized by the Institute of Electrical and Electronics Engineers (IEEE) and has been enjoying increasing popularity in the electronics design community. The original VHDL standard was written in 1987 and updated in 1993 (IEEE Std. 1076-1993). This standard and other related ones continue to undergo revision. The current status of Std. 1076 can be determined from the IEEE Standards web site at http://www.standards.ieee.org.

Entity and Architecture

**KEY TERMS**

**Entity**  A VHDL structure that defines the inputs and outputs of a design.

**Architecture**  A VHDL structure than defines the relationship between input, output, and internal signals or variables in a design.

**Port**  A name assigned to an input or output of a VHDL design entity.

**Mode (of a port)**  The kind of port, such as input or output.

**Signal**  A name given to an internal connection in a VHDL architecture.

**Variable**  A block of working memory used for internal calculation or storage in a VHDL architecture.

**Type**  A set of characteristics associated with a VHDL port name, signal, or variable that determines the allowable values of the port, signal, or variable.

**Library**  A collection of VHDL design units that have been previously compiled.

**Package**  A group of VHDL design elements that can be used by more than one VHDL file.

**IEEE Standard 1164**  The standard which defines a variety of VHDL types and operations, including the STD_LOGIC and STD_LOGIC_VECTOR types.

**Concurrent**  Simultaneous.

**Concurrent signal assignment**  A relationship between an input and output port or signal in which the output is changed as soon as there is a change in input. If the file has more than one concurrent signal assignment, they are all evaluated simultaneously.

**Selected signal assignment statement**  A concurrent signal assignment in VHDL in which a value is assigned to a signal, depending on the alternative values of another signal or variable.

**Comment**  Explanatory text in a VHDL (or other computer language) file that is ignored by the computer at compile time.

**Vector**  A group of digital signals or variables, usually related numerically, that can be treated as a single multibit variable.

**Bit string literal**  A group of bits assigned to the elements of a vector, enclosed in double quotes (e.g., “001011”).
Every VHDL file requires at least two structures: an entity declaration and an architecture body. The entity declaration defines the external aspects of the VHDL function; that is, the input and output names and the name of the function. The architecture body defines the internal aspects; that is, how the inputs and outputs behave with respect to one another and with respect to other signals or functions that are internal only.

Let us examine the structure of a VHDL design for the majority vote circuit defined in Figure 4.1. The complete VHDL file for the majority vote circuit is shown next. The double dashes before the first two lines are to indicate that these lines are comments. There are also a few other comments to illustrate the use of VHDL.

```vhdl
--maj_vot2.vhd
--VHDL implementation of a majority vote circuit
--Library contains standard VHDL logic types
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
--Entity defines inputs and outputs
ENTITY maj_vot2 IS
PORT (a, b, c : IN STD LOGIC;
y : OUT STD LOGIC);
END maj_vot2;

--Architecture describes input/output relationship
ARCHITECTURE majority OF maj_vot2 IS
BEGIN
  y <= (a and b) or (b and c) or (a and c);
END majority;
```

VHDL is not case-sensitive, so statements written in lowercase and uppercase are equivalent. For example, \((Y <= A \text{ AND } B;)\) is equivalent to \((y <= a \text{ and } b;)\). However, Altera’s style guidelines for VHDL suggest that all keywords, devices, constants, and primitives be capitalized and everything else be written in lowercase letters. The VHDL style guideline can be referred to in the MAX+PLUS II Help menu.

The name of the entity, `maj_vot2`, is given in the first and last lines of the entity declaration. The VHDL file that contains this entity must be named `maj_vot2.vhd`. Figure 4.28 shows how the design entity looks if it is converted to a symbol for use in a Graphic Design File.

The Boolean equation for a 3-input majority vote circuit is \(Y = AB + BC + AC\). In the architecture body, we can write this operation as:

\[y <= (a \text{ and } b) \text{ or } (b \text{ and } c) \text{ or } (a \text{ and } c);\]

The operator \(<=\) assigns the value of the right hand side of the equation to the left hand side. Whenever there is a change in a, b, or c, the statement is re-evaluated and the new value is assigned to y. Note that VHDL logical operators (such as `and` and `or`) have equal precedence, so we must make the order of precedence explicit with parentheses.

The Boolean equation above is an example of a concurrent signal assignment statement. Concurrent means “simultaneous.” The implication is that any number of concurrent signal assignments can be listed in a VHDL architecture body and the order in which they are evaluated does not depend on the order in which they are written, since all statements are concurrent. In this way, a concurrent structure imitates combinational hardware, where
a change in one input that is common to several circuits makes all circuits change at the same time.

Enclosed in the entity declaration is a port definition. A port is a connection from the PLD to the outside world. Figure 4.29 shows the possible modes of a port. Mode IN refers to a port that is only for input. Mode OUT is output only. Mode INOUT is a bidirectional port, in which data can flow in either direction, based on the status of a control input. Mode BUFFER is a special case of OUT that has a feedback connection back into the CPLD logic that can be used as part of another Boolean expression.

Figure 4.30 shows the difference between BUFFER and OUT modes. Port x (defined by \( x = a \text{ and } b \)) must be of mode BUFFER because it is fed back and used as part of the expression for port y (defined by \( y = x \text{ or } c \)). Port y can be of mode OUT since it has no feedback, only an output.

In addition to defining the port modes, the entity declaration also defines what type each port is. The type of a port, signal, or variable defines what values it is allowed to have. Three common types in VHDL are BIT, STD_LOGIC, and INTEGER. Multibit extensions of these types include BIT_VECTOR and STD_LOGIC_VECTOR.

Ports, signals and variables of type BIT can have a value of ‘0’ or ‘1’. When using these values, they must be enclosed in single quotes.

The STD_LOGIC (standard logic) type, also called IEEE Std.1164 Multi-Valued Logic, has been defined to give a broader range of output values than just ‘0’ and ‘1’. Any port, signal, or variable of type STD_LOGIC or STD_LOGIC_VECTOR can have any of the values listed below.

- ‘U’, -- Uninitialized
- ‘X’, -- Forcing Unknown
- ‘0’, -- Forcing 0
- ‘1’, -- Forcing 1
- ‘Z’, -- High Impedance
‘W’, -- Weak Unknown
‘L’, -- Weak 0
‘H’, -- Weak 1
‘-’ -- Don’t care

“Forcing” levels are deemed to be the equivalent of a gate output. “Weak” levels are
specified by a pull-up or pull-down resistor. (“Weak” levels are usually used in circuit
modeling, where it is important to distinguish between gate outputs and pull-up/down.
These levels will not be of importance to us.) The ‘Z’ state is used as the high-impedance
state of a tristate buffer.

The majority of applications can be handled by ‘X’, ‘0’, ‘1’, and ‘Z’ values.
To use STD_LOGIC in a VHDL file, you must include the following reference to the
VHDL library called ieee and the std_logic_1164 package before the entity declaration:

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

<table>
<thead>
<tr>
<th>Type</th>
<th>Values</th>
<th>How Written</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIT</td>
<td>0 or 1</td>
<td>Single quotes</td>
<td>‘0’, ‘1’</td>
</tr>
<tr>
<td>INTEGER</td>
<td>Whole numbers</td>
<td>No quotes</td>
<td>4095, 7, -120, -1</td>
</tr>
<tr>
<td>BIT_VECTOR</td>
<td>Multiple instances of 0 or 1</td>
<td>Double quotes</td>
<td>“100110”</td>
</tr>
</tbody>
</table>
| STD_LOGIC_VECTOR | Multiple instances of U, X, 0, 1, Z, W, L, H, - | Double quotes | “1001100”,
|                  |                                 |             | “00ZZ11”,
|                  |                                 |             | “ZZZZZZZZ”            |

Why use STD_LOGIC rather than BIT, if we only use ‘0’ and ‘1’ values? The usual
reason is for compatibility with existing VHDL components that might be used in our de-
sign entities. For example, the Altera Library of Parameterized Modules (LPM) contains

FIGURE 4.31
2-line-to-4-line Decoder
predesigned components that are written using STD_LOGIC types. To include these components in a VHDL design, the design must be written with STD_LOGIC types, as well.

The INTEGER type can take on whole-number values. When used in a VHDL file, an integer is written without quotes. Table 4.1 summarizes the BIT, STD_LOGIC, and INTEGER types, as well as the BIT_VECTOR and STD_LOGIC_VECTOR types.

---

**EXAMPLE 4.3**

Figure 4.31 shows the logic diagram of a 2-line-to-4-line decoder. The circuit detects the presence of a particular binary code and makes one and only one output HIGH, depending on the value of the 2-bit number $D_1D_0$. Write a VHDL file that describes the decoder.

**Solution** The circuit has two inputs and four outputs, which are numerically related. We could describe the two inputs as separate names, as we could the four outputs. Or, we could show the inputs and outputs as two groups of related ports, called vectors. The elements of the vector can be treated separately or as a group.

**Case 1: separate variables**

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY decode1 IS
  PORT(
    d1, d0 : IN STD_LOGIC;
    y0, y1, y2, y3 : OUT STD_LOGIC);
END decode1;

ARCHITECTURE decoder1 OF decode1 IS
BEGIN
  y0 <= (not d1) and (not d0);
  y1 <= (not d1) and (    d0);
  y2 <= (    d1) and (not d0);
  y3 <= (    d1) and (    d0);
END decoder1;
```

**Case 2: vectors (elements treated separately)**

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY decode2 IS
  PORT ( 
    d : IN   STD_LOGIC_VECTOR (1 downto 0);
    y : OUT STD_LOGIC_VECTOR (3 downto 0));
END decode2;

ARCHITECTURE decoder2 OF decode2 IS
BEGIN
  y(0) <= (not d(1)) and (not d(0));
  y(1) <= (not d(1)) and (    d(0));
  y(2) <= (    d(1)) and (not d(0));
  y(3) <= (    d(1)) and (    d(0));
END decoder2;
```

In Case 2, we specify the length of the vector by the construct `(3 downto 0)`, indicating that Y3 is the leftmost bit in the vector. We could also use the constructs `(0 to 3)`, `(4 downto 1)`, or `(1 to 4)`, depending on our requirements. Each individual element of the vector is specified by a number in parentheses.

**Case 3: vectors (elements treated as a group)**

```vhdl
-- decode2a.vhd
-- 4-channel decoder
```
— Makes one and only one output HIGH for each
— binary combination of (d1, d0).

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY decode2a IS
    PORT (d : IN STD_LOGIC_VECTOR (1 downto 0);
          y : OUT STD_LOGIC_VECTOR (3 downto 0));
END decode2a;

ARCHITECTURE decoder OF decode2a IS
BEGIN
    — Choose a signal assignment for y
    — based on binary value of d
    — Default case: all outputs deactivated
    WITH d SELECT
        y <= "0001" WHEN "00",
            "0010" WHEN "01",
            "0100" WHEN "10",
            "1000" WHEN "11",
            "0000" WHEN others;
END decoder;

In Case 3, we use a selected signal assignment statement to assign a value to all
bits of vector y for each combined value of vector d. For example, when d(1) = 0 and
d(0) = 0, the values assigned to y are: y(3) = 1, y(2) = 0, y(1) = 0, y(0) = 0. Similar assignments are made for other values of d. The result is a construct that acts much like a truth table of the decoder circuit. The others clause is necessary to define a default case.

FIGURE 4.32
MAX+PLUS II Template Menu

FIGURE 4.33
VHDL Template Dialog Box
since the STD_LOGIC_VECTOR type contains values other than ‘0’ and ‘1’.

The multibit values assigned to the vectors, called **bit string literals**, must be enclosed in double quotes.

### VHDL Templates in MAX+PLUS II

MAX+PLUS II offers a shortcut to creating VHDL structure in a **Template Menu**. Figure 4.32 shows this menu, which is available in the MAX+PLUS II Text Editor window. To choose a template, select the one desired from the **VHDL Template** dialog box, shown in Figure 4.33.

Choosing the Entity Declaration template results in the following text:

```vhdl
ENTITY __entity_name IS
    GENERIC (__parameter_name : string := __default_value;
              __parameter_name : integer := __default_value);
    PORT (
        __input_name, __input_name : IN STD_LOGIC;
        __input_vector_name : IN STD_LOGIC_VECTOR (__high downto __low);
        __bidir_name, __bidir_name : INOUT STD_LOGIC;
        __output_name, __output_name : OUT STD_LOGIC);
END __entity_name;
```

To convert this into a valid entity for our use, we delete the lines we do not need and substitute input and output names into the template. For our majority vote circuit, we had inputs called $A$, $B$, and $C$ and an output called $Y$. Thus, we can modify the template to yield the entity declaration:

```vhdl
ENTITY maj_vot2 IS
    PORT (a, b, c : IN STD_LOGIC;
          y           : OUT STD_LOGIC);
END maj_vot2;
```

#### Integrating VHDL and Graphical Design Components

We can create a default symbol for the VHDL majority vote function, much as we did for the same function in the Graphic Design File. In the Text Editor **File** menu, select **Create**
Default Symbol. We can integrate this new symbol into a two-level majority vote circuit, as shown in Figure 4.34. This circuit contains primitives (AND gate, input pins, and output pin), a `gdf` symbol (maj_vote), and a symbol created from a VHDL file (MAJ_VOT2). Double-clicking on either symbol will bring forward its original design file.

4.7 Creating a Physical Design

**KEY TERMS**

| Assignment and Configuration File (acf) | A MAX+PLUS II file that contains information about the configuration options for a project, including assigned device and pin numbers. |

The previous sections have concentrated on the design aspects of a project. Of course, the ultimate goal of this procedure is to create a physical version of the design. Before we can program our majority vote circuit into hardware, we must assign the input and output pin numbers on the target CPLD. At that point we can recompile the design file and program the CPLD.

**Assigning Pin Numbers**

Before proceeding with this step, make sure that you have assigned a device part number to the design. Save the file and set the project to the current file.

To assign a pin number, click on the pin to highlight it, then right-click to see the pop-up menu in Figure 4.35. Choose Assign, then Pin/Location/Chip. You could also do this from the Assign menu at the top of the screen.
FIGURE 4.36
Pin/Location/Chip Assignment Dialog Box

Table 4.2  Pin Assignment for a Majority Vote Circuit

<table>
<thead>
<tr>
<th>Pin Name</th>
<th>Pin Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>12</td>
</tr>
<tr>
<td>B1</td>
<td>16</td>
</tr>
<tr>
<td>C1</td>
<td>18</td>
</tr>
<tr>
<td>A2</td>
<td>15</td>
</tr>
<tr>
<td>B2</td>
<td>17</td>
</tr>
<tr>
<td>C2</td>
<td>21</td>
</tr>
<tr>
<td>Y</td>
<td>4</td>
</tr>
</tbody>
</table>

FIGURE 4.37
Pin Assignments in ACF (Before Copying)
We can assign pin numbers in the dialog box in Figure 4.36.

Type A1 in the Node Name box, 12 in the Pin box and click Add. Type B1 in the Node Name box, assign this name to pin 16, and click Add. Repeat this procedure until all names are assigned, as in Table 4.2. When all assignments are complete, click OK.

We can also assign pin numbers by editing the Assignment and Configuration File (acf), as shown in Figures 4.37 and 4.38. This technique works especially well if you need to assign pin numbers to a sequence of numerically related inputs and outputs.

Figure 4.37 shows the acf with four pin assignments made. We can add the others easily by using a copy-and-paste procedure. Highlight the line you wish to copy and copy it to the Windows clipboard (use Copy in the File menu or the Copy icon on the toolbar or Ctrl-C). Paste three copies into the acf and modify them so that they represent the remaining required pin assignments, as shown in Figure 4.38.

Figure 4.39 shows the input pin assignments as they appear in the gdf file.

---

**FIGURE 4.38**
Pin Assignments in ACF (After Copying)

**FIGURE 4.39**
Pin Assignments as Seen in gdf File
Programming CPLDs on the Altera UP-1 Circuit Board

**KEY TERMS**

**ByteBlaster**  An Altera ribbon cable and connector used to program or configure Altera CPLDs via the parallel port (LPT port) of an IBM PC or compatible.

**JTAG**  Joint Test Action Group. A standards body that developed the format (called IEEE Std. 1149.1) for testing and programming devices while they are installed in a system.

**ISP**  In-system programmability. The ability of a PLD (such as a MAX7000S) to be programmed without removing it from a circuit board.

**ICR**  In-circuit reconfigurability. The ability of a PLD (such as a FLEX10K) to be configured without removing it from a circuit board.

---

The CPLDs on the Altera UP-1 circuit board are programmed via the programming software in MAX+PLUS II and a ribbon cable called the **ByteBlaster**. The ByteBlaster, shown in Figure 4.40, connects to the parallel port of a PC running MAX+PLUS II to a 10-pin male socket that complies with the **JTAG** standard. This standard specifies a four-wire interface, originally developed for testing chips without removing them from a circuit board, but can also be used to program or configure PLDs.
PLDs that can be programmed or configured while installed on a circuit board are called **in-system programmable (ISP)** or **in-circuit reconfigurable (ICR)**. ISP is used to refer to nonvolatile devices, such as MAX7000S; ICR refers to volatile devices, such as FLEX10K.

The JTAG interface has four wires, as well as power and ground connections, as shown in Figure 4.41. Data are sent to a device from a JTAG controller (i.e., the PC) via the TDI (Test Data In) line. Data return from the device via TDO (Test Data Out). The data transfer is controlled by TMS (Test Mode Select). The process is driven from one step to the next by TCK (Test Clock).

Multiple devices can be programmed in a **JTAG Chain**, as shown in Figure 4.42. This connection allows both CPLDs on the Altera UP-1 Board to be programmed at the same time. The UP-1 board also has a female 10-pin socket labeled JTAG out, which allows two or more boards to be chained together. The choice of programming one or more CPLDs, or...
the CPLDs on one or more UP-1 boards, is determined by the placement of four on-board jumpers. These jumper positions are explained in the *Altera University Program Design Laboratory Package User Guide*. A copy of the *User Guide* is included in Appendix A for reference and is available at Altera’s Web site.

The operation of the JTAG port is controlled automatically by MAX+PLUS II, so further details are not necessary at this time. For further information on the JTAG interface, refer to *Altera Application Note 39, JTAG Boundary-Scan Testing in Altera Devices*, included in the Altera Documentation folder on the accompanying CD.

**MAX+PLUS II Programmer**

To program a device on the Altera UP-1 board, set the jumpers to program the EPM7128S or configure the EPF10K20, as shown in the *Altera University Program Design Laboratory Package User Guide*. Connect the ByteBlaster cable from the parallel port of the PC running MAX+PLUS II to the 10-pin JTAG header. (You may have to run a 25-wire cable...
(male-D-connector-to-female-D-connector) to make it reach.) Plug an AC adapter (9-volt dc output) into the power jack of the UP-1 board.

Open the top-level file of the project you wish to download to the UP-1 board (e.g., `maj_vote.gdf`). Set the project to the current file. Invoke the MAX+PLUS II Programmer from the MAX+PLUS II menu or click the Programmer button (the icon showing the blue ribbon cable) on the MAX+PLUS II toolbar.

If you have never programmed a device with your copy of MAX+PLUS II, you will need to set up the hardware configuration. Click Hardware Setup in the Options menu to get the dialog box in Figure 4.43.

Select ByteBlaster in the Hardware Type box. Ensure that Parallel Port is the same as the port the ByteBlaster is plugged into (usually LPT1:). Click OK. (If you have a choice, configure your parallel port as an Enhanced Communications Port (ECP) in your computer’s CMOS setup. For most users this step is not necessary, as the port is already configured this way.)
FIGURE 4.47
Select Programming File Dialog Box

If the current project was compiled with the MAX7000S device selected, the pof file for the project will automatically be available. The **Programmer** dialog box will appear as in Figure 4.44. To download, click **Program**.

If the project was compiled for the FLEX10K device and the device is to be configured via a ByteBlaster, it must be configured via the **Multi-Device JTAG Chain** available in the JTAG menu. Select the JTAG menu, shown in Figure 4.45, and choose **Multi-Device JTAG Chain Setup**.

In the **Multi-Device JTAG Chain Setup** window, shown in Figure 4.46, select the pull-down menu for the device name. Select **EPF10K20**. Choose **Delete All** to clear the box of any previous programming file names. Choose the **Select Programming File** button.

The **Select Programming File** dialog box will appear, as in Figure 4.47. Find and select the file \max2work\maj_v...
ote\maj_vote.sof. Click OK. Choose the Add button in the JTAG setup box to add the SRAM Object File (sof) to the list. Choose the Detect JTAG Chain Info button to set up the hardware for programming. Choose OK. Click the Configure button in the Programmer dialog box to download the binary information to the FLEX10K CPLD on the UP-1 board.

**SUMMARY**

1. A programmable logic device (PLD) is a digital device that is shipped blank and whose function is determined by the end user.
2. PLDs offer design flexibility, reduce board space and package count, and can be used to develop digital designs more quickly than fixed-function logic.
3. Some types of PLDs include PAL (programmable array logic), GAL (generic array logic), EPLD (erasable PLD), CPLD (complex PLD), FPGA (field-programmable gate array).
4. Complex PLDs (CPLDs) are devices with several program-mable sections that are interconnected inside the chip.
5. PLD design and programming requires special software, such as Altera’s MAX+PLUS II.
6. PLD designs can be entered by schematic capture (Graphic Design Files) or text-based languages, such as Altera Hardware Description Language (AHDL) and VHSIC Hardware Description Language (VHDL).
7. MAX+PLUS II organizes PLD design files in a project. Since many operations in MAX+PLUS II are performed on a project, you should set the project to the current file (File menu) whenever you change windows and make a modification to a design file.
8. Save your work every time you pause for thought.
9. A MAX+PLUS II Graphic Design File (gdf) consists of graphical symbols of components that are interconnected by lines drawn between inputs and outputs of the components.
10. Circuit inputs and outputs in a gdf have special symbols. The input and output pins must be named, but need not be numbered in the first stages of a design.

11. The MAX+PLUS II compiler translates the design information from a gdf or text file into binary data that can be downloaded into a PLD. For a MAX7000S, the compiler generates a Programmer Object File (pof) to program the device. For a FLEX10K, an SRAM Object File (sof) is generated to configure the device.
12. MAX7000S devices are nonvolatile; they stay programmed when the power is removed from the chip. FLEX10K devices are volatile; they lose their programming data when power is removed.
13. If a CPLD part number is not specified, the MAX+PLUS II compiler will automatically select one. It is good practice to assign the part number of the device before compiling, as this can affect the accuracy of certain parts of the design process, such as simulation. The CPLDs on the Altera UP-1 board are EPM7128SLC84-7 and EPF10K20RC240-4.
14. Some useful compiler options are: Design Doctor (checks for good design practice), Timing SNF Extractor (compiles data required for timing simulations), and Smart Recompile (allows part of the compile process to be skipped if only part of a design has changed).
15. Compiler messages can be in green text (Info), blue text (Warning; possible problems, but not fatal), or red text (Error; fatal, compiling stops).
16. MAX+PLUS II files can be arranged in a design hierarchy. That is, a MAX+PLUS II file can contain components that are complete MAX+PLUS II designs in and of themselves.
17. A file that contains other designs, but is not part of a higher-level design, is called the top level of a hierarchy.
18. If the top level of a hierarchy is a gdf, lower-level designs are embedded in the gdf as default symbols that are created from the original design files of the components.
19. MAX+PLUS II looks for default symbols in the present working directory, then in the directories specified as user libraries.
20. VHDL (VHSIC Hardware Description Language) is a text-based programming language used to model and program digital circuits.
21. Every VHDL file requires an entity declaration, which describes the external aspects of the design (inputs and outputs), and an architecture body, which describes the relation-
20. A port in VHDL is an input or output. A signal is an internal connection, like a wire. A variable is a piece of working memory reserved by the VHDL file.
21. The simplest way to relate inputs and outputs in a VHDL design is with a concurrent signal assignment statement, which has the form: x <= (a and b) or c; The port or signal on the left side is assigned the value of the logic expression on the right side. (Variables are assigned with a different operator.)
22. A port, signal, or variable can have a multiple-bit construction of type BIT_VECTOR or STD_LOGIC_VECTOR. These structures are called vectors and can be referred to a separate elements (e.g., y(3) <= d(1) and d(0) ;) or as a group (e.g., y <= "1000").
23. A selected signal assignment statement can act as a truth table in VHDL. It assigns alternative values to one or more outputs, depending on the alternative values on one or more inputs.
24. VHDL constructs and statements can be selected in generic form from a template menu in MAX+PLUS II.
25. VHDL designs can be embedded in a gdf as default symbols.
26. Pin numbers must be assigned to a design before it can be downloaded to a CPLD. Pins can be assigned in the Pin/Location/Chip dialog box (accessed by highlighting a pin symbol and right-clicking) or by editing the project’s Assignment and Configuration File (acf).
27. An Altera CPLD can be programmed directly from a PC parallel port via a ByteBlaster cable.
28. The ByteBlaster cable implements a programming interface specified by a standard (IEEE Std. 1149.1) of the Joint Test Action Group (JTAG).
29. A JTAG port is a 4-wire interface for loading test and programming information into one or more JTAG-compliant devices. It consists of an input (TDI), output (TDO), mode select (TMS), and clock (TCK).

**Glossary**

AHDL (Altera Hardware Description Language)  Altera’s proprietary text-entry design tool for PLDs.
Altera UP-1 Board  A circuit board, part of Altera’s University Program Design Laboratory Package, containing two CPLDs and a number of input and output devices.
Architecture  A VHDL structure than defines the relationship between input, output, and internal signals or variables in a design.
ASICS (application specific integrated circuits)  Integrated circuits that are constructed for a specific design purpose. The term could refer to a PLD, although it usually means a custom-designed fixed function device.
Assignment and Configuration File (acf)  A MAX+PLUS II file that contains information about the configuration options for a project, including assigned device and pin numbers.

**Bit string literal**  A group of bits assigned to the elements of a vector, enclosed in double quotes (e.g., "001011").
**ByteBlaster**  An Altera ribbon cable and connector used to program or configure Altera CPLDs via the parallel port (LPT port) of an IBM PC or compatible.
**Comment**  Explanatory text in a VHDL (or other computer language) file that is ignored by the computer at compile time.
**Compile**  The process used by CPLD design software to interpret design information (such as a drawing or text file) and create required programming information for a CPLD.
**Complex PLD (CPLD)**  A digital device consisting of several programmable sections with internal interconnections between the sections.
**Concurrent**  Simultaneous.
**Concurrent signal assignment**  A relationship between an input and output port or signal in which the output is changed as soon as there is a change in input. If the file has more than one concurrent signal assignment, they are all evaluated simultaneously.
**Default symbol**  A graphical symbol that represents a PLD design as a block, showing only the design’s inputs and outputs. The symbol can be used as a component in any Graphic Design File.
**Design entry**  The process of using software tools to describe the design requirements of a PLD. Design entry can be done by entering a schematic or a text file that describes the required digital function.
**Download**  Program a PLD from a computer running PLD design and programming software.
**Entity**  A VHDL structure that defines the inputs and outputs of a design.
**Fitting**  Assigning internal PLD circuitry, as well as input and output pins, for a PLD design.
**Graphic Design File (gdf)**  A PLD design file in which the digital design is entered as a schematic.
**Hardware description language**  A computer language used to design digital circuits by entering text-based descriptions of the circuits.
**Hierarchical design**  A PLD design that is ordered in layers or levels. The highest level of design contains components that are themselves complete designs. These components may, in turn, have lower-level designs embedded within them.
**ICR**  In-circuit reconfigurability. The ability of a PLD (such as a FLEX10K) to be configured without removing it from a circuit board.
**IEEE Standard 1164**  The standard which defines a variety of VHDL types and operations, including the STD_LOGIC and STD_LOGIC_VECTOR types.
**ISP**  In-system programmability. The ability of a PLD (such as a MAX7000S) to be programmed without removing it from a circuit board.
**JTAG**  Joint Test Action Group. A standards body that developed the format (called IEEE Std. 1149.1) for testing and programming devices while they are installed in a system.
**JTAG Chain**  Multiple JTAG-compliant devices whose TDI and TDO ports form a continuous chain connection. Such a chain allows multi-device programming.

**Library**  A collection of VHDL design units that have been previously compiled.

**MAX+PLUS II**  CPLD design and programming software owned by Altera Corporation.

**Mode (of a port)**  The kind of port, such as input or output.

**Nonvolatile**  Able to retain stored information after power is removed.

**Programming**  Transferring design information from the computer running PLD design software to the actual PLD chip.

**Project**  A set of MAX+PLUS II files associated with a particular PLD design.

**Schematic capture**  A technique of entering CPLD design information by using a CAD (computer aided design) tool to draw a logic circuit as a schematic. The schematic can then be interpreted by design software to generate programming information for the CPLD.

**Selected signal assignment statement**  A concurrent signal assignment in VHDL in which a value is assigned to a signal, depending on the alternative values of another signal or variable.

**Signal**  A name given to an internal connection in a VHDL architecture.

**Simulation**  Verifying design function by specifying a set of inputs and observing the resultant outputs. Simulation is generally shown as a series of input and output waveforms.

**Software tools**  Specialized computer programs used to perform specific functions such as design entry, compiling, fitting, and so on. (Sometimes just called “tools”.)

**SRAM Object File (sof)**  Binary file used to configure a PLD of the Altera FLEX series.

**Suite (of software tools)**  A related collection of tools for performing specific tasks. MAX+PLUS II is a suite of tools for designing and programming digital functions in a PLD.

**Syntax**  The “grammar” of a computer language (i.e., the rules of construction of language statements).

**Target device**  The specific PLD for which a digital design is intended.

**TCK**  Test Clock. The JTAG signal that drives the JTAG downloading process from one state to the next.

**Table 4.3  Pin Assignments for Multiplexer Circuit**

<table>
<thead>
<tr>
<th>Function</th>
<th>Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>12</td>
</tr>
<tr>
<td>S0</td>
<td>16</td>
</tr>
<tr>
<td>D0</td>
<td>15</td>
</tr>
<tr>
<td>D1</td>
<td>17</td>
</tr>
<tr>
<td>D2</td>
<td>21</td>
</tr>
<tr>
<td>D3</td>
<td>25</td>
</tr>
<tr>
<td>Y</td>
<td>4</td>
</tr>
</tbody>
</table>

**TDI**  Test Data In. In a JTAG port, the serial input data to a device.

**TDO**  Test Data Out. The JTAG signal, the serial output data from a device.

**FIGURE 4.48**  Problem 4.4

4-to-1 Multiplexer

**FIGURE 4.49**  Problem 4.9

4-channel Demultiplexer
TMS Test Mode Select. The JTAG signal that controls the downloading of test or programming data.

Top level (of a hierarchy) The file in a hierarchy that contains components specified in other design files and is not itself a component of a higher-level file.

<table>
<thead>
<tr>
<th>Function</th>
<th>Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>12</td>
</tr>
<tr>
<td>S0</td>
<td>16</td>
</tr>
<tr>
<td>D</td>
<td>15</td>
</tr>
<tr>
<td>Y0</td>
<td>4</td>
</tr>
<tr>
<td>Y1</td>
<td>6</td>
</tr>
<tr>
<td>Y2</td>
<td>8</td>
</tr>
<tr>
<td>Y3</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 4.5 Pin Assignments for Equality Comparator

<table>
<thead>
<tr>
<th>Function</th>
<th>Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>12</td>
</tr>
<tr>
<td>A2</td>
<td>16</td>
</tr>
<tr>
<td>B1</td>
<td>15</td>
</tr>
<tr>
<td>B2</td>
<td>17</td>
</tr>
<tr>
<td>AEQB</td>
<td>4</td>
</tr>
</tbody>
</table>

Variable A block of working memory used for internal calculation or storage in a VHDL architecture.

Vector A group of digital signals or variables, usually related numerically, that can be treated as a single multi-bit variable.

VHDL (VHSIC Hardware Description Language) An industry-standard computer language used to model digital circuits and produce programming data for PLDs.

Volatile A device is volatile if it does not retain its stored information.

VHSIC Very high speed integrated circuit
formation after the power to the device is removed.

PROBLEMS

Table 4.6  Pin Assignments for Full Adder

<table>
<thead>
<tr>
<th>Function</th>
<th>Pin</th>
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<tbody>
<tr>
<td>A</td>
<td>12</td>
</tr>
<tr>
<td>B</td>
<td>15</td>
</tr>
<tr>
<td>CARRY IN</td>
<td>33</td>
</tr>
<tr>
<td>SUM</td>
<td>6</td>
</tr>
<tr>
<td>CARRY OUT</td>
<td>4</td>
</tr>
</tbody>
</table>

Section 4.1  What Is a PLD?

4.1  List some of the advantages of programmable logic over fixed-function logic.

4.2  What does CPLD stand for? How is it different from the term PLD?

4.3  List some types of PLDs other than CPLDs.

4.4  Figure 4.48 shows a 4-to-1 multiplexer circuit. (The circuit switches one of four digital inputs to a single output, depending on the states of two “select inputs.”) State the number of 74HC type devices required to make this circuit. You may use the following devices: 74HC04 hex inverter; 74HC11 triple 3-input AND gate; 74HC4002 dual 4-input NOR gate (there are no 4-input OR devices available in the 74HC family). State how many devices are required to make two multiplexers.

Section 4.3  Graphic Design File

Section 4.4  Compiling MAX+PLUS II Files

4.5  Briefly describe the difference between a design file and a project in MAX+PLUS II.

4.6  State two ways to set the MAX+PLUS II project to the current file.

4.7  State the definitions of the following terms:

a. primitives
b. instance

4.8  Use MAX+PLUS II to create a Graphic Design File for the multiplexer circuit shown in Figure 4.48. Save the file as drive:\max2work\chapt4\problems\4to1mux.gdf. Assign pins as in Table 4.3. Set the project to the current file and compile.

4.9  Figure 4.49 shows the circuit for a 4-channel demultiplexer, which switches a digital input to one of four outputs, depending in the states of two “select inputs.”

Use MAX+PLUS II to create a Graphic Design File for the demultiplexer circuit. Save the file as drive:\max2work\chapt4\problems\4ch_dmux.gdf. Assign pins as in Table 4.4. Set the project to the current file and compile.

4.10 Repeat Problem 4.9 for the 2-bit equality comparator in Figure 4.50. This circuit generates a HIGH output when the two 2-bit numbers \( A_2A_1 \) and \( B_2B_1 \) are equal. Save the file as drive:\max2work\chapt4\problems\eq_comp.gdf. Use the pin assignments in Table 4.5.

4.11 Use MAX+PLUS II to create a Graphic Design File for the half-adder circuit shown in Figure 4.51. The half-adder adds 2 bits to generate a sum and a carry output. Save the file as drive:\max2work\chapt4\problems\halfadd.gdf. Create a default symbol for the file and compile, after setting the project to the current file. Do not assign pin numbers at this time.

4.12 Use MAX+PLUS II to create a Graphic Design File for the full adder circuit shown in Figure 4.52. The full adder combines two bits \( A \) and \( B \), plus an input carry from a previous stage to generate a sum and a carry output. Save the file as drive:\max2work\chapt4\problems\fulladd.gdf. Assign pin numbers as shown in Table 4.6. Set the project to the current file and compile.

4.13 Examine the half adder circuit in Figure 4.51 and the full adder circuit in Figure 4.52. You should find two half adders in the full adder circuit. Use the half adder symbol you created in Problem 4.11 to create a full adder as a hierarchical design, consisting of two half adders and other logic. Save the file as drive:\max2work\chapt4\problems\halfadd.gdf.
A number of standard combinational logic functions have been developed for digital circuits that represent many of the useful tasks that can be performed with digital circuits.

Decoders detect the presence of particular binary states and can activate other circuits based on their input values or can convert an input code to a different output code. Encoders generate a binary or binary coded decimal (BCD) code corresponding to an active input.

Multiplexers and demultiplexers are used for data routing. They select a transmission path for incoming or outgoing data, based on a selection made by a set of binary-related inputs.

CHAPTER 5

Combinational Logic Functions

OUTLINE

5.1 Decoders
5.2 Encoders
5.3 Multiplexers
5.4 Demultiplexers
5.5 Magnitude Comparators
5.6 Parity Generators and Checkers

CHAPTER OBJECTIVES

Upon successful completion of this chapter you will be able to:

- Design binary decoders using logic gates.
- Create decoder designs in MAX+PLUS II, using Graphic Design Files or VHDL.
- Create MAX+PLUS II simulation files to verify the operation of combinational circuits.
- Design BCD-to-seven-segment and hexadecimal-to-seven-segment decoders, including special features such as ripple blanking, using VHDL and Graphic Design Files in MAX+PLUS II.
- Use MAX+PLUS II Graphic Design Files and VHDL to generate the design for a 3-bit binary and a BCD priority encoder.
- Describe the circuit and operation of a simple multiplexer and program these functions in VHDL.
- Draw logic circuits for multiplexer applications, such as single-channel data selection, multibit data selection, waveform generation, and time-division multiplexing (TDM).
- Describe demultiplexer circuits and program them using VHDL.
- Define the operation of a CMOS analog switch and its use in multiplexers and demultiplexers.
- Define the operation of a magnitude comparator and program its function in VHDL.
- Explain the use of parity as an error-checking system and draw simple parity-generation and checking circuits.
Magnitude comparators determine whether one binary number is less than, greater than, or equal to another binary number.

Parity generators and checkers are used to implement a system of checking for errors in groups of data.

## 5.1 Decoders

**Key Terms**

**Decoder** A digital circuit designed to detect the presence of a particular digital state.

The general function of a decoder is to activate one or more circuit outputs upon detection of a particular digital state. The simplest decoder is a single logic gate, such as a NAND or AND, whose output activates when all its inputs are HIGH. When combined with one or more inverters, a NAND or AND can detect any unique combination of binary input values.

An extension of this type of decoder is a device containing several such gates, each of which responds to a different input state. Usually, for an $n$-bit input, there are $2^n$ logic gates, each of which decodes a different combination of input variables. A variation is a BCD device with 4 input variables and 10 outputs, each of which activates for a different BCD input.

Some types of decoders translate binary inputs to other forms, such as the decoders that drive seven-segment numerical displays, those familiar figure-8 arrangements of LED or LCD outputs (“segments”). The decoder has one output for every segment in the display. These segments illuminate in unique combinations for each input code.

### Single-Gate Decoders

The simplest decoder is a single gate, sometimes in combination with one or more inverters, used to detect the presence of one particular binary value. Figure 5.1 shows two such decoders, both of which detect an input $D_3D_2D_1D_0 = 1111$.

![Diagram of Single-Gate Decoders](image)

**FIGURE 5.1**

**Single-Gate Decoders**

The decoder in Figure 5.1a generates a logic HIGH when its input is 1111. The decoder in Figure 5.1b responds to the same input, but makes the output LOW instead.

In Figure 5.1, we designate $D_3$ as the most significant bit of the input and $D_0$ the least significant bit. We will continue this convention for multi-bit inputs.

In Boolean expressions, we will indicate the active levels of inputs and outputs separately. For example, in Figure 5.1, the inputs to both gates are the same, so we write $D_3D_2D_1D_0$ for the inputs of both gates. The gates in Figures 5.1a and b have outputs with opposite active levels, so we write the output variables as complements ($Y$ and $\bar{Y}$).

### EXAMPLE 5.1

Figure 5.2 shows three single-gate decoders. For each one, state the output active level and the input code that activates the decoder. Also write the Boolean expression of each output.
5.1 • Decoders

Solution  Each decoder is a NAND or AND gate. For each of these gates, the output is active when all inputs are HIGH. Because of the inverters, each circuit has a different code that fulfils this requirement.

Figure 5.2a: Output: Active LOW
Input code: $D_3D_2D_1D_0 = 1001$
$$\bar{Y} = D_3\bar{D}_2\bar{D}_1D_0$$

Figure 5.2b: Output: Active LOW
Input code: $D_2D_1D_0 = 001$
$$\bar{Y} = \bar{D}_2\bar{D}_1D_0$$

Figure 5.2c: Output: Active HIGH
Input code: $D_3D_2D_1D_0 = 1010$
$$Y = D_3\bar{D}_2D_1\bar{D}_0$$

Single-gate decoders are often used to activate other digital circuits under various operating conditions, particularly if there is a choice of circuits to activate. For example, single-gate decoders are used to enable peripheral devices in a personal computer (PC). A combination of binary values, called the address, specifies a unique set of conditions to enable a particular peripheral device.

EXAMPLE 5.2

A PC has two serial port cards called COM1 and COM2. Each card is activated when either one of two control inputs called $I/OR$ (Input/Output Read) and $I/OW$ (Input/Output Write) are active and a unique 10-bit address is present. $I/OR$ and $I/OW$ are active-LOW. The address is specified by bits $A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0$, which can be represented by three hexadecimal digits. The decoder outputs, $COM1\_Enable$ and $COM2\_Enable$ are both active-LOW.

The card for COM1 activates when ($I/OR$ OR $I/OW$ is LOW) AND the address is between 3F8H and 3FFH.

The card for COM2 activates when ($I/OR$ OR $I/OW$ is LOW) AND the address is between 2F8H and 2FFH.

Create a Graphic Design File in MAX+PLUS II that implements the specified decoder.

Solution  The lowest address that activates COM1 is

$$A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0 = 3F8H = 11\ 1111\ 1000$$

The highest COM1 address is

$$A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0 = 3FFH = 11\ 1111\ 1111$$
Since any address in this range is valid, we can represent the last three bits, $A_2A_1A_0$, as don’t care states. Thus, for COM1, we should decode the address:

$$A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0 = \text{11 1111 1XXX}$$

Similarly, for COM2:
- Low address: $A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0 = \text{2F8H} = \text{10 1111 1000}$
- High address: $A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0 = \text{2FFH} = \text{10 1111 1111}$
- Decode: $A_9A_8A_7A_6A_5A_4A_3A_2A_1A_0 = \text{10 1111 1XXX}$

Figure 5.3 shows the gdf representation of the decoder circuit, including inputs for the control signals $I_{OR}$ and $I_{OW}$.

**FIGURE 5.3**

Example 5.2
COM Port Decoders

### SECTION 5.1A REVIEW PROBLEM

5.1 Draw a single-gate decoder that detects the input state $D_3D_2D_1D_0 = \text{1100}$

a. with active-HIGH indication  

b. with active-LOW indication

### Multiple-Output Decoders

Decoder circuits often are constructed with multiple outputs. In effect, such a device is a collection of decoding gates controlled by the same inputs. A decoder circuit with $n$ inputs can activate up to $m = 2^n$ load circuits. Such a decoder is usually described an $n$-line-to-$m$-line decoder.
Figure 5.4 shows the logic circuit of a 2-line-to-4-line decoder. The circuit detects the presence of a particular state of the 2-bit input $D_1D_0$, as shown by the truth table in Table 5.1. One and only one output is HIGH for any input combination, provided the enable input $G$ is LOW. The active input of each line is shown in boldface. The subscript of the active output is the same as the value of the 2-bit input. For example, if $D_1D_0 = 10$, output $Y_2$ is active since 10 (binary) = 2 (decimal).

<table>
<thead>
<tr>
<th>$\bar{G}$</th>
<th>$D_1$</th>
<th>$D_0$</th>
<th>$Y_0$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
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<td>0</td>
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</table>

If we are using the decoder to activate one of four output loads, it is possible that there are situations where we want no output to be active. In such a case, we can deactivate all outputs (make them all LOW) by setting $\bar{G}$ HIGH.

We can create the 2-line-to-4-line decoder of Figure 5.4 as a graphic or text file in MAX+PLUS II and create a symbol for it that can be used in higher-level graphic files. Figure 5.5 shows the symbol for the decoder.
Figure 5.6 shows the circuit for a 3-line-to-8-line decoder, again with an active-LOW enable, \( G \). In this case, the decoder outputs are active LOW. One and only one output is active for any given combination of \( D_2D_1D_0 \). Table 5.2 shows the truth table for this decoder. Again if the enable line is HIGH, no output is active.

**Table 5.2**  Truth Table of a 3-to-8 Decoder with Enable

<table>
<thead>
<tr>
<th>( \bar{G} )</th>
<th>( D_2 )</th>
<th>( D_1 )</th>
<th>( D_0 )</th>
<th>( \bar{Y}_0 )</th>
<th>( \bar{Y}_1 )</th>
<th>( \bar{Y}_2 )</th>
<th>( \bar{Y}_3 )</th>
<th>( \bar{Y}_4 )</th>
<th>( \bar{Y}_5 )</th>
<th>( \bar{Y}_6 )</th>
<th>( \bar{Y}_7 )</th>
</tr>
</thead>
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</tbody>
</table>

**EXAMPLE 5.3**

Figure 5.7 shows a partial Graphic Design File, created in MAX+PLUS II, that shows how a 3-line-to-8-line decoder, such as the one shown in Figure 5.6, can be used in a microcomputer memory system as an address decoder. Each block labeled 8k_sram is a memory chip capable of holding 8192 (8K) bytes of data. Since there are eight such devices, the
The whole system can hold \( 8 \times 8192 = 65,536 \) (64K) bytes. (Although this amount of memory may seem small by the standards of a desktop computer, it may be typical of a small stand-alone computer system (called an embedded system or a microcontroller) that is used in control applications.)

Each 8K block is enabled by a LOW at its \( G \) input. Briefly explain the function of the decoder in the system.

Solution Since only one decoder output is LOW at any one time, the decoder allows only one memory block to be active at any one time. The active block is chosen by inputs \( ADDR_{15:13} \), which are connected to \( D_2D_1D_0 \) on the decoder. The active memory block is the one connected to the \( y \) output whose subscript matches the binary value of these inputs. For example, when \( ADDR_{15:13} = 110 \), the block connected to \( y6 \) is active.

If the decoder is the same as the one in Figure 5.6, no outputs will be active, and therefore no memory block will be enabled, when \( \overline{G} = 1 \). (Note that the MAX+PLUS II Graphic Editor cannot represent an input or output with an inversion bar. Some conventions would represent an active-LOW terminal with an “n” prefix, indicating “NOT” (e.g., \( nG \)). This is a matter of personal choice, but without such an indication it is not possible to tell the active level of an input or output from the MAX+PLUS II Graphic Design File.)

The decoders in Figure 5.6 and 5.7 have identical functions, but the symbol in Figure 5.7 shows the \( D \) inputs and \( Y \) outputs as multibit vectors or busses. Figure 5.7 also shows how the individual signals in a bus can be connected to separate parts of the circuit in a MAX+PLUS II Graphic Design File.

To make the connections, draw and label a line extending from each terminal. To label a line, highlight the line by clicking on it with the left mouse button, then right-click. Select Enter Node/Bus Name from the pop-up menu and enter the text. Lines that have the same names are automatically connected by their text references. If a line is a multiple line,
it must have signal designators in brackets (e.g., \(y[0..7]\)). Individual signals from a bus must be numbered in a way that corresponds to the multiple-bit line (e.g., \(y_0, y_1, y_2\), and so on).

SECTION 5.1B REVIEW PROBLEM

5.2 How many inputs are required for a binary decoder with 16 outputs? How many inputs are required for a decoder with 32 outputs?

Simulation of a 2-Line-to-4-Line Decoder

**KEY TERMS**

- **Timing diagram**: A diagram showing how two or more digital waveforms in a system relate to each other over time.
- **Simulation**: The verification, using timing diagrams, of the logic of a digital design before programming it into a PLD.
- **Stimulus waveforms**: A set of user-defined input waveforms in a simulator file designed to imitate input conditions of a digital circuit.
- **Response waveforms**: A set of output waveforms generated by a simulator for a particular digital design in response to a set of stimulus waveforms.
- **Propagation delay**: Time difference between a change on a digital circuit input and a change on an output in response to the input change.

An important part of the CPLD design process is simulation of the design. A simulation tool allows us to see whether the output responses to a set of circuit inputs are what we expected in our initial design idea. The simulator works by creating a timing diagram. We specify a set of input (stimulus) waveforms. The simulator looks at the relationship between inputs and outputs, as defined by the design file, and generates a set of response outputs.

Figure 5.8 shows a set of simulation waveforms created for the 2-line-to-4-line decoder in Figure 5.4. The inputs D1 and D0 are combined as a single 2-bit value, to which an increasing binary count is applied as a stimulus. The decoder output waveforms are observed individually to determine the decoder’s response. Once we have entered the design in the MAX+PLUS II Graphic Editor and compiled it, we can create the waveforms as follows.

**FIGURE 5.8**
Simulation Waveforms for a 2-to-4 Decoder with Enable

From the File menu, select New. On the resultant dialog box, select Waveform Editor File, with a default file extension scf. From the File menu, choose Save As, then enter `drive:\max2work\chapt05\decoders\2to4dcdr.scf`. 

2to4dcdr.gdf
2to4dcdr.scf
We specify the inputs and outputs we want to view by selecting **Enter Nodes from SNF** on the **Node** menu, shown in Figure 5.9. In the dialog box that pops up (Figure 5.10), there are two boxes labelled **Available Nodes & Groups** and **Selected Nodes & Groups**, with an arrow (=>) pointing from one to the other. Select the **List** button to show the “available” signals and click the arrow to transfer them all to the “selected” box. Click **OK** to close the box.

Figure 5.11 shows the simulation waveforms in their uninitialized (default) states. Inputs and outputs are shown by symbols in front of the signal names. Inputs are at logic 0 and outputs are indicated as X or unknown values.
We now set the timing length of the simulation. The default value is 1 μs, written 1.0us. For this example, we will leave the end time at the default value. However, if we want to change it, we select End Time (File menu, Figure 5.12) and enter the new time for the end of simulation in the dialog box of Figure 5.13. Click OK.

The End Time dialog sets the end of the simulation. We should also set the Grid Size, which determines the size of the smallest time division in the simulation. To do so, select Grid Size from the Options menu, shown in Figure 5.14. In the dialog box of Figure 5.15, enter the value 20ns and click OK. (We will use this value for many of our simulations.)
because it corresponds to one half period of the oscillator on the Altera UP-1 board. In the simulator, one full period requires two grid spaces.)

When we created the simulation file, the D inputs were entered as separate waveforms. We can join these waveforms to make a Group. Highlight both D1 and D0 by clicking on one name and dragging the mouse to the next name, as in Figure 5.16. From the Node menu or the pop-up menu in Figure 5.17, select Enter Group. The dialog box shown in Figure 5.18 appears, containing the most likely name derived from the highlighted group. Either type a new group name or accept the original name by clicking OK.

**FIGURE 5.16**
Highlighting a Group

**FIGURE 5.17**
Pop-up Menu (Enter Group)

**FIGURE 5.18**
Enter Group Dialog Box
As a decoder stimulus, we will define an increasing binary count on the D inputs. Highlight the input group by clicking in the Value column. Use the Overwrite Count toolbar button to create an increasing binary count on the group, D[1..0]. Fill in the dialog box as shown in Figure 5.19 and click OK. The count is increased every 40 ns (2 × 20 ns), as shown in Figure 5.20.

Save the file. From the MAX+/PLUS II menu, bring the Simulator to the front and click Start. When the simulation is finished (almost immediately), click Open SCF and maximize the window. From the View menu, select Fit in Window or select the toolbar button for this function.

The simulator output, shown in Figure 5.21, shows the result of a repeating binary count at the decoder input when the outputs are always enabled. The outputs activate in a repeating sequence, from Y0 to Y3.

You will notice that the D inputs change exactly on the grid lines, but the Y outputs change slightly after. This is due to propagation delay, defined as the time between an
input change and the time an output changes in response to that input. In the EPM7128SLC84-7 CPLD, for which this simulation is created, propagation delay is about 7 nanoseconds. (The MAX+PLUS II simulator accounts for the propagation delay in different CPLDs.) Later simulations in this chapter will not necessarily show the delay, as the timing chosen may be very long compared to delay times.

To see the result of the enable input, highlight the G waveform from approximately 500 ns to 1 µs by dragging the mouse along this part of the waveform. Overwrite the highlighted part by clicking the Overwrite with HIGH button. When we run the simulation again, we get the waveforms shown in Figure 5.8.

VHDL Binary Decoder

**KEY TERMS**

**Selected signal assignment statement** A concurrent signal assignment in VHDL in which a value is assigned to a signal, depending on the alternative values of another signal or variable.

**Conditional signal assignment statement** A concurrent VHDL construct that assigns a value to a signal, depending on a sequence of conditions being true or false.

In Chapter 4, we saw an example of how we can use VHDL to define the function of a 2-line-to-4-line decoder. For reference the description is replicated below, with the difference that the input and output ports are defined as BIT rather than STD_LOGIC types. (This is sufficient for a combinational circuit like a decoder, as the only I/O (input/output) values required are ‘0’ and ‘1’. If we use BIT types, we do not require a reference to the IEEE library, as we do to define STD_LOGIC types.)

```vhdl
ENTITY decode1 IS
  PORT(
    d1, d0     : IN BIT;
    y0, y1, y2, y3 : OUT BIT);
END decode1;
ARCHITECTURE decoder1 OF decode1 IS
BEGIN
  y0  <= (not d1) and (not d0);
  y1  <= (not d1) and (    d0);
  y2  <= (    d1) and (not d0);
  y3  <= (    d1) and (    d0);
END decoder1;
```

The above formulation has no enable input. If we wish to include the enable function, we must modify the entity declaration to include that input and change the signal assignment statements, as well. The new VHDL code is as follows.

```vhdl
ENTITY decode2 IS
  PORT(
    d1, d0, g      : IN BIT;
    y0, y1, y2, y3 : OUT BIT);
END decode2;
ARCHITECTURE decoder2 OF decode2 IS
BEGIN
  y0  <= (not d1) and (not d0) and (not g);
  y1  <= (not d1) and (    d0) and (not g);
  y2  <= (    d1) and (not d0) and (not g);
  y3  <= (    d1) and (    d0) and (not g);
END decoder2;
```
In addition to coding the Boolean expressions directly, we can use two types of concurrent signal assignments to create decoder circuits: the **selected signal assignment statement** and the **conditional signal assignment statement**. Both the Altera VHDL manual and the Help menu in MAX+PLUS II have a section on “Golden Rules” for VHDL. The VHDL Golden Rules suggest that you should use a selected signal assignment rather than a conditional signal assignment, if possible. This is because, in certain cases, the selected signal assignment uses the internal circuitry of the CPLD more efficiently.

The selected signal assignment has the form:

```
label: WITH __expression SELECT
__signal <=__expression WHEN __constant_value,
__expression WHEN __constant_value,
__expression WHEN __constant_value,
__expression WHEN __constant_value;
```

The signal indicated in the second line of the statement template is assigned one of several expressions, depending on the constant value of the expression in the first line. The label is optional. Examine the selected signal statement below:

```
circuit: WITH mode SELECT
    y   <=  q      WHEN "00"
       not q WHEN "01",
       p     WHEN "11",
       '1'  WHEN others;
```

Signal `y` is assigned one of three values, `p`, `q`, or `not q`, depending on the status of a two-bit variable called `mode`. Note that the value of `y` for the case when `mode` = "10" is not explicitly stated. This is covered by the last clause (WHEN others), which defines a default value for signal `y` of logic 1.

The following VHDL code implements a 2-line-to-4-line decoder using a selected signal assignment statement.

```
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY decode3 IS
    PORT(
        d  : IN   STD_LOGIC_VECTOR (1 downto 0);
        y : OUT  STD_LOGIC_VECTOR (3 downto 0));
END decode3;
ARCHITECTURE decoder OF decode3 IS
BEGIN
    WITH d SELECT
        y <=  "0001" WHEN "00",
             "0010" WHEN "01",
             "0100" WHEN "10",
             "1000" WHEN "11",
             "0000" WHEN others;
END decoder;
```

The selected signal assignment statement evaluates input `d`. For every possible combination of the 2-bit input vector, `d`, a particular value is assigned to the 4-bit vector, `y`. (For example, for the case `d = 10` (= `2_{10}`), the output `y` is HIGH: `y` = 100.)

The default case (“WHEN others”) is required because of the multivalued logic type STD_LOGIC VECTOR. Since a STD_LOGIC VECTOR can have values other than ‘0’ and ‘1’, the values listed for `d` don’t cover all possible cases. The default output (which will never occur if we only use ‘0’ and ‘1’ inputs) is chosen such that no output is active in the
default case. The default case would not be required if we chose to use BIT_VECTOR, rather than STD_LOGIC_VECTOR, since the listed combinations of \( d \) cover all possible combinations of a BIT_VECTOR. However, it is a good practice to include the default case, in order to account for all possible contingencies.

In order to include an enable input \((g)\) in a decoder, we can increase the input vector size to include the \( g \) input, as shown in the following code.

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY decode3a IS
  PORT(
    d  : IN    STD_LOGIC_VECTOR (1 downto 0);
    g  : IN    STD_LOGIC;
    y  : OUT   STD_LOGIC_VECTOR (3 downto 0));
END decode3a;

ARCHITECTURE decoder OF decode3a IS
  SIGNAL inputs : STD_LOGIC_VECTOR (2 downto 0);
BEGIN
  inputs(2) <= g;
  inputs(1 downto 0) <= d;
  WITH inputs SELECT
    y <=    "0001" WHEN "000",
          "0010" WHEN "001",
          "0100" WHEN "010",
          "1000" WHEN "011",
          "0000" WHEN others;
END decoder;
```

To include \( g \) and \( d \) in a single vector, we create a signal called `inputs`, a vector with three elements in the sequence \( g, d(1), d(0) \). When assigning the \( d \) to the last two elements of `inputs`, we must be explicit about which elements of `inputs` we want to use. Since \( d \) only contains two elements and we are assigning them to two elements of `inputs`, we don’t need to list the elements of \( d \) explicitly.

We can use a selected signal assignment statement to evaluate all inputs, including \( g \), and assign outputs accordingly. When \( g = 0 \), the decoder outputs are assigned the same as they were in the example without the enable input. The cases where \( g = 1 \) are covered by the `others` clause. In this default case, all decoder outputs are LOW (inactive).

Another way to include an enable input is to use a conditional signal assignment statement, which makes an assignment based on a Boolean expression. This template for the conditional signal assignment statement is:

```vhdl
__signal <= __expression WHEN __boolean_expression ELSE __expression WHEN __boolean_expression ELSE __expression;
```

The first Boolean expression in the statement is evaluated. If it is true, the corresponding expression is assigned to the signal. If false, the next Boolean expression is evaluated, and so on until a true Boolean expression is found. If none are true, the signal is assigned a default expression, listed last in the statement.

The VHDL code below implements the decoder with an active-LOW enable. If \( g \) is LOW, one decoder output activates, depending on the value of \( d \). Note that the \( d \) inputs are defined as type INTEGER, rather than BIT_VECTOR or STD_LOGIC_VECTOR. In this situation, we don’t need to specify the number of inputs; the compiler automatically defines the required inputs \( d_1 \) and \( d_0 \) when fitting the design to the selected CPLD. Also, since \( d \) is of type INTEGER, we write its value in the selected signal assignment statement directly, without quotes.
LIBRARY ieee;
USE ieee.std_logic_1164.all;

ENTITY decode4g IS
PORT(
    d  : IN   INTEGER RANGE 0 to 3;
    g  : IN   STD_LOGIC;
    y  : OUT  STD_LOGIC_VECTOR (0 to 3));
END decode4g;

ARCHITECTURE a OF decode4g IS
BEGIN
    y <=   "1000" WHEN (d=0 and g='0') ELSE
           "0100" WHEN (d=1 and g='0') ELSE
           "0010" WHEN (d=2 and g='0') ELSE
           "0001" WHEN (d=3 and g='0') ELSE
           "0000";
END a;

MAX+PLUS II Report File

In the Altera Golden Rules, we are told to choose a selected signal assignment over a conditional signal assignment because it uses the CPLD resources more efficiently. How do we check this assertion? Is it always true? This information is stored in a MAX+PLUS II report file (rpt), which is created at compile time.

The compile process of MAX+PLUS II goes on behind the scenes; until now we have not enquired about the result of this process. One of many functions of the compiler is to reduce the design information in a graphic or text file to a series of Boolean equations that can be programmed into a PLD.

For example, the report file decode3a.rpt, for the file that uses the selected signal assignment, gives us the following information under the EQUATIONS heading.

** EQUATIONS **

d0     : INPUT;
d1     : INPUT;
g      : INPUT;

-- Node name is 'y0'
-- Equation name is 'y0', location is LC117, type is output.
y0     = LCELL( _EQ001 $  GND);
_EQ001 = !d0 & !d1 & !g;

-- Node name is 'y1'
-- Equation name is 'y1', location is LC115, type is output.
y1     = LCELL( _EQ002 $  GND);
_EQ002 =  d0 & !d1 & !g;

-- Node name is 'y2'
-- Equation name is 'y2', location is LC118, type is output.
y2     = LCELL( _EQ003 $  GND);
_EQ003 = !d0 &  d1 & !g;

-- Node name is 'y3'
-- Equation name is 'y3', location is LC120, type is output.
y3     = LCELL( _EQ004 $  GND);
_EQ004 =  d0 &  d1 & !g;

Each output is designated as a node. Let us examine the equation of one node in detail so that we will know how to interpret the others.
The Boolean format in the report file uses different operators than VHDL. They are as follows:

\( ! = \text{NOT} \)
\( & = \text{AND} \)
\( # = \text{OR} \)
\( $ = \text{XOR} \)

Thus, the equation given as \(_{EQ001} = !d_0 & !d_1 & !g\) is equivalent to the Boolean expression \(_{EQ001} = d_0 \cdot d_1 \cdot g\).

In the expression \((y_0 = \text{LCELL (}_{EQ001} \; \text{XOR} \; \text{GND});\)), equation \(_{EQ001}\) is XORed with GND (logic 0) and applied to an LCELL (logic cell) primitive to yield \(y_0\). The LCELL represents one output of the CPLD. The XOR function is a way to either invert or not invert a logic function by setting one XOR input to GND (noninverting) or VCC (inverting). Thus \(_{EQ001}\) is applied to a CPLD output without inversion.

A comment in the report file indicates that \(y_0\) is assigned to logic cell LC117 (out of 128), which corresponds to pin 75 (out of 84) on the CPLD. Other equations are assigned to other LCELLs with other Boolean functions, as appropriate. Every pin number on the CPLD package is permanently connected to a specific LCELL. The compiler chooses the LCELL/pin assignments automatically; if we desire specific pin number assignments, we must assign them explicitly before compiling.

How does this compare with the report file for the design with the conditional signal assignment? If you examine decode4g.rpt, you will find that the Boolean equations are exactly the same. Thus, we can conclude that for a simple function, such as a 2-line-to-4-line decoder with enable, the two statement forms are easy enough for the compiler to interpret both in the most efficient way.

### Seven-Segment Decoders

**KEY TERMS**

- **Seven-segment display**: An array of seven independently controlled light-emitting diode (LED) or liquid crystal display (LCD) elements, shaped like a figure-8, which can be used to display decimal digits and other characters by turning on the appropriate elements.

- **Common anode display**: A seven-segment LED display where the anodes of all the LEDs are connected to the circuit supply voltage. Each segment is illuminated by a logic LOW at its cathode.

- **Common cathode display**: A seven-segment display in which the cathodes of all LEDs are connected together and grounded. A logic HIGH illuminates a segment when applied to its anode.

**Display**

The **seven-segment display**, shown in Figure 5.22, is a numerical display device used to show digital circuit outputs as decimal digits (and sometimes hexadecimal digits or other alphabetic characters). It is called a seven-segment display because it consists of seven luminous segments, usually LEDs or liquid crystals, arranged in a figure-8. We can display any decimal digit by turning on the appropriate elements, designated by lowercase letters, \(a\) through \(g\). It is conventional to designate the top segment as \(a\) and progress clockwise around the display, ending with \(g\) as the center element.

Figure 5.23 shows the usual convention for decimal digit display. Some variation from this convention is possible. For example, we could have drawn the digits 6 and 9 with “tails” (i.e., with segment \(a\) illuminated for 6 or segment \(d\) for 9). By convention, we
display digit 1 by illuminating segments $b$ and $c$, although segments $e$ and $f$ would also work.

The electrical requirements for an LED circuit are simple. Since an LED is a diode, it conducts when its anode is positive with respect to its cathode, as shown in Figure 5.24a. A decoder/driver for an LED display will illuminate an element by completing this circuit, either by supplying $V_{CC}$ or ground. A series resistor limits the current to prevent the diode from burning out and to regulate its brightness. If the anode is +5 volts with respect to cathode, the resistor value should be in the range of 220 $\Omega$ to 470 $\Omega$.

Seven-segment displays are configured as common anode or common cathode, as shown in Figures 5.24b and c. In a common cathode display, the cathodes of all LEDs are connected together and brought out to one or more pin connections on the display package. The cathode pins are wired externally to the circuit ground. We illuminate the segments by applying logic HIGHs to individual anodes.

Similarly, the common anode display has the anodes of the segments brought out to one or more common pins. These pins must be tied to the circuit power supply ($V_{CC}$). The segments illuminate when a decoder/driver makes their individual cathodes LOW. Figure 5.25 shows how the diodes could be physically laid out in a common anode display.

The two types of displays allow the use of either active HIGH or active LOW circuits to drive the LEDs, thus giving the designer some flexibility. However, it should be noted that the majority of seven-segment decoders are for common-anode displays.
EXAMPLE 5.4

Sketch the segment patterns required to display all 16 hexadecimal digits on a seven-segment display. What changes from the patterns in Figure 5.23 need to be made?

Solution

The segment patterns are shown in Figure 5.26.

![Segment Patterns](image)

Hex digits B and D must be displayed as lowercase letters, b and d, to avoid confusion between B and 8 and between D and 0. To make 6 distinct from b, 6 must be given a tail (segment a) and to make 6 and 9 symmetrical, 9 should also have a tail (segment d).

Decoder

**KEY TERMS**

**BCD** Binary coded decimal. A code in which each individual digit of a decimal number is represented by a 4-bit binary number (e.g., 905 (decimal) = 1001 0000 0101 (BCD)).

A BCD-to-seven-segment decoder is a circuit with a 4-bit input for a BCD digit and seven outputs for segment selection. To display a number, the decoder must translate the input bits to a combination of active outputs. For example, the input digit $D_3D_2D_1D_0 = 0000$ must illuminate segments $a$, $b$, $c$, $d$, $e$, and $f$ to display the digit 0. We can make a truth
table for each of the outputs, showing which must be active for every digit we wish to display. The truth table for a common-anode decoder (active LOW outputs) is given in Table 5.3.

<table>
<thead>
<tr>
<th>Digit</th>
<th>( D_3 )</th>
<th>( D_2 )</th>
<th>( D_1 )</th>
<th>( D_0 )</th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( d )</th>
<th>( e )</th>
<th>( f )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>0</td>
<td>1</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>0</td>
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<td>0</td>
<td>0</td>
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</tr>
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<tr>
<td>8</td>
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</tr>
<tr>
<td>9</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
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</tr>
<tr>
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<td>1</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

The illumination of each segment is determined by a Boolean function of the input variables, \( D_3D_2D_1D_0 \). From the truth table, the function for segment \( a \) is

\[
a = \overline{D_3}D_2\overline{D_1}D_0 + \overline{D_3}D_2D_1\overline{D_0} + \overline{D_3}D_2D_1D_0
\]

(Since the display is active-LOW, this means segment \( a \) is OFF for digits 1, 4, and 6.)

If we assume that inputs 1010 to 1111 are never going to be used (“don’t care states”, symbolized by \( X \)), we can make any of these states produce HIGH or LOW outputs, depending on which is most convenient for simplifying the segment functions. Figure 5.27a shows a Karnaugh map simplification for segment \( a \). The resultant function is

\[
a = \overline{D_3}D_2\overline{D_1}D_0 + D_3D_2D_1D_0
\]

The corresponding partial decoder is shown in Figure 5.27b.

We could do a similar analysis for each of the other segments, but if we are programming the decoder function into a CPLD, it is just as simple to write the truth table directly into a selected signal assignment statement, as shown in the VHDL code that follows.

```vhdl
-- bcd_7seg.vhd
-- BCD-to-seven-segment decoder

ENTITY bcd_7seg IS
    PORT(
        d3, d2, d1, d0 : IN BIT;
        a, b, c, d, e, f, g : OUT BIT);
END bcd_7seg;

ARCHITECTURE seven_segment OF bcd_7seg IS
    SIGNAL input : BIT_VECTOR (3 downto 0);
    SIGNAL output: BIT_VECTOR (6 DOWNTO 0);
    BEGIN
        input <= d3 & d2 & d1 & d0;
        WITH input SELECT
            output <= "0000001" WHEN "0000",
                     "1001111" WHEN "0001",
```
“0010010” WHEN “0010”,
“0000110” WHEN “0011”,
“1001100” WHEN “0100”,
“0100100” WHEN “0101”,
“1100000” WHEN “0110”,
“0001111” WHEN “0111”,
“0000000” WHEN “1000”,
“0001100” WHEN “1001”,
“1111111” WHEN others;

-- Separate the output vector to make individual pin outputs.
  a   <= output(6);
  b   <= output(5);
  c   <= output(4);
  d   <= output(3);
  e   <= output(2);
  f   <= output(1);
  g   <= output(0);
END seven_segment;

FIGURE 5.27
Decoding Segment a

The inputs D3D2D1D0 are defined separately, then concatenated (linked in sequence) by the & operator to make a BIT_VECTOR called input. This is equivalent to the following four concurrent signal assignments:

```vhdl
input (3) <= d3;
input (2) <= d2;
input (1) <= d1;
input (0) <= d0;
```
Why not simply define \( \mathbf{d} \) as a vector? If we wish to create a graphic symbol for the seven-segment decoder, the above method creates a symbol shown with four separate inputs, rather than a single thick line for a 4-bit bus input. The design will work either way.

For each value of input, a signal assignment defines the output vector, each bit of which represents the value of one segment. For example, the first clause ("0000001" WHEN "0000") sets all segments ON except segment \( g \), thus displaying the digit ’0’.

As a variation, we could define a signal called \( \mathbf{d}_\text{inputs} \) of type \text{INTEGER} with RANGE 0 to 9. The WHEN clauses would evaluate the integer values 0 to 9, as follows.

```vhdl
WITH \mathbf{d}_\text{inputs} \text{ SELECT}
  \text{output} <= "0000001" \text{ WHEN 0},
    "1001111" \text{ WHEN 1},
    "0010010" \text{ WHEN 2},
    "0001110" \text{ WHEN 3},
    "1001100" \text{ WHEN 4},
    "0100100" \text{ WHEN 5},
    "0100000" \text{ WHEN 6},
    "0001111" \text{ WHEN 7},
    "0000000" \text{ WHEN 8},
    "0000100" \text{ WHEN 9},
    "1111111" \text{ WHEN others}; —— blank
```

**Ripple Blanking**

<table>
<thead>
<tr>
<th>KEY TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ripple blanking</strong></td>
</tr>
<tr>
<td><strong>RBI</strong></td>
</tr>
<tr>
<td><strong>RBO</strong></td>
</tr>
<tr>
<td><strong>PROCESS</strong></td>
</tr>
<tr>
<td><strong>Sensitivity list</strong></td>
</tr>
<tr>
<td><strong>CASE statement</strong></td>
</tr>
<tr>
<td><strong>IF statement</strong></td>
</tr>
</tbody>
</table>

A feature often included in seven-segment decoders is **ripple blanking**. The ripple blanking feature allows for suppression of leading or trailing zeros in a multiple digit display, while allowing zeros to be displayed in the middle of a number.

Each display decoder has a ripple blanking input (\( \text{RBI} \)) and a ripple blanking output (\( \text{RBO} \)), which are connected in cascade, as shown in Figure 5.28. If the decoder input \( D_3D_2D_1D_0 \) is 0000, it displays digit 0 if \( \text{RBI} = 1 \) and shows a blank if \( \text{RBI} = 0 \).

If \( \text{RBI} = 1 \) OR \( D_3D_2D_1D_0 \) is (NOT 0000), then \( \text{RBO} = 1 \). When we cascade two or more displays, these conditions suppress leading or trailing zeros (but not both) and still display internal zeros.

To suppress leading zeros in a display, ground the \( \text{RBI} \) of the most significant digit decoder and connect the \( \text{RBO} \) of each decoder to the \( \text{RBI} \) of the next least significant digit. Any zeros preceding the first nonzero digit (9 in this case) will be blanked, as \( \text{RBI} = 0 \) AND \( D_3D_2D_1D_0 = 0000 \) for each of these decoders. The 0 inside the number 904 is displayed since its \( \text{RBI} = 1 \).
Trailing zeros are suppressed by reversing the order of $RBI$ and $RBO$ from the above example. $RBI$ is grounded for the least significant digit and the $RBO$ for each decoder cascade to the $RBI$ of the next most significant digit.

We can implement the ripple blanking feature in a VHDL file by modifying the file for a standard BCD- or hexadecimal-to-seven-segment decoder to include a CASE statement within a PROCESS. A PROCESS is a construct containing statements that are executed if a signal in the sensitivity list of the PROCESS changes. The general form of a PROCESS is:

```
PROCESS (sensitivity list)
BEGIN
    statements;
END PROCESS;
```

A CASE statement can be one of the constructs used inside a process if we want to select among several alternatives. It takes the following form:
## Combinational Logic Functions

---

**CASE statement within a PROCESS**

```vhdl
PROCESS (__signal_name, __signal_name, __signal_name)
BEGIN
  CASE __expression IS
    WHEN __constant_value =>
      __statement;
      __statement;
    WHEN __constant_value =>
      __statement;
      __statement;
    WHEN OTHERS =>
      __statement;
      __statement;
  END CASE;
END PROCESS;
```

Whether the digit “0” is displayed or suppressed is conditional upon the value of \( R \). This can be tested by an **IF statement** within the PROCESS. An IF statement executes one or more VHDL statements, depending on the state of a test condition. It has the following syntax.

```vhdl
IF __expression THEN
  __statement;
  __statement;
ELSIF __expression THEN
  __statement;
  __statement;
ELSE
  __statement;
  __statement;
END IF;
```

The following VHDL code demonstrates the ripple blanking function.

```vhdl
ENTITY sevsegrb IS
  PORT(
    nRBI, d3, d2, d1, d0 : IN BIT;
    a, b, c, d, e, f, g, nRBO : OUT BIT);
END sevsegrb;
ARCHITECTURE seven_segment OF sevsegrb IS
  SIGNAL input: BIT_VECTOR (3 DOWNTO 0);
  SIGNAL output: BIT_VECTOR (6 DOWNTO 0);
BEGIN
  input  <=  d3 & d2 & d1 & d0;
  --  Process Statement
  PROCESS (input, nRBI)
  BEGIN
    IF (input = "0000" and nRBI = '0') THEN
      -- 0 suppressed
      output  <= "1111111";
      nRBO    <= '0';
    ELSIF (input = "0000" and nRBI = '1') THEN
      -- 0 displayed
      output  <= "0000001";
      nRBO    <= '1';
    ELSE
      CASE input IS
        WHEN "0001"   => output <= "1001111"; -- 1
```
WHEN "0010" => output <= "0010010"; —— 2
WHEN "0011" => output <= "0000110"; —— 3
WHEN "0100" => output <= "1001100"; —— 4
WHEN "0101" => output <= "0100100"; —— 5
WHEN "0110" => output <= "0100000"; —— 6
WHEN "0111" => output <= "0001111"; —— 7
WHEN "1000" => output <= "0000000"; —— 8
WHEN "1001" => output <= "0000100"; —— 9
WHEN others => output <= "1111111"; —— blank
END CASE;
nRBO <= '1';
END IF;

-- Separate the output vector to make individual pin outputs.
a <= output(6);
b <= output(5);
c <= output(4);
d <= output(3);
e <= output(2);
f <= output(1);
g <= output(0);

END PROCESS;
END seven_segment;

\section{SECTION 5.1C REVIEW PROBLEM}

5.3 When would it be logical to suppress trailing zeros in a multiple-digit display and when should trailing zeros be displayed?

\section{5.2 Encoders}

\begin{center}
\textbf{KEY TERMS}
\end{center}

\begin{itemize}
\item \textbf{Encoder} A circuit that generates a binary code at its outputs in response to one or more active input lines.
\item \textbf{Priority encoder} An encoder that generates a binary or BCD output corresponding to the subscript of the active input having the highest priority. This is usually defined as the input with the largest subscript value.
\end{itemize}

The function of a digital encoder is complementary to that of a digital decoder. A decoder activates a specified output for a unique digital input code. An encoder operates in the reverse direction, producing a particular digital code (e.g., a binary or BCD number) at its outputs when a specific input is activated.

Figure 5.29 shows an 3-bit binary encoder. The circuit generates a unique 3-bit binary output for every active input provided only one input is active at a time.

The encoder has only 8 permitted input states out of a possible 256. Table 5.4 shows the allowable input states, which yield the Boolean equations used to design the encoder. These Boolean equations are:

\[
\begin{align*}
Q_2 &= D_7 + D_6 + D_3 + D_4 \\
Q_1 &= D_7 + D_6 + D_3 + D_2 \\
Q_0 &= D_7 + D_5 + D_3 + D_1 
\end{align*}
\]

The $D_0$ input is not connected to any of the encoding gates, since all outputs are in their LOW (inactive) state when the 000 code is selected.
Priority Encoder

The shortcoming of the encoder circuit shown in Figure 5.29 is that it can generate wrong codes if more than one input is active at the same time. For example, if we make $D_3$ and $D_5$ HIGH at the same time, the output is neither 011 or 101, but 111; the output code does not correspond to either active input.

One solution to this problem is to assign a priority level to each input and, if two or more are active, make the output code correspond to the highest-priority input. This is called a **priority encoder**. Highest priority is assigned to the input whose subscript has the largest numerical value.

### Example 5.5

Figures 5.30a through c show a priority encoder with three different combinations of inputs. Determine the resultant output code for each figure. Inputs and outputs are active HIGH.

<table>
<thead>
<tr>
<th>$D_7$</th>
<th>$D_6$</th>
<th>$D_5$</th>
<th>$D_4$</th>
<th>$D_3$</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>0</td>
<td>1</td>
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<td>0</td>
<td>0</td>
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<td>1</td>
<td>0</td>
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<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Figure 5.29**
3-bit Encoder (No Input Priority)

**Figure 5.30**
Example 5.5
Priority Encoder Inputs
Solution

**Figure 5.30a:** The highest-priority active input is $D_5$. $D_4$ and $D_1$ are ignored. $Q_2Q_1Q_0 = 101$.

**Figure 5.30b:** The highest-priority active input is $D_4$. $D_1$ is ignored. $Q_2Q_1Q_0 = 100$.

**Figure 5.30c:** The highest-priority active input is $D_7$. All other inputs are ignored. $Q_2Q_1Q_0 = 111$.

**NOTE**

The encoding principle of a priority encoder is that a low-priority input must not change the code resulting from a higher-priority input.

For example, if inputs $D_3$ and $D_5$ are both active, the correct output code is $Q_2Q_1Q_0 = 101$. The code for $D_3$ would be $Q_2Q_1Q_0 = 011$. Thus, $D_3$ must not make $Q_1 = 1$. The Boolean expressions for $Q_2$, $Q_1$, and $Q_0$ covering only these two codes are:

- $Q_2 = D_5$ (HIGH if $D_5$ is active.)
- $Q_1 = D_3\bar{D}_5$ (HIGH if $D_3$ is active AND $D_5$ is NOT active.)
- $Q_0 = D_3 + D_5$ (HIGH if $D_3$ OR $D_5$ is active.)

The truth table of a 3-bit priority encoder is shown in Table 5.5.

<table>
<thead>
<tr>
<th>$D_7$</th>
<th>$D_6$</th>
<th>$D_5$</th>
<th>$D_4$</th>
<th>$D_3$</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Restating the encoding principle, a bit goes HIGH if it is part of the code for an active input AND it is NOT kept LOW by an input with a higher priority. We can use this principle to develop a mechanical method for generating the Boolean equations of the outputs.

1. Write the codes in order from highest to lowest priority, as in Table 5.6.

<table>
<thead>
<tr>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
<th>Code Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
2. Examine each code. For a code with value \( n \), add a \( D_n \) term to each \( Q \) equation where there is a 1. For example, for code 111, add the term \( D_7 \) to the equations for \( Q_2 \), \( Q_1 \), and \( Q_0 \). For code 110, add the term \( D_6 \) to the equations for \( Q_2 \) and \( Q_1 \). (Steps 1 and 2 generate the nonpriority encoder equations listed earlier.)

3. Modify any \( D_n \) terms to ensure correct priority. Every time you write a \( D_n \) term, look at the previous lines in the table. For each previous code with a 0 in the same column as the 1 that generates \( D_n \), use an AND function to combine \( D_n \) with a corresponding \( \overline{D} \). For example, code 101 generates a \( D_5 \) term in the equations for \( Q_2 \) and \( Q_0 \). The term in the \( Q_2 \) equation need not be modified because there are no previous codes with a 0 in the same column. The term in the \( Q_0 \) equation must be modified since there is a 0 in the \( Q_0 \) column for code 110. This generates the term \( D_6D_5 \).

The equations from the 3-bit encoder of Figure 5.29 are modified by the priority encoding principle as follows:

\[
\begin{align*}
Q_2 &= D_7 + D_6 + D_5 + D_4 \\
Q_1 &= D_7 + D_6 + \overline{D}_5D_4D_3 + \overline{D}_5D_3D_2 \\
Q_0 &= D_7 + \overline{D}_6D_5 + \overline{D}_6D_5D_3 + \overline{D}_6D_4D_2D_1
\end{align*}
\]

**VHDL Priority Encoder**

The most obvious way to program a priority encoder in VHDL is to use the equations derived in the previous section in a set of concurrent signal assignment statements, as follows.

--- hi_pri8a.vhd

```vhdl
ENTITY hi_pri8a IS
  PORT(
    d : IN BIT_VECTOR(7 downto 0);
    q : OUT BIT_VECTOR (2 downto 0));
END hi_pri8a;
ARCHITECTURE a OF hi_pri8a IS
BEGIN
  q(2)   <= d(7) or d(6) or d(5) or d(4);
  q(1)   <= d(7) or d(6) 
           or ((not d(5)) and (not d(4)) and d(3)) 
           or ((not d(5)) and (not d(4)) and d(2));
  q(0)   <= d(7) or ((not d(6)) and d(5)) 
           or ((not d(6)) and (not d(4)) and d(3)) 
           or ((not d(6)) and (not d(4)) and (not d(2)) and d(1));
END a;
```

Although this code works, it is not terribly elegant, nor does it give any insight into the operation of the encoder circuit. Also, if we expand our encoder output by one or more bits, the equations become more cumbersome with each new bit and soon become impractically large and susceptible to typing errors. A VHDL conditional signal assignment statement is an ideal alternative for use in a priority encoder circuit. A section of VHDL code using this format is shown below.

--- hi_pri8b.vhd

```vhdl
ENTITY hi_pri8b IS
  PORT(
    d : IN BIT_VECTOR (7 downto 0);
    q : OUT INTEGER RANGE 0 to 7));
END hi_pri8b;
```
ARCHITECTURE a OF hi_pri8b IS
BEGIN
— Conditional Signal Assignment
encoder:
q <= 7 WHEN d(7)='1' ELSE
6 WHEN d(6)='1' ELSE
5 WHEN d(5)='1' ELSE
4 WHEN d(4)='1' ELSE
3 WHEN d(3)='1' ELSE
2 WHEN d(2)='1' ELSE
1 WHEN d(1)='1' ELSE
0;
END a;

Output q is defined as type INTEGER. Since it ranges from 0 to 7, the MAX+PLUS II VHDL compiler will automatically assign three outputs: \( Q_2 \), \( Q_1 \), and \( Q_0 \). The conditional signal assignment statement evaluates the first WHEN clause to determine if its condition \((d(7) = '1')\) is true. If so, it assigns q the value of 7 \((Q_2Q_1Q_0 = 111)\). If the first condition is false, the next WHEN clause is evaluated, assigning q the value 6 \((Q_2Q_1Q_0 = 110)\) if true, and so on until all WHEN clauses have been evaluated. If no clause is true, then the default value \((0: Q_2Q_1Q_0 = 000)\) is assigned to the output.

In the conditional signal assignment, the highest-priority condition is examined first. If it is true, the output is assigned according to that condition and no further conditions are evaluated. If the first condition is false, the condition of next priority is evaluated, and so on until the end. Thus, a low-priority input cannot alter the code resulting from an input of higher priority, as required by the priority encoding principle.

The effect is similar to that of an IF statement, where a sequence of conditions is evaluated, but only one output assignment is made. However, an IF statement must be used within a PROCESS statement, if we choose to use it. The IF statement for a priority encoder is as shown below.

PROCESS (d)
BEGIN
IF (d(7) = '1') THEN
q <= 7;
ELSIF (d(6) = '1') THEN
q <= 6;
ELSIF (d(5) = '1') THEN
q <= 5;
ELSIF (d(4) = '1') THEN
q <= 4;
ELSIF (d(3) = '1') THEN
q <= 3;
ELSIF (d(2) = '1') THEN
q <= 2;
ELSIF (d(1) = '1') THEN
q <= 1;
ELSE
q <= 0;
END IF;
END PROCESS;

Figure 5.31 shows the simulation of an 3-bit priority encoder. The \( d \) inputs are shown separately, so that we can easily determine which inputs are active. The \( q \) outputs are grouped so as to show the encoded output value as a hexadecimal number.

**BCD Priority Encoder**

A BCD priority encoder, illustrated in Figure 5.32, accepts ten inputs and generates a BCD code (0000 to 1001), corresponding to the highest-priority active input. The truth table for this circuit is shown in Table 5.7, with a simulation of the circuit shown in Figure 5.33.
Figure 5.31
Simulation File for a 3-bit Priority Encoder

Table 5.7  Truth Table of a BCD Priority Encoder

<table>
<thead>
<tr>
<th>$D_9$</th>
<th>$D_8$</th>
<th>$D_7$</th>
<th>$D_6$</th>
<th>$D_5$</th>
<th>$D_4$</th>
<th>$D_3$</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$Q_3$</th>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>1</td>
<td>X</td>
<td>X</td>
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<td>0</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Figure 5.32
BCD Priority Encoder

Figure 5.33
Simulation File for a BCD Priority Encoder
5.3 Multiplexers

**KEY TERMS**

- **Multiplexer**: A circuit that directs one of several digital signals to a single output, depending on the states of several select inputs.
- **Data inputs**: The multiplexer inputs that feed a digital signal to the output when selected.
- **Select inputs**: The multiplexer inputs that select a digital input channel.
- **Double-subscript notation**: A naming convention where two or more numerically related groups of signals are named using two subscript numerals. Generally, the first digit refers to a group of signals and the second to an element of a group. (e.g., \( X_{03} \) represents element 3 of group 0 for a set of signal groups, \( X \)).

A multiplexer (abbreviated MUX) is a device for switching one of several digital signals to an output, under the control of another set of binary inputs. The inputs to be switched are called the **data inputs**; those that determine which signal is directed to the output are called the **select inputs**.

Derivation of the BCD priority encoder equations and development of a VHDL description of the circuit are left as exercises in the end-of-chapter problems.

**SECTION 5.2 REVIEW PROBLEM**

5.4 State the main limitation of the 3-bit binary encoder shown in Figure 5.29. How can the encoder be modified to overcome this limitation?

Figure 5.34 shows the logic circuit for a 4-to-1 multiplexer, with data inputs labelled \( D_0 \) to \( D_3 \) and the select inputs labelled \( S_0 \) and \( S_1 \). By examining the circuit, we can see that the 4-to-1 MUX is described by the following Boolean equation:

\[
Y = D_0 \bar{S}_1 \bar{S}_0 + D_1 \bar{S}_1 S_0 + D_2 S_1 \bar{S}_0 + D_3 S_1 S_0
\]
For any given combination of $S_1S_0$, only one of the above four product terms will be enabled. For example, when $S_1S_0 = 10$, the equation evaluates to:

$$Y = (D_0 \cdot 0) + (D_1 \cdot 0) + (D_2 \cdot 1) + (D_3 \cdot 0) = D_2$$

The MUX equation can be described by a truth table as in Table 5.8. The subscript of the selected data input is the decimal equivalent of the binary combination $S_1S_0$.

Figure 5.35 shows two symbols used for a 4-to-1 multiplexer. The first symbol shows the data and select inputs as individual lines. The second symbol shows the data inputs as a single 4-bit bus line and the select inputs as a 2-bit bus.

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_0$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$D_0$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$D_1$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$D_2$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$D_3$</td>
</tr>
</tbody>
</table>

Table 5.8 4-to-1 MUX Truth Table

In general, a multiplexer with $n$ select inputs will have $m = 2^n$ data inputs. Thus, other common multiplexer sizes are 8-to-1 (for 3 select inputs) and 16-to-1 (for 4 select inputs). Data inputs can also be multiple-bit busses, as in Figure 5.36. The slash through a thick data line and the number 4 above the line indicate that it represents four related data signals. In this device, the select inputs switch groups of data inputs, as shown in the truth table in Table 5.9.

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_0$</th>
<th>$Y_3$</th>
<th>$Y_2$</th>
<th>$Y_1$</th>
<th>$Y_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$D_{03}D_{02}D_{01}D_{00}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$D_{13}D_{12}D_{11}D_{10}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$D_{23}D_{22}D_{21}D_{20}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$D_{33}D_{32}D_{31}D_{30}$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.9 Truth Table for a 4-to-1 4-bit Bus MUX

The naming convention shown in Table 5.9, known as double-subscript notation, is used frequently for identifying variables that are bundled in numerically related groups, the elements of which are themselves numbered. The first subscript identifies the group that a variable belongs to; the second subscript indicates which element of the group a variable represents.

### Multiplexing of Time-Varying Signals

We can observe the function of a multiplexer by using time-varying waveforms, such as a series of digital pulses. If we apply a different digital signal to each data input, and step the
select inputs through an increasing binary sequence, we can see the different input waveforms appear at the output in a predictable sequence, as shown by the simulation waveforms in Figure 5.37. The frequencies shown in the simulation were chosen to make as great a contrast as possible between adjacent inputs so that the different selected inputs could easily be seen.

In Figure 5.37, we initially see the $D_0$ waveform appearing at the $Y$ output when $S_1S_0 = 00$, followed in sequence by the $D_1$, $D_2$, and $D_3$ waveforms when $S_1S_0 = 01$, 10, and 11, respectively. (The $S_1S_0$ input combination is shown as a single hexadecimal value between 0 and 3, labelled $S[1..0]$.)

This simulation can be created in the MAX+PLUS II simulator by defining a base clock pulse length (e.g., 40 ns) and assigning that to one of the inputs ($D_1$ in this case). Other input waveforms are set to periods of 2, 4, and 8 times the base waveform period (for $D_3$, $D_2$, and $D_0$, respectively). The select input count waveforms are set to allow three cycles of the longest waveform ($D_0$) to appear at $Y$ when selected.

**VHDL Implementation of Multiplexers**

A multiplexer can be represented in MAX+PLUS II as a Graphic Design File, similar to the diagram of Figure 5.34, or in a hardware description language such as VHDL.

Several different VHDL constructs can be used to define a multiplexer. We can use a concurrent signal assignment statement, a selected signal assignment statement, or a CASE statement within a PROCESS. We will briefly look at each form for a 4-to-1 multiplexer. Later, you will be required to extend these constructs to larger multiplexer circuits.

**Concurrent Signal Assignment**

Recall that the concurrent signal assignment statement takes the form:

```vhdl
<signal> <= <expression>;
```

We can use this to encode the Boolean expression that describes a 4-to-1 MUX. The VHDL file that incorporates this statement is as follows.

```vhdl
-- mux4.vhd
-- 4-to-1 multiplexer
-- Directs one of four input signals (d0 to d3) to output,
-- depending on status of select bits (s1, s0).
```
ENTITY mux4 IS
PORT(
    d0, d1, d2, d3 : IN BIT;
    s              : IN BIT_VECTOR (1 downto 0);
    y              : OUT BIT);
END mux4;
ARCHITECTURE mux4to1 OF mux4 IS
BEGIN
    -- Concurrent Signal Assignment
    y<= ((not s(1)) and (not s(0)) and d0)
        or ((not s(1)) and (    s(0)) and d1)
        or ((    s(1)) and (not s(0)) and d2)
        or ((    s(1)) and (    s(0)) and d3);
END mux4to1;

While the concurrent signal assignment is fairly easy to use, it becomes cumbersome
for larger multiplexers, such as 8-to-1 or greater.
The entity declaration will be identical for the other VHDL examples. The only
change we will make will be to replace the concurrent signal assignment in the architecture
body with some other VHDL construct.

Selected Signal Assignment Statement
This construct has the following form (the label is optional):

    __label:
    WITH __expression SELECT
    __signal <= __expression WHEN __constant_value,
                 __expression WHEN __constant_value,
                 __expression WHEN __constant_value,
                 __expression WHEN __constant_value;

The 4-to-1 MUX can be described in VHDL as follows, using a selected signal as-

ENTITY mux4sel IS
PORT(
    d0, d1, d2, d3 : IN BIT;
    s              : IN BIT_VECTOR (1 downto 0);
    y              : OUT BIT);
END mux4sel;
ARCHITECTURE mux4to1 OF mux4sel IS
BEGIN
    M: WITH s SELECT
    y  <= d0 WHEN "00",
         d1 WHEN "01",
         d2 WHEN "10",
         d3 WHEN "11";
END mux4to1;

The selected signal assignment evaluates the expression in the WITH clause (in this
case, the 2-bit vector, s) and, depending on its value, selects an expression to assign to y.
Thus, if \( s_1s_0 = 00 \), \( y = d_0 \). If \( s_1s_0 = 01 \), then \( y = d_1 \), and so on for the remaining values
of \( s_1s_0 \).
5.3 • Multiplexers

CASE Statement within a PROCESS
In our MUX example, we could use a CASE statement as follows:

ENTITY mux4case IS
  PORT(
    d0, d1, d2, d3 : IN BIT;
    s              : IN BIT_VECTOR (1 downto 0);
    y              : OUT BIT);
END mux4case;
ARCHITECTURE mux4to1 OF mux4case IS
BEGIN
  -- CASE statement within a PROCESS
  -- Monitor select inputs and execute if they change
  PROCESS (s)
  BEGIN
    CASE s IS
      WHEN "00"    =>  y  <= d0;
      WHEN "01"    =>  y  <= d1;
      WHEN "10"    =>  y  <= d2;
      WHEN "11"    =>  y  <= d3;
      WHEN others  =>  y  <= '0';
    END CASE;
  END PROCESS;
END mux4to1;

If the select inputs change, the PROCESS statements are executed. The CASE statement evaluates the select input vector, s, and chooses a signal assignment based on its value. It is good design practice to include a default case (the “others” clause) even when there are no obvious other cases. A default case is essential when using STD_LOGIC types rather than BIT types, as ‘0’ and ‘1’ values do not cover all possible cases for STD_LOGIC signals. (Recall from Chapter 4 that STD_LOGIC is a nine-valued logic type, incorporating such things as “Don’t Care” (‘-’), “Unknown” (‘X’), and “High Impedance” (‘Z’), as well as ‘0’ and ‘1’.)

Multiplexer Applications
Multiplexers are used for a variety of applications, including selection of one data stream out of several choices, switching multiple-bit data from several channels to one multiple-bit output, sharing data on one output over time, and generating bit patterns or waveforms.

Single-Channel Data Selection
The simplest way to use a multiplexer is to switch the select inputs manually in order to direct one data source to the MUX output. Example 5.6 shows a pair of single-pole single-throw (SPST) switches supplying the select input logic for this type of application.

EXAMPLE 5.6
Figure 5.38 shows a digital audio switching system. The system shown can select a signal from one of four sources (compact disc (CD) players, labelled $CD_0$ to $CD_3$) and direct it to a digital signal processor (DSP) at its output. We assume we have direct access to the audio signals in digital form.

Make a table listing which digital audio source in Figure 5.38 is routed to the DSP for each combination of the multiplexer select inputs, $S_1$ and $S_0$. 

mux4case.vhd
Solution

Table 5.10  Sources Selected by a 4-to-1 MUX in Figure 5.38

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_0$</th>
<th>Selected Input</th>
<th>Selected Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$D_0$</td>
<td>$CD_0$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$D_1$</td>
<td>$CD_1$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>$D_2$</td>
<td>$CD_2$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$D_3$</td>
<td>$CD_3$</td>
</tr>
</tbody>
</table>

Multi-Channel Data Selection

Example 5.6 assumes that the output of a multiplexer is a single bit or stream of bits. Some applications require several bits to be selected in parallel, such as when data would be represented on a numerical display.

Figure 5.39 shows a circuit, based on a quadruple (4-channel) 2-to-1 multiplexer, that will direct one of two BCD digits to a seven-segment display. The bits $D_{03}D_{02}D_{01}D_{00}$ act as a 4-bit group input, since the first digit of all four subscripts is 0. When the MUX select input ($S$) is 0, these inputs are all connected to the outputs $Y_3Y_2Y_1Y_0$. Similarly, when the select input is 1, inputs $D_{13}D_{12}D_{11}D_{10}$ are connected to the $Y$ outputs.

The seven-segment display in Figure 5.39 will display “4” if $S = 0$ ($D_0$ inputs selected) and “9” if $S = 1$ ($D_1$ inputs selected).
EXAMPLE 5.7

Draw the symbol for a multiplexer that will select one of four 4-bit channels and direct it to a 4-bit output. Create a VHDL file that implements this function and a simulation showing the operation of the device.

Solution  Figure 5.40 shows the symbol for the 4-channel, 4-bit multiplexer. This symbol is shown with the data inputs and outputs in bus form. The data inputs are labelled in groups $D_0$ to $D_3$, which contain the individual inputs $[D_{03}, D_{02}, D_{01}, D_{00}]$ to $[D_{33}, D_{32}, D_{31}, D_{30}]$.

A VHDL file describing this function is listed below.

```vhdl
-- quad4to1.vhd
ENTITY quad4to1 IS
  PORT(
    s   : IN   INTEGER RANGE 0 to 3;
    d0  : IN   BIT_VECTOR (3 downto 0);
    d1  : IN   BIT_VECTOR (3 downto 0);
    d2  : IN   BIT_VECTOR (3 downto 0);
    d3  : IN   BIT_VECTOR (3 downto 0);
    y   : OUT  BIT_VECTOR (3 downto 0));
END quad4to1;
ARCHITECTURE mux4 OF quad4to1 IS
BEGIN
  -- Selected Signal Assignment
  MUX4:   WITH s SELECT
    y <= d0 WHEN 0,
        d1 WHEN 1,
        d2 WHEN 2,
        d3 WHEN 3;
END mux4;
```

Figure 5.41 shows a set of simulation waveforms for the multiplexer. The $D$ inputs are shown in groups of four, the value of each shown as a steady hexadecimal value. The select inputs are grouped, showing an increasing 2-bit binary count as a hexadecimal value (0 to 3, then repeating). As the $S$ inputs select each group of $D$ inputs, their combined value is directed to the $Y$ output group.
Time-Dependent Multiplexer Applications

**KEY TERMS**

Counter  A digital circuit whose output produces a fixed sequence of binary states when an input called the clock receives a series of pulses. The output advances by one for each clock pulse (e.g., the output state of a 4-bit binary counter progresses in order from 0000 to 1111, then repeats).

Clock  A signal that controls the operation of a sequential digital circuit, such as a counter, by advancing its outputs to the next state when it receives a pulse.

Positive edge  The point on a digital waveform where the logic level of the waveform makes a LOW-to-HIGH transition.

A time-dependent multiplexer application is one that uses the MUX input channels one after the other in a repeating time sequence. We can create such an application by applying a set of changing binary signals to the MUX select inputs. For this function, we can use a 3-bit binary counter to generate a binary sequence that goes from 000 to 111 (8 states) and repeats indefinitely, the outputs advancing by one with every pulse applied to the clock input of the counter.

![Timing Diagram of a 3-bit Counter](image)

**FIGURE 5.42**
Timing Diagram of a 3-bit Counter

Figure 5.42 shows the timing diagram of a 3-bit counter. The outputs $Q_2, Q_1, Q_0$ change every time the clock signal makes a transition from LOW to HIGH. If you read the $Q$
waveforms from bottom to top, you will see that they generate a repeating binary sequence (000, 001, 010, 011, 100, 101, 110, 111, 000 . . .).

If we connect the counter outputs $Q_2Q_1Q_0$ to the select inputs of an 8-to-1 MUX, as in Figure 5.43, we will select the channels in sequence, one after the other. The counter is labelled CTR DIV 8 because its most significant bit output has a frequency equal to the clock frequency divided by eight. The triangle on the clock input indicates that it is active when the clock waveform makes a transition from one logic level to another. Since there is no inverting bubble on the clock input, we know that the active clock transition is from LOW to HIGH (i.e., a positive edge).

Waveform Generation. A multiplexer and counter can be used as a programmable waveform generator. The output waveform can be programmed to any pattern by switching the logic levels on the data inputs. This is an easy way to generate an asymmetrical waveform, a task which is more complicated using other digital circuits. The circuit can also generate symmetrical waveforms by alternating the logic levels of consecutive groups of inputs.

EXAMPLE 5.8

Draw a circuit that uses an 8-to-1 multiplexer to generate a programmable 8-bit repeating pattern. Draw the timing diagram of the select inputs and the output waveform for the following pattern of data inputs.

<table>
<thead>
<tr>
<th>$D_7$</th>
<th>$D_6$</th>
<th>$D_5$</th>
<th>$D_4$</th>
<th>$D_3$</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$D_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Solution Figure 5.44a shows the waveform generator circuit. The output waveform with respect to the counter inputs is shown in Figure 5.44b. This pattern is relatively difficult to generate by other means since it has several unequal HIGH and LOW sequences in one period.
EXAMPLE 5.9

The programmable waveform generator in Figure 5.44 generates a symmetrical pulse waveform having a frequency of 1 kHz when the data inputs are set as follows.

\[
\begin{array}{cccccccc}
D_7 & D_6 & D_5 & D_4 & D_3 & D_2 & D_1 & D_0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
\end{array}
\]

How should the switches be set to generate a symmetrical 2 kHz waveform? A symmetrical 4 kHz waveform?

Solution

Pattern for 2 kHz:

\[
\begin{array}{cccccccc}
D_7 & D_6 & D_5 & D_4 & D_3 & D_2 & D_1 & D_0 \\
0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
\end{array}
\]
Pattern for 4 kHz:

<table>
<thead>
<tr>
<th>D_7</th>
<th>D_6</th>
<th>D_5</th>
<th>D_4</th>
<th>D_3</th>
<th>D_2</th>
<th>D_1</th>
<th>D_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Time Division Multiplexing**

**KEY TERMS**

**Time division multiplexing (TDM)** A technique of using one transmission line to send many signals simultaneously by making them share the line for equal fractions of time.

**Time slot** A period of time during which a transmitted data element has sole access to a transmission path.

**Bit multiplexing** A TDM technique in which one bit is sent from each channel during the channel’s assigned time slot.

**Byte (or word) multiplexing** A TDM technique in which a byte (or word) is sent from each channel during its assigned time slot. (A byte is eight bits; a word is a group of bits whose size varies with the particular system.)

**Time division multiplexing** is a method of improving the efficiency of a transmission system by sharing one transmission path among many signals. For example, if we wish to send four 4-bit numbers over a single transmission line, we can transmit the bits one after the other, as shown in Figure 5.45.

In Figure 5.45, we see the least significant bit of the 4-bit word $p_0$ transmitted, followed by the LSB of $p_1$, $p_2$, then $p_3$. After that, the second bit of each word is transmitted in sequence, then all the third bits, and finally, all MSBs in sequence. Each bit is assigned a **time slot** in the sequence. During that time, the bit has sole access to the transmission line. When its time elapses, the next bit is sent and so on in sequence, until the channel assignment returns to the original location. This technique, known as **bit multiplexing**, can be implemented by a circuit similar to the waveform generator shown in Figure 5.44. Rather than fixed switch inputs, the data inputs would be some data source, such as a digitized audio signal.

We can also arrange our circuit so that one byte (8 bits) or one word (a group of bits) is sent through a selected channel. In this case, we must keep the channel selected for enough clock pulses to transmit the byte or word, then move to the next one. This technique is called **byte (or word) multiplexing**. Figure 5.46 shows a data stream of four 4-bit words that are word-multiplexed down a data transmission path.

**FIGURE 5.46**

4 × 4 Data Stream (Word Multiplexing)
Telephone companies use TDM to maximize the use of their phone lines. Speech or data is digitally encoded for transmission. Each speech or data channel becomes a multiplexer data input which shares time with all other channels on a single phone line. A counter on the MUX selects the speech channels one after the other in a continuous sequence. The counter must switch the channels fast enough so that there is no apparent interruption of the transmitted conversation or data stream.

**EXAMPLE 5.10**

Draw a diagram of a circuit that uses an 8-to-1 multiplexer to share one telephone line among eight digitized speech channels.

Write a VHDL file for the multiplexer and create a simulation to show its operation.

**Solution**  Figure 5.47 shows the required multiplexer circuit. Each channel is connected to a data input and a 3-bit binary counter is connected to the select inputs.

![Multplexer Circuit](image)

**FIGURE 5.47**

Example 5.10

Time-Division Multiplexing of Telephone Channels

The VHDL code for the multiplexer is:

```vhdl
ENTITY mux_8ch IS
  PORT(
    sel : IN BIT_VECTOR (2 downto 0);
    d   : IN BIT_VECTOR (7 downto 0);
    y   : OUT BIT);
END mux_8ch;

ARCHITECTURE a OF mux_8ch IS
BEGIN
  -- Selected Signal Assignment
  MUX8: WITH sel SELECT
    y <= d(0) WHEN "000",
```
d(1) WHEN "001",
d(2) WHEN "010",
d(3) WHEN "011",
d(4) WHEN "100",
d(5) WHEN "101",
d(6) WHEN "110",
d(7) WHEN "111";

END a;

The simulation is shown in Figure 5.48. For clarity, digital data are present on the MUX inputs just before and after they are switched to the $Y$ output. The output shows the channel data in sequence, starting with channel 0.

![Figure 5.48](image)

**Figure 5.48**
Simulation for an 8-bit Time-Division Multiplexer

### SECTION 5.3 REVIEW PROBLEM

5.5 What defines whether a multiplexer application is time-dependent or not? What additional component can be added to make a MUX application time-dependent?

### 5.4 Demultiplexers

**Key Terms**

**Demultiplexer**  A circuit that uses a binary decoder to direct a digital signal from a single source to one of several destinations.

A demultiplexer performs the reverse function of a multiplexer. A multiplexer (MUX) directs one of several input signals to a single output; a demultiplexer (DMUX) directs a single input signal to one of several outputs. In both cases, the selected input or output is chosen by the state of an internal decoder.

Figure 5.49 shows the logic circuit for a 1-to-4 demultiplexer. Compare this to Figure 5.4, a 4-output decoder. This circuits are the same except that the active-LOW enable input has been changed to an active-HIGH data input. The circuit in Figure 5.49 could still be used as a decoder, except that its enable input would be active-HIGH.
Each AND gate in the demultiplexer enables or inhibits the signal output according to the state of the select inputs, thus directing the data to one of the output lines. For instance, \( S_1 S_0 = 10 \) directs incoming digital data to output \( Y_2 \).

Figure 5.50 illustrates the use of a single device as either a decoder or a demultiplexer. In Figure 5.50a, input \( D \) is tied HIGH. When an output is selected by \( S_1 \) and \( S_0 \), it goes HIGH, acting as a decoder with active-HIGH outputs. In Figure 5.50b, \( D \) acts as a demultiplexer data input. The data are directed to the output selected by \( S_1 \) and \( S_0 \).

Since a single device can be used either way, this implies that any of the VHDL binary decoder designs used in this chapter can also be used as demultiplexers.

A decoder/demultiplexer can have active-LOW outputs, but only if the \( D \) input is also active-LOW. This is important because the demultiplexer data must be inverted twice to retain its original logic values.

Demultiplexing a TDM Signal

In Example 5.10, we saw how a multiplexer could be used to send 8 digital channels across a single line, multiplexed over time. Obviously, such a system is not of much value if the signals cannot be sorted out at the receiving end. The received digital data must be demultiplexed and sent to their appropriate destinations.

\[ \text{FIGURE 5.49} \\
4\text{-bit Decoder/Demultiplexer} \]

\[ \text{FIGURE 5.50} \\
\text{Same Device Used as a Decoder or Demultiplexer} \]
The process is the reverse of multiplexing; data are sent to an output selected by a counter at the DMUX select inputs. (We assume that the counters at the MUX and DMUX select inputs are somehow synchronized or possibly, if located close together, are the same counter.)

**EXAMPLE 5.11**

Draw a demultiplexing circuit that will take the multiplexed output of the circuit in Figure 5.47 and distribute it to 8 different local telephone circuits. Write a VHDL file for the demultiplexer and create a simulation file that shows its operation. Use active-LOW outputs for the demultiplexer. How does this affect the outputs when they are not transmitting data?

**Solution**  Figure 5.51 shows the original multiplexing circuit connecting to the new demultiplexing circuit. The diagram indicates that the two sides of the circuit are separated by some distance. The clock is shared between both sides of the circuit, but is generated on the MUX side. Both sides share a common ground. Each side of the circuit has its own 3-bit counter.

**FIGURE 5.51**

Example 5.11
Time-Division Multiplexing and Demultiplexing

The VHDL code for the demultiplexer is as follows. (This is the same implementation as a 3-line-to-8-line decoder with an enable input.)

```vhdl
-- dmux8.vhd
-- 1-to-8 demultiplexer/decoder
-- Decoder: set d to '0'; outputs are activated by
-- binary combination of s.
-- Demultiplexer: apply data stream to d; data directed to
-- y output with subscript same as value of s.
-- Outputs and d are active-LOW. DMUX data are inverted twice
-- to keep them true.
```
ENTITY dmux8 IS
PORT(
s : IN INTEGER Range 0 to 7;
d : IN BIT;
y : OUT BIT_VECTOR (0 to 7));
END dmux8;

ARCHITECTURE a OF dmux8 IS
SIGNAL output : BIT_VECTOR (0 to 7);
BEGIN
PROCESS (d, s)
BEGIN
IF  (d = '1') THEN
  output <= "11111111";
ELSE
  CASE s IS
    WHEN 0 => output <= "01111111";
    WHEN 1 => output <= "10111111";
    WHEN 2 => output <= "11011111";
    WHEN 3 => output <= "11101111";
    WHEN 4 => output <= "11110111";
    WHEN 5 => output <= "11111011";
    WHEN 6 => output <= "11111101";
    WHEN 7 => output <= "11111110";
    WHEN OTHERS => output <= "11111111";
  END CASE;
END IF;
y <= output;
END PROCESS;
END a;

The simulation, shown in Figure 5.52, has as its input data the output of the original MUX simulation in Figure 5.48. Data are distributed to the outputs in sequence. Compare the DMUX output data to the MUX input data in Figure 5.48.

Note that idle channels sit HIGH. This is opposite from the status of the idle MUX lines and may affect circuit operation. If so, a DMUX with active-HIGH outputs and active-HIGH enable should be used.
CMOS Analog Multiplexer/Demultiplexer

**KEY TERMS**

**CMOS analog switch** A CMOS device that will pass an analog or digital signal in either direction, when enabled. Also called a transmission gate. There is no TTL equivalent.

An interesting device used in some CMOS medium-scale integration multiplexers and demultiplexers, as well as other applications, is the CMOS analog switch, or transmission gate. This device has the property of allowing signals to pass in two directions, instead of only one, thus allowing both positive and negative voltages and currents to pass. It also has no requirement that the voltages be of a specific value such as +5 volts. These properties make the device suitable for passing analog signals.

**FIGURE 5.53** Line Drivers

Figure 5.53 shows several symbols, indicating the development of the transmission gate concept. Figures 5.53a and b show amplifiers whose output and input are clearly defined by the direction of the triangular amplifier symbol. A signal has one possible direction of flow. Figure 5.53b includes an active-LOW gating input, which can turn the signal on and off.

Figure 5.53c shows two opposite-direction overlapping amplifier symbols, with a gating input to enable or inhibit the bidirectional signal flow. The signal through the transmission gate may be either analog or digital.

Analog switches are available in packages of four switches with part numbers such as 4066B (standard CMOS) or 74HC4066 (high-speed CMOS).

Several available CMOS MUX/DMUX chips use analog switches to send signals in either direction. Figure 5.54 illustrates the design principle as applied to a 4-channel MUX/DMUX.

**FIGURE 5.54** 4-Channel CMOS MUX/DMUX

If four signals are to be multiplexed, they are connected to inputs $D_0$ to $D_3$. The decoder, activated by $S_1$ and $S_0$, selects which one of the four switches is enabled. Figure 5.54 shows Channel 2 active ($S_1S_0 = 10$).
Since all analog switch outputs are connected together, any selected channel connects to $Y$, resulting in a multiplexed output. To use the circuit in Figure 5.54 as a demultiplexer, the inputs and outputs are merely reversed.

**EXAMPLE 5.12**

A CMOS 4097B dual 8-channel MUX/DMUX can be used simultaneously as a multiplexer on one half of the device and as a demultiplexer on the other side.

**FIGURE 5.55**

Example 5.12
4097B MUX/DMUX as a Time Division MUX/DMUX
A circuit in a recording studio uses one side of a 4097B MUX/DMUX to multiplex 8 digital audio channels into a digital signal processor (DSP), using time division multiplexing. The other half of the 4097B takes the processed signals from a DSP output and distributes them to 8 channels on a digital audio tape (DAT) unit. Draw the circuit.

**Solution** Figure 5.55 shows a possible circuit. The counter can be part of the DSP. An audio source channel is selected by the counter inputs, data are sent to the DSP, where they are processed and sent to the same channel of the DAT. The counter advances by one, selecting a new channel and repeating the process.

Some analog MUX/DMUX devices in high-speed CMOS include: 74HC4051 8-channel MUX/DMUX, 74HC4052 dual 4-channel MUX/DMUX, and 74HC4053 triple 2-channel MUX/DMUX.

### 5.5 Magnitude Comparators

**Table 5.11 XNOR Truth Table**

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Y$</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>0</td>
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<td>1</td>
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</table>

**Figure 5.56**

Exclusive NOR Gate

**Figure 5.57**

2-bit Magnitude Comparator

If we are interested in finding out whether or not two binary numbers are the same, we can use a **magnitude comparator**. The simplest comparison circuit is the Exclusive NOR gate, whose circuit symbol is shown in Figure 5.56 and whose truth table is given in Table 5.11.

The output of the XNOR gate is 1 if its inputs are the same ($A = B$, symbolized $AEQB$) and 0 if they are different. For this reason, the XNOR gate is sometimes called a coincidence gate.

We can use several XNORs to compare each bit of two multi-bit binary numbers. Figure 5.57 shows a 2-bit comparator with one output that goes HIGH if all bits of $A$ and $B$ are identical.

If the most significant bit (MSB) of $A$ equals the MSB of $B$, the output of the upper XNOR is HIGH. If the least significant bits (LSBs) are the same, the output of the lower XNOR is HIGH. If both these conditions are satisfied, then $A = B$, which is indicated by a HIGH at the AND output. This general principle applies to any number of bits:

$$AEQB = (A_{n-1} \oplus B_{n-1}) \cdot (A_{n-2} \oplus B_{n-2}) \ldots (A_1 \oplus B_1) \cdot (A_0 \oplus B_0)$$

for two $n$-bit numbers, $A$ and $B$.

Some magnitude comparators also include an output that activates if $A$ is greater than $B$ (symbolized $A > B$ or $AGTB$) and another that is active when $A$ is less than $B$ (symbolized $A < B$ or $ALTB$). Figure 5.58 shows the comparator of Figure 5.57 expanded to include the “greater than” and “less than” functions.

Let us analyze the $AGTB$ circuit. The $AGTB$ function has two AND-shaped gates that compare $A$ and $B$ bit-by-bit to see which is larger.
1. The 2-input gate examines the MSBs of A and B. If \( A_1 = 1 \) AND \( B_1 = 0 \), then we know that \( A > B \). (This implies one of the following inequalities: 10 > 00; 10 > 01; 11 > 00; or 11 > 01.)

2. If \( A_1 = B_1 \), then we don’t know whether or not \( A > B \) until we compare the next most significant bits, \( A_0 \) and \( B_0 \). The 3-input gate makes this comparison. Since this gate is enabled by the XNOR, which compares the two MSBs, it is only active when \( A_1 = B_1 \). This yields the term \((A_1 \oplus B_1)A_0B_0\) in the Boolean expression for the \( AGTB \) function.

3. If \( A_1 = B_1 \) AND \( A_0 = 1 \) AND \( B_0 = 0 \), then the 3-input gate has a HIGH output, telling us, via the OR gate, that \( A > B \). (The only possibilities are \( 01 > 00 \) and \( 11 > 10 \).)

Similar logic works in the \( ALTB \) circuit, except that inversion is on the \( A \), rather than the \( B \) bits. Alternatively, we can simplify either the \( AGTB \) or the \( ALTB \) function by using a NOR function. For instance, if we have developed a circuit to indicate \( AEQB \) and \( ALTB \), we can make the \( AGTB \) function from the other two, as follows:

\[
AGTB = AEQB + ALTB
\]

This Boolean expression implies that if \( A \) is not equal to or less than \( B \), then it must be greater than \( B \).

Figure 5.59 shows a 4-bit comparator with \( AEQB \), \( ALTB \), and \( AGTB \) outputs.

The Boolean expressions for the outputs are:

\[
AEQB = (A_3 \oplus B_3)(A_2 \oplus B_2)(A_1 \oplus B_1)(A_0 \oplus B_0)
\]

\[
ALTB = A_3B_3 + (A_3 + B_3)A_2B_2 + (A_3 \oplus B_3)(A_2 \oplus B_2)A_1B_1 + (A_3 \oplus B_3)(A_2 \oplus B_2)A_0B_0
\]

\[
AGTB = AEQB + ALTB
\]

This comparison technique can be expanded to as many bits as necessary. A 4-bit comparator requires four AND-shaped gates for its \( ALTB \) function. We can interpret the Boolean expression for this function as follows.

\( A < B \) if:

1. The MSB of \( A \) is less than the MSB of \( B \), OR
2. The MSBs are equal, but the second bit of \( A \) is less than the second bit of \( B \), OR
3. The first two bits are equal, but the third bit of \( A \) is less than the third bit of \( B \), OR
4. The first three bits are equal, but the LSB of \( A \) is less than the LSB of \( B \)

Expansion to more bits would use the same principle of comparing bits one at a time, beginning with the MSBs.
EXAMPLE 5.13

Application

A digital thermometer has two input probes. A circuit in the thermometer converts the measured temperature at each probe to an 8-bit number, as shown by the block in Figure 5.59.

In addition to measuring the temperature at each input, the thermometer has a comparison function that indicates whether the temperature at one input is greater than, equal to, or less than the temperature at the other input.

Draw a logic diagram showing how a magnitude comparator could be connected to light a green LED for $AGTB$, an amber LED for $AEQB$, and a red LED for $ALTB$.

Solution  Figure 5.61 shows the logic diagram of the magnitude comparator connected...
to the thermometer’s digital output.

When one of the comparator outputs goes HIGH, it sets the output of the corresponding inverter LOW. This provides a current path to ground for the indicator LED for that output, causing it to illuminate.

**VHDL Magnitude Comparators**

The most obvious way to create a VHDL representation of a magnitude comparator is to use a concurrent signal assignment statement for each comparing function. For example, the following VHDL code can represent the 2-bit magnitude comparator of Figure 5.57:

```vhdl
compare2.vhd

ENTITY compare2 IS
PORT(
a, b : IN BIT_VECTOR (1 downto 0);
agtb, aeqb, altb : OUT BIT);
END compare2;

ARCHITECTURE a OF compare2 IS
BEGIN
altb <= (not (a(1)) and b(1))
or ((not (a(1) xor b(1))) and (not (a(0)) and b(0)));
aeqb <= (not (a(1) xor b(1))) and (not (a(0) xor b(0)));
agtb <= (a(1) and not (b(1)))
or ((not (a(1) xor b(1))) and (a(0) and not (b(0))));
END a;

A simulation for this file is shown in Figure 5.62. The comparison outputs go HIGH to indicate \( A = B \), \( A < B \), or \( A > B \).

Although this approach works, it is not a very good one. Due to the complexity of the Boolean equations for \( \text{ALTB} \) and \( \text{AGTB} \), it is difficult to type them without making errors. (Try it!) The difficulty increases greatly with the number of required inputs.
The following code for a 4-bit comparator illustrates a much more efficient method. Since VHDL allows inputs to be represented as integers, we can define the required size of inputs $A$ and $B$ and compare them using IF statements. For every comparison, we assign an output vector consisting of bits for $ALTB$, $AEQB$, and $AGTB$ one of the values 110, 101, or 011, for active-LOW outputs. For example, if $A = 12$ and $B = 9$, then the output vector would be 011 (i.e., $A > B$). An active-LOW output will illuminate a LOW-sense LED, such as those on the Altera UP-1 board.

![compare4.vhd]

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY compare4 IS
    PORT(
        a, b : IN  INTEGER RANGE 0 TO 15;
        agtb, aeqb, altb : OUT STD_LOGIC);
END compare4;
ARCHITECTURE a OF compare4 IS
    SIGNAL compare : STD_LOGIC_VECTOR (2 downto 0);
    BEGIN
        PROCESS (a,b)
        BEGIN
            IF a<b THEN
                compare <= "110";
            ELSIF a=b THEN
                compare <= "101";
            ELSIF a>b THEN
                compare <= "011";
            ELSE
                compare <= "111";
            END IF;
            agtb   <=  compare(2);
            aeqb   <=  compare(1);
            altb   <=  compare(0);
        END PROCESS;
    END a;
```

The beauty of this method is that the number of input bits can be changed by modifying one number: the range of the INTEGER-type input. For example, a 12-bit comparator is identical to the 4-bit comparator in the previous VHDL code, except that the inputs have a range of 0 to 4095 ($= 2^{12} - 1$). Using this method, we can program an EPM7128S CPLD.
with a comparator up to 28 bits wide (range of 0 to 268,435,455). If we do, however, there is no room for anything else.

EXAMPLE 5.14

Write a VHDL file that uses IF statements to compare two 8-bit numbers \( A \) and \( B \). The design should have outputs for \( AEQB \), \( ALTB \), and \( AGTB \).

Solution

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY compare8 IS
    PORT(
        a, b : IN INTEGER RANGE 0 TO 255;
        agtb, aeqb, altb : OUT STD_LOGIC);
END compare8;

ARCHITECTURE a OF compare8 IS
    SIGNAL compare : STD_LOGIC_VECTOR (2 downto 0);
    BEGIN
        PROCESS (a,b)
        BEGIN
            IF a<b THEN
                compare <= "110";
            ELSIF a=b THEN
                compare <= "101";
            ELSIF a>b THEN
                compare <= "011";
            ELSE
                compare <= "111";
            END IF;
            agtb <= compare(2);
            aeqb <= compare(1);
            altb <= compare(0);
        END PROCESS;
    END a;
```

5.6 Parity Generators and Checkers

**KEY TERMS**

Parity A system that checks for errors in a multi-bit binary number by counting the number of 1s.

Even parity An error-checking system that requires a binary number to have an even number of 1s.

Odd parity An error-checking system that requires a binary number to have an odd number of 1s.

Parity bit A bit appended to a binary number to make the number of 1s even or odd, depending on the type of parity.

When data are transmitted from one device to another, it is necessary to have a system of checking for errors in transmission. These errors, which appear as incorrect bits, occur as a result of electrical limitations such as line capacitance or induced noise.
Parity error checking is a way of encoding information about the correctness of data before they are transmitted. The data can then be verified at the system’s receiving end. Figure 5.63 shows a block diagram of a parity error-checking system.

The parity generator in Figure 5.63 examines the outgoing data and adds a bit called the **parity bit** that makes the number of 1s in the transmitted data odd or even, depending on the type of parity. Data with **EVEN parity** have an even number of 1s, including the parity bit, and data with **ODD parity** have an odd number of 1s.

The data receiver “knows” whether to expect EVEN or ODD parity. If the incoming number of 1s matches the expected parity, the parity checker responds by indicating that correct data have been received. Otherwise, the parity checker indicates an error.

**EXAMPLE 5.16**

Data are transmitted from a PC serial port to a modem in groups of 7 data bits plus a parity bit. What should the parity bit, $P$, be for each of the following data if the parity is **EVEN**? If the parity is **ODD**?

a. 0110110
b. 1000000
c. 0010101

**Solution**

a. 0110110 Four 1s in data. (4 is an even number.)
   - EVEN parity: $P = 0$
   - ODD parity: $P = 1$

b. 1000000 One 1 in data. (1 is an odd number.)
   - EVEN parity: $P = 1$
   - ODD parity: $P = 0$

c. 0010101 Three 1s in data. (3 is an odd number.)
   - EVEN parity: $P = 1$
   - ODD parity: $P = 0$

An Exclusive OR gate can be used as a parity generator or a parity checker. Figure 5.64 shows the gate, and Table 5.12 is the XOR truth table. Notice that each line of the XOR truth table has an even number of 1s if we include the output column.

Figure 5.65 shows the block diagram of a circuit that will generate an **EVEN** parity bit from 2 data bits, $A$ and $B$, and transmit the three bits one after the other, that is, serially, to a data receiver.
Figure 5.66 shows a parity checker for the parity generator in Figure 5.65. Data are received serially, but read in parallel. The parity bit is re-created from the received values of $A$ and $B$, and then compared to the received value of $P$ to give an error indication, $P'$. If $P$ and $A \oplus B$ are the same, then $P' = 0$ and the transmission is correct. If $P$ and $A \oplus B$ are different, then $P' = 1$ and there has been an error in transmission.

**EXAMPLE 5.17**

The following data and parity bits are transmitted four times: $ABP = 101$.

1. State the type of parity used.
2. The transmission line over which the data are transmitted is particularly noisy and the data arrive differently each time as follows:
   a. $ABP = 101$
   b. $ABP = 100$
   c. $ABP = 111$
   d. $ABP = 110$

   Indicate the output $P'$ of the parity checker in Figure 5.66 for each case and state what the output means.

**Solution**

1. The system is using EVEN parity.
2. The parity checker produces the following responses:
   a. $ABP = 101$
      \[A \oplus B = 1 \oplus 0 = 1\]
      \[P' = (A \oplus B) \oplus P = 1 \oplus 1 = 0\]
      Data received correctly.
   b. $ABP = 100$
      \[A \oplus B = 1 \oplus 0 = 1\]
      \[P' = (A \oplus B) \oplus P = 1 \oplus 0 = 1\]
      Transmission error. (Parity bit incorrect.)
   c. $ABP = 111$
The second and third cases in Example 5.17 show that parity error-detection cannot tell which bit is incorrect.

The fourth case points out the major flaw of parity error detection: An even number of errors cannot be detected. This is true whether the parity is EVEN or ODD. If a group of bits has an even number of 1s, a single error will change that to an odd number of 1s, but a double error will change it back to even. (Try a few examples to convince yourself this is true.)

An ODD parity generator and checker can be made using an Exclusive NOR, rather than an Exclusive OR, gate. If a set of transmitted data bits require a 1 for EVEN parity, it follows that they require a 0 for ODD parity. This implies that EVEN and ODD parity generators must have opposite-sense outputs.

Example 5.18

Modify the circuits in Figures 5.65 and 5.66 to operate with ODD parity. Verify their operation with the data bits $AB = 11$ transmitted twice and received once as $AB = 11$ and once as $AB = 01$.

Solution  Figure 5.67a shows an ODD parity generator and Figure 5.67b shows an ODD parity checker. The checker circuit still has an Exclusive OR output since it presents the same error codes as an EVEN parity checker. The parity bit is re-created at the receive end of the transmission path and compared with the received parity bit. If they are the same, $P' = 0$ (correct transmission). If they are different, $P' = 1$ (transmission error).

Verification:

Generator:

Data: $AB = 11$  Parity: $P = A \oplus B = 1 \oplus 1 = 1$

Checker:

Received data: $AB = 11$

$P' = (A \oplus B) \oplus P = (1 \oplus 1) \oplus 1 = 1 \oplus 1 = 0$  (Correct transmission)
Generator:

Data: \( AB = 11 \) \hspace{1cm} Parity: \( P = A \oplus B = 1 \oplus 1 = 1 \)

Checker:

Received data: \( AB = 01 \)
\[
P' = (A \oplus B) \oplus P = (0 \oplus 1) \oplus 1 = 0 \oplus 1 = 1 \quad \text{(Incorrect transmission)}
\]

Parity generators and checkers can be expanded to any number of bits by using an XOR gate for each pair of bits and combining the gate outputs in further stages of 2-input XOR gates. The true form of the generated parity bit is \( P_E \), the EVEN parity bit. The complement form of the bit is \( P_O \), the ODD parity bit.

Table 5.13 shows the XOR truth table for 4 data bits and the ODD and EVEN parity bits. The EVEN parity bit \( P_E \) is given by \((A \oplus B) \oplus (C \oplus D)\). The ODD parity bit \( P_O \) is given by \((A \oplus B) \oplus (C \oplus D)\).

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>( A \oplus B )</th>
<th>( C \oplus D )</th>
<th>( P_E )</th>
<th>( P_O )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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\( P_O \) is given by \( \bar{P}_E = (A \oplus B) \oplus (C \oplus D) \). For every line in Table 5.13, the bit combination \( A B C D P_E \) has an even number of 1s and the group \( A B C D P_O \) has an odd number of 1s.

**EXAMPLE 5.19**

Use Table 5.13 to draw a 4-bit parity generator and a 4-bit parity checker that can generate and check either EVEN or ODD parity, depending on the state of one select input.

**FIGURE 5.68**
Example 5.19
4-bit Parity Generator
EXAMPLE 5.20

Draw the circuit for an 8-bit EVEN/ODD parity generator.

Solution  An 8-bit parity generator is an expanded version of the 4-bit generator in the previous example. The circuit is shown in Figure 5.70.

SECTION 5.6 REVIEW PROBLEM

5.6 Data (including a parity bit) are detected at a receiver configured for checking ODD parity. Which of the following data do we know are incorrect? Could there be errors in the remaining data? Explain.

a. 010010
b. 011010
c. 1110111
d. 1010111
e. 1000101
SUMMARY

1. A decoder detects the presence of a particular binary code. The simplest decoder is an AND or NAND gate, which can detect a binary code when combined with the right combination of input inverters.

2. Multiple-output decoders are implemented by a series of single-gate decoders, each of which responds to a different input code.

3. For an \( n \)-input decoder, there can be as many as \( 2^n \) unique outputs.

4. MAX+PLUS II can simulate the function of a digital circuit by generating a set of output waveforms in response to a defined set of input waveforms.

5. VHDL constructs such as selected signal assignment statements and conditional signal assignments can describe decoders. Both statement types assign alternative values to a VHDL port or signal, based on the state of another port or signal.

6. A selected signal assignment statement has the form:

\[
\text{label: WITH } \_\text{expression} \text{ SELECT}
\]

\[
\text{\_signal } \leq \_\text{expression WHEN } \_\text{constant_value,}
\text{\_expression WHEN } \_\text{constant_value,}
\text{\_expression WHEN } \_\text{constant_value,}
\text{\_expression WHEN } \_\text{constant_value;}
\]

7. A conditional signal assignment statement has the form:

\[
\text{\_signal } \leq \_\text{expression WHEN } \_\text{boolean_expression ELSE}
\text{\_expression WHEN } \_\text{boolean_expression ELSE}
\text{\_expression;}
\]

8. SIGNALs act as internal connections in a VHDL design entity. They can be single lines or vectors and are declared before the BEGIN clause of an ARCHITECTURE body.

9. The report file of a MAX+PLUS II project contains design and configuration information, including the Boolean equations that the compiler derives from the design entry file(s) of the project.

10. A seven-segment display is an array of seven luminous segments (usually LED or LCD), arranged in a figure-8 pattern, used to display numerical digits.

11. The segments in a seven-segment display are designated by lowercase letters \( a \) through \( g \). The sequence of labels goes clockwise, starting with segment \( a \) at the top and ending with \( g \) in the center.

12. Seven-segment displays are configured as common anode (active-LOW inputs) or common cathode (active-HIGH segments).

13. A seven-segment decoder can be described with a truth table or Boolean equation for each segment function. Since the segment functions do not simplify very much, it is often easier to program a CPLD with a VHDL truth table, in the form of a selected signal assignment statement, rather than with the Boolean equations of the decoder.

14. A multiplexer (MUX) is a circuit that directs a signal or group of signals (called the data inputs) to an output, based on the status of a set of select inputs.

15. Generally, for \( n \) select inputs in a multiplexer, there are \( m = 2^n \) data inputs. Such a multiplexer is referred to as an \( m \)-to-1 multiplexer.

16. The selected data input in a MUX is usually denoted by a subscript that is the decimal equivalent of the combined binary value of the select inputs. For example, if the select inputs in an 8-to-1 MUX are set to \( S_2S_1S_0 = 100 \), data input \( D_4 \) is selected since 100 (binary) = 4 (decimal).

17. A MUX can be designed to switch groups of signals to a multi-bit output. The inputs can be denoted by double subscript notation, where the first subscript indicates the number of the signal group and the second subscript the element in the group. For example, a MUX can have a 4-bit set of inputs called \( D_{03}D_{02}D_{01}D_{00} \) and another 4-bit input group called \( D_{13}D_{12}D_{11}D_{10} \), each of which can be switched to a 4-bit output called \( Y_{31}Y_{21}Y_{11}Y_{01} \) by the state of one select input.

18. A multiplexer can be used in time-dependent applications if a binary counter is applied to its select inputs.

19. Some examples of time-dependent MUX applications are waveform or bit pattern generation and time-division multiplexing (TDM).

20. In time division multiplexing, several digital signals share a single transmission path by allotting a time slot for every signal, during which that signal has sole access to the transmission path.

21. TDM can be configured for bit multiplexing, in which a channel transmits one bit each time it is selected, or byte (or word) multiplexing, in which a channel transmits and entire byte or word each time it is selected.

22. A demultiplexer (DMUX) receives data from a single source and directs the data to one of several outputs, which is selected by the status of a set of select inputs.

23. A decoder with an enable input can act as a demultiplexer if the enable input of the decoder is used as a data input for a demultiplexer.

24. A TDM signal can be demultiplexed by applying a binary count to the DMUX’s select inputs at the same rate as the count is applied to the select input of the multiplexer that originally sent the data.

25. A CMOS analog multiplexer or demultiplexer works by using a decoder to enable a set of analog data transmission switches. It can be used in either direction.

26. A magnitude comparator determines whether two binary numbers are equal and, if not, which one is greater.

27. The simplest equality comparator is an XNOR gate, whose output is HIGH if both inputs are the same.

28. A pair of multiple-bit numbers can be compared by a set of XNOR gates whose outputs are ANDed. The circuit compares the two numbers bit-by-bit.

29. Given two numbers \( A \) and \( B \), the Boolean function \( \overline{A}B \overline{n} \), if true, indicates that the \( n \)th bit of \( A \) is less than the \( n \)th bit of \( B \).

30. Given two numbers \( A \) and \( B \), the Boolean function \( A\overline{B} \overline{n} \), if true, indicates that the \( n \)th bit of \( A \) is greater than the \( n \)th bit of \( B \).
31. The less-than and greater-than functions can be combined with an equality comparator to determine, bit-by-bit, how two numbers compare in magnitude to one another.

32. A magnitude comparator can be best implemented in VHDL by using INTEGER types for the inputs and using IF statements to compare their respective magnitudes.

33. Parity checking is a system of error detection that works by counting the number of 1s in a group of bits.

34. Even parity requires a group of bits to have an even number of 1s. Odd parity requires a group of bits to have an odd number of 1s. This is achieved by appending a parity bit to the data whose value depends on the number of 1s in the data bits.

35. An XOR gate is the simplest even parity generator. Each line in its truth table has an even number of 1s, if the output column is included.

36. An XNOR gate can be used to generate an odd parity bit from two data bits.

37. A parity checker consists of a parity generator on the receive end of a transmission system and a comparator to determine if the locally generated parity bit is the same as the transmitted parity bit.

38. Parity generators and checkers can be expanded to any number of bits by using an XOR gate for each pair of bits and combining the gate outputs in further stages of 2-input XOR gates.

**GLOSSARY**

**BCD**  Binary coded decimal. A code in which each individual digit of a decimal number is represented by a 4-bit binary number. (e.g., 905 (decimal) = 1001 0000 0101 (BCD)).

**Bit multiplexing**  A TDM technique in which one bit is sent from each channel during its assigned time slot.

**Byte (or word) multiplexing**  A TDM technique in which a byte (or word) is sent from each channel during its assigned time slot. (A byte is eight bits; a word is a group of bits whose size varies with the particular system.)

**CASE statement**  A VHDL construct in which there is a choice of statements to be executed, depending on the value of a signal or variable.

**Clock**  A signal that controls the operation of a sequential digital circuit, such as a counter, by advancing its outputs to the next state when it receives a pulse.

**CMOS analog switch**  A CMOS device that will pass an analog or digital signal in either direction, when enabled. Also called a transmission gate. There is no TTL equivalent.

**Common anode display**  A seven-segment LED display where the anodes of all the LEDs are connected to the circuit supply voltage. Each segment is illuminated by a logic LOW at its cathode.

**Common cathode display**  A seven-segment display in which the cathodes of all the LEDs are connected together and grounded. A logic HIGH illuminates a segment when applied to its anode.

**Conditional signal assignment statement**  A concurrent VHDL construct that assigns a value to a signal, depending on a sequence of conditions being true or false.

**Counter**  A digital circuit whose output produces a fixed sequence of binary states when an input called the clock receives a series of pulses. The output advances by one for each clock pulse (e.g., the output state of a 4-bit binary counter progresses in order from 0000 to 1111, then repeats).

**Data inputs**  The multiplexer inputs that feed a digital signal to the output when selected.

**Decoder**  A digital circuit designed to detect the presence of a particular digital state.

**Demultiplexer**  A circuit that uses a binary decoder to direct a digital signal from a single source to one of several destinations.

**Double-subscript notation**  A naming convention where two or more numerically related groups of signals are named using two subscript numerals. Generally, the first digit refers to a group of signals and the second to an element of a group. (e.g., $X_{03}$ represents element 3 of group 0 for a set of signal groups, $X$.)

**Encoder**  A circuit that generates a digital code at its outputs in response to one or more active input lines.

**Even parity**  An error-checking system that requires a binary number to have an even number of 1s.

**IF statement**  A VHDL construct within a process that executes a series of statements, if a Boolean test condition is true.

**Magnitude comparator**  A circuit that compares two n-bit binary numbers, indicates whether or not the numbers are equal, and, if not, which one is larger.

**Multiplexer**  A circuit that directs one of several digital signals to a single output, depending on the states of several select inputs.

**Odd parity**  An error-checking system that requires a binary number to have an odd number of 1s.

**Parity**  A system that checks for errors in a multi-bit binary number by counting the number of 1s.

**Parity bit**  A bit appended to a binary number to make the number of 1s even or odd, depending on the type of parity.

**Positive edge**  The point on a digital waveform where the logic level of the waveform makes a LOW-to-HIGH transition.

**Priority encoder**  An encoder that generates a binary or BCD output corresponding to the subscript of the active input having the highest priority. This is usually defined as the input with the largest subscript value.

**PROCESS**  A VHDL construct that contains statements that are executed if there is a change in a signal in its sensitivity list.

**Propagation delay**  Time difference between a change on a digital circuit input and a change on an output in response to the input change.

**RBI**  Ripple blanking input.

**RBO**  Ripple blanking output.

**Response waveforms**  A set of output waveforms generated by a simulator tool for a particular digital design in response to a set of stimulus waveforms.

**Ripple blanking**  A technique used in a multiple-digit numerical display that suppresses leading or trailing zeros in the display, but allows internal zeros to be displayed.
Select inputs  The multiplexer inputs which select a digital input channel.

Selected signal assignment statement  A concurrent signal assignment in VHDL in which a value is assigned to a signal, depending on the alternative values of another signal or variable.

Sensitivity list  A list of signals in a PROCESS statement that are monitored to determine whether the PROCESS should be executed.

Seven-segment display  An array of seven independently controlled light-emitting diode (LED) or liquid crystal display (LCD) elements, shaped like a figure-8, which can be used to display decimal digits and other characters by turning on the appropriate elements.

Simulation  The verification of the logic of a digital design before programming it into a PLD.

Stimulus waveforms  A set of user-defined input waveforms on a simulator file designed to imitate input conditions of a digital circuit.

Time division multiplexing (TDM)  A technique of using one transmission line to send many signals simultaneously by making them share the line for equal fractions of time.

Time slot  A period of time during which a transmitted data element has sole access to a transmission path.

Timing diagram  A diagram showing how two or more digital waveforms in a system relate to each other over time.

PROBLEMS

Section 5.1    Decoders

5.1  When a HIGH is on the outputs of each of the decoding circuits shown in Figure 5.71, what is the binary code appearing at the inputs? Write the Boolean expression for each decoder output.

![Decoding Circuits Diagram]

**FIGURE 5.71**  
Problem 5.1  
Decoding Circuits

5.2  Draw the decoding circuit for each of the following Boolean expressions:

a. \( \overline{Y} = \overline{D}_3D_2\overline{D}_1D_0 \)

b. \( \overline{Y} = \overline{D}_3D_2D_1D_0 \)

c. \( Y = D_3D_2D_1D_0 \)

d. \( \overline{Y} = D_3D_2\overline{D}_1D_0 \)

e. \( Y = D_3D_2\overline{D}_1D_0 \)

5.3  Use a Graphic Design File in MAX+PLUS II to draw the logic diagram of a 2-line-to-4-line decoder with active-HIGH outputs and an active-LOW enable input. Create a simulation file to show the operation of the circuit.

5.4  Use a Graphic Design File in MAX+PLUS II to draw the logic diagram of a 3-line-to-8-line decoder with active-HIGH outputs and an active-LOW enable input. Create a simulation file to show the operation of the circuit.

5.5  For a generalized \( n \)-line-to-\( m \)-line decoder, state the value of \( m \) if \( n \) is:

a. 5

b. 6

c. 8

Write the equation giving the general relation between \( n \) and \( m \).

5.6  A microcomputer system has a RAM capacity of 128 megabytes (MB), split into 16 MB portions. Each RAM device is enabled by a low at a \( G \) input. Draw a logic diagram showing how a binary decoder can select one particular RAM device.

5.7  Briefly describe the difference between a selected signal assignment statement and a conditional signal assignment statement in VHDL. State which one is the preferred statement in VHDL files and why.

5.8a.  Write a VHDL file for a 3-line-to-8-line decoder with active-LOW outputs and no enable. Use a selected signal assignment statement. Assign the device as an EPM 7128SLC84.

b. Write the Boolean equations for the decoder in part a, as reported in the decoder’s MAX+PLUS II report file. (Use the form \((x \cdot \overline{y} \cdot \overline{z} + x \cdot y \cdot z)\) rather than \((!x \& !y \& !z \# x \& y \& z)\.)

c. Change the decoder in part a so that its outputs are active-HIGH. Compile the design and examine the resulting report file to find the Boolean equations of the modified design. Write the equations and state
5.9 Create a MAX+PLUS II simulation file for the decoder in Problem 5.8.

5.10 Write a VHDL file for a 3-line-to-8-line decoder with active-LOW outputs and an active-LOW enable input.

5.11 Create a MAX+PLUS II simulation file for the decoder in Problem 5.10.

5.12 Write a truth table for a hexadecimal-to-seven-segment decoder for a common anode display. Use the digit patterns of Figure 5.26 as a model.

5.13 Use the truth table derived in Problem 5.12 to derive the Boolean equations for each segment driver. Simplify the equations as much as possible, using any convenient method.

5.14 Write a VHDL file for the hexadecimal-to-seven-segment decoder described in Problem 5.12.

5.15 Modify the VHDL file for the hexadecimal-to-seven-segment decoder from Problem 5.14 to add a ripple-blanking feature.

5.16 Draw a diagram consisting of four seven-segment displays, each driven by a BCD-to-seven-segment decoder with ripple blanking. The circuit should be configured to suppress all leading zeros. Show the displayed digits and RBO/RBI logic levels for each of the following displayed values: 100, 217, 1024.

Section 5.2 Encoders

5.17 Figure 5.72 shows a BCD priority encoder with three different sets of inputs. Determine the resulting output code for each input combination. Inputs and outputs are active HIGH.

5.18 Derive the Boolean equations for the outputs of a BCD priority encoder, based on the encoding principle stated in Section 5.2. Show all work.

5.19 Create a Graphic Design File in MAX+PLUS II for a BCD priority encoder, based on the equations in Problem 5.18. Also generate a simulation for this function.

5.20 Write a VHDL file that implements the function of a BCD priority encoder. Create a simulation file for this function. Write the Boolean equations of the encoder, as shown in the encoder’s report file. State how the equations from the report file compare to the equations you derived in Problem 5.18.

5.21 Write a VHDL file that implements the function of a 4-bit binary priority encoder. Create a simulation file for this function.

Section 5.3 Multiplexers

5.22 Make a table listing which digital audio source in Figure 5.73 is routed to output $Y$ for each combination of the multiplexer select inputs. (CD = compact disc; DAT = digital audio tape.)

5.23 Draw symbols for an 8-to-1 and a 16-to-1 multiplexer. Write the truth table for each multiplexer, showing which data input is selected for every binary combination of the select inputs.

5.24 Make a Graphic Design File in MAX+PLUS II for an 8-to-1 multiplexer circuit. Also create a simulation that shows the operation of the device.

5.25 Write the Boolean expression describing an 8-to-1 multiplexer. Evaluate the equation for the case where input $D_5$ is selected.

5.26 Draw the symbol for a quadruple 8-to-1 multiplexer (i.e., a MUX with eight switched groups of 4 bits each). Write the truth table for this device, showing which data inputs are selected for every binary combination of the select inputs. Use double-subscript notation.
5.27 Write a VHDL file for the quadruple 8-to-1 multiplexer in Problem 5.26. Create a MAX+PLUS II simulation for the design to verify its operation.

5.28 Draw the symbol for an octal 4-to-1 multiplexer (i.e., a MUX with four switched groups of 8 bits each). Write the truth table for this device, showing which data inputs are selected for every binary combination of the select inputs. Use double-subscript notation.

5.29 Write a VHDL file for the octal 4-to-1 multiplexer in the Problem 5.28. Create a MAX+PLUS II simulation for the design to verify its operation. Write its Boolean equations from the project report file.

5.30 Write a VHDL file for an 8-to-1 multiplexer using a concurrent signal assignment statement to encode the multiplexer’s Boolean equation directly. Would this be a good method for encoding a larger device, such as a 16-to-1 multiplexer? Explain your answer.

5.31 Write a VHDL file for an 8-to-1 multiplexer using a selected signal assignment statement. Would this be a good method for encoding a larger device, such as a 16-to-1 multiplexer? Explain your answer.

5.32 Write a VHDL file for a 16-to-1 multiplexer using the method you believe to be most efficient.

5.33 Draw the circuit of a programmable waveform generator based on an 8-to-1 multiplexer. Draw a timing diagram of this circuit for the following input data:
   a. \( D_7D_6D_5D_4D_3D_2D_1D_0 = 01100101 \)
   b. \( D_7D_6D_5D_4D_3D_2D_1D_0 = 01010101 \)

5.34 The data pattern in Problem 5.34b generates a symmetrical 12 kHz waveform. Write the data patterns required to produce a 6 kHz waveform and a 3 kHz waveform at the output of a MUX-based programmable waveform generator.

Section 5.4 Demultiplexers

5.35 Make a Graphic Design File in MAX+PLUS II for a 1-to-4 demultiplexer circuit with active-LOW outputs and an active-LOW enable input. Create a simulation that shows how this device can be used as a demultiplexer or decoder.

5.36 Make a Graphic Design File in MAX+PLUS II for a 1-to-8 demultiplexer circuit with active-HIGH outputs. Create a simulation that shows the operation of the device.

5.37 Write a VHDL file that implements the function of a 1-to-16 demultiplexer.

5.38 Briefly state what characteristics of an analog switch make it suitable for transmitting analog signals.

5.39 Draw a diagram showing how eight analog switches can be connected to a decoder to form an 8-channel MUX/DMUX circuit. Briefly explain why the same circuit can be used as a multiplexer or as a demultiplexer.

5.40 Draw a circuit showing how a 74HC4052 dual 4-channel analog MUX/DMUX can be used to multiplex four transmitted digital audio channels onto a phone line and demultiplex four received audio channels from another phone line.
**Section 5.5  Magnitude Comparators**

5.41 Briefly explain the operation of the ALTB portion of the 2-bit magnitude comparator shown in Figure 5.58.

5.42 Draw the ALTB portion of a 4-bit magnitude comparator as a Graphic Design File in MAX+PLUS II. Create a simulation for the circuit and briefly explain its operation.

5.43 Use MAX+PLUS II to create a 3-bit magnitude comparator that has outputs for AEQB, AGTB, and ALTB functions. Create a simulation that shows the operation of this circuit.

5.44 Write the Boolean expressions for the AEQB, ALTB, and AGTB outputs of a 6-bit magnitude comparator.

5.45 Write a VHDL file that implements the functions A < B, A = B and A > B for two 16-bit numbers.

5.46 Write a VHDL file that implements the following six comparison functions in a single device for two 4-bit inputs A and B: A < B, A ≤ B, A = B, A ≠ B, A ≥ B, and A > B. Make the outputs indicate active-LOW.

5.47 Create a simulation that verifies the operation of the six-function comparator in Problem 5.46.

**Section 5.6  Parity Generators and Checkers**

5.48 What parity bit, P, should be added to the following data if the parity is EVEN? If the parity is ODD?
   a. 1111100
   b. 1010110
   c. 0001101

5.49 The following data are transmitted in a serial communication system (P is the parity bit). What parity is being used in each case?
   a. ABCDEFGHP = 010000101
   b. ABCDEFGHP = 011000101
   c. ABCDP = 01101
   d. ABCDEP = 101011
   e. ABCDEP = 111011

5.50 The data ABCDEFGHP = 110001100 are transmitted in a serial communication system. Give the output P’ of a receiver parity checker for the following received data. State the meaning of the output P’ for each case.
   a. ABCDEFGHP = 110101100
   b. ABCDEFGHP = 110001101
   c. ABCDEFGHP = 110001100
   d. ABCDEFGHP = 110010100
   e. ABCDEFGHP = 110010100

5.51 Use MAX+PLUS II to create a Graphic Design File for a 5-bit parity generator with a switchable EVEN/ODD output. Create a simulation file to show the operation of the device.

5.52 Use MAX+PLUS II to create a Graphic Design File for a 5-bit parity checker corresponding to the parity generator in Problem 5.51. Create a simulation file to show the operation of the device.

---

**Answers to Section Review Problems**

Section 5.1a

5.1 The decoders are shown in Figure 5.74.

![Decoders](image1)

**FIGURE 5.74**

Decoders

Section 5.1b

5.2 A decoder with 16 outputs requires 4 inputs. A decoder with 32 outputs requires 5 inputs.

Section 5.1c

5.3 Trailing zeros could logically be suppressed after a decimal point or if there are digits displaying a power-of-ten exponent (e.g., 455. or 4.55 02), that is, if the zeros are nonsignificant. The zeros should be displayed if they set the location of the decimal point (e.g., 450).

Section 5.2

5.4 The encoder in Figure 5.29 can have only one input active at any time. If more than one input is active, it may generate incorrect output codes. The circuit can be modified according to the priority encoding principle, as expressed by the Boolean equations for the 3-bit priority encoder, to ensure that a low-priority input is not able to modify the code generated by a higher-priority input.

Section 5.3

5.5 A multiplexer application is time-dependent if its channels are selected in a repeating sequence. This can be accomplished by connecting a binary counter to the select inputs of the multiplexer.

Section 5.6

5.6 Parts a and c are certainly incorrect because each has an even number of 1s. Items b, d, and e could have an even number of errors, which is undetectable by parity checking.
There are two ways of performing binary arithmetic: with unsigned binary numbers or with signed binary numbers. Signed binary numbers incorporate a bit defining the sign of a number; unsigned binary numbers do not. Several ways of writing signed binary numbers are true-magnitude form, which maintains the magnitude of the number in binary value, and 1’s complement and 2’s complement forms, which modify the magnitude but are more suited to digital circuitry.
Hexadecimal arithmetic is used for calculations that would be awkward in binary due to the large number of bits involved. Important applications of hexadecimal arithmetic are found in microcomputer systems.

In addition to positional number systems, binary numbers can be used in a variety of nonpositional number codes, which can represent numbers, letters, and computer control codes. Binary coded decimal (BCD) codes represent decimal digits as individually encoded groups of bits. Gray code is a binary code used in special applications. American Standard Code for Information Interchange (ASCII) represents alphanumeric and control code characters in a 7- or 8-bit format.

There are a number of different digital circuits for performing digital arithmetic, most of which are based on the parallel binary adder, which in turn is based on the full adder and half adder circuits. The half adder adds two bits and produces a sum and a carry. The full adder also allows for an input carry from a previous adder stage. Parallel adders have many full adders in cascade, with carry bits connected between the stages.

Specialized adder circuits are used for adding and subtracting binary numbers, generating logic functions, and adding numbers in binary-coded decimal (BCD) form.

6.1 Digital Arithmetic

Digital arithmetic usually means binary arithmetic, or perhaps BCD arithmetic. Binary arithmetic can be performed using signed binary numbers, in which the MSB of each number indicates a positive or negative sign, or unsigned binary numbers, in which the sign is presumed to be positive.

The usual arithmetic operations of addition and subtraction can be performed using signed or unsigned binary numbers. Signed binary arithmetic is often used in digital circuits for two reasons:

1. Calculations involving real-world quantities require us to use both positive and negative numbers.
2. It is easier to build circuits to perform some arithmetic operations, such as subtraction, with certain types of signed numbers than with unsigned numbers.

Unsigned Binary Arithmetic

Operand A number upon which an arithmetic function operates (e.g., in the expression \( x + y = z \), \( x \) and \( y \) are the operands).

Augend The number in an addition operation to which another number is added.

Addend The number in an addition operation that is added to another.

Sum The result of an addition operation.

Carry A digit that is “carried over” to the next most significant position when the sum of two single digits is too large to be expressed as a single digit.
**Sum bit (single-bit addition)**  The least significant bit of the sum of two 1-bit binary numbers.

**Carry bit**  A bit that holds the value of a carry (0 or 1) resulting from the sum of two binary numbers.

### Addition

When we add two numbers, they combine to yield a result called the **sum**. If the sum is larger than can be contained in one digit, the operation generates a second digit, called the **carry**. The two numbers being added are called the **augend** and the **addend**, or more generally, the **operands**.

For example, in the decimal addition $9 + 6 = 15$, 9 is the augend, 6 is the addend, and 15 is the sum. Since the sum cannot fit into a single digit, a carry is generated into a second digit place.

Four binary sums give us all of the possibilities for adding two $n$-bit binary numbers:

- $0 + 0 = 00$
- $1 + 0 = 01$
- $1 + 1 = 10$  \( (1_{10} + 1_{10} = 2_{10}) \)
- $1 + 1 + 1 = 11$  \( (1_{10} + 1_{10} + 1_{10} = 3_{10}) \)

Each of these results consists of a **sum bit** and a **carry bit**. For the first two results above, the carry bit is 0. The final sum in the table is the result of adding a carry bit from a sum in a less significant position.

When we add two 1-bit binary numbers in a logic circuit, the result always consists of a sum bit and a carry bit, even when the carry is 0, since each bit corresponds to a measurable voltage at a specific circuit location. Just because the value of the carry is 0 does not mean it has ceased to exist.

---

**EXAMPLE 6.1**

Calculate the sum $10010 + 1010$.

**SOLUTION**

\[
\begin{array}{c}
1 \\
10010 \\
+ 1010 \\
11100
\end{array}
\]

---

**EXAMPLE 6.2**

Calculate the sum $10111 + 10010$.

**SOLUTION**

\[
\begin{array}{c}
11 \\
1111 \\
10111 \\
+ 10010 \\
101001
\end{array}
\]

---

**SECTION 6.1A REVIEW PROBLEMS**

6.1 Add $11111 + 1001$.

6.2 Add $10011 + 1101$. 
Subtraction

**KEY TERMS**

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Difference</td>
<td>The result of a subtraction operation.</td>
</tr>
<tr>
<td>Minuend</td>
<td>The number in a subtraction operation from which another number is subtracted.</td>
</tr>
<tr>
<td>Subtrahend</td>
<td>The number in a subtraction operation that is subtracted from another number.</td>
</tr>
<tr>
<td>Borrow</td>
<td>A digit brought back from a more significant position when the subtrahend digit is larger than the minuend digit.</td>
</tr>
</tbody>
</table>

In unsigned binary subtraction, two operands, called the subtrahend and the minuend, are subtracted to yield a result called the difference. In the operation \( x = a - b \), \( x \) is the difference, \( a \) is the minuend, and \( b \) is the subtrahend. To remember which comes first, think of the minuend as the number that is diminished (i.e., something is taken away from it).

Unsigned binary subtraction is based on the following four operations:

\[
\begin{align*}
0 - 0 &= 0 \\
1 - 0 &= 1 \\
1 - 1 &= 0 \\
10 - 1 &= 1 \quad (2_{10} - 1_{10} = 1_{10})
\end{align*}
\]

The last operation shows how to obtain a positive result when subtracting a 1 from a 0: borrow 1 from the next most significant bit.

**Borrowing Rules:**

1. If you are borrowing from a position that contains a 1, leave behind a 0 in the borrowed-from position.
2. If you are borrowing from a position that already contains a 0, you must borrow from a more significant digit that contains a 1. All 0s up to that point become 1s, and the last borrowed-from digit becomes a 0.

---

**EXAMPLE 6.3**

Subtract 1110 - 1001.

**SOLUTION**

\[
\begin{array}{c|c|c}
\text{New 2nd LSB} & \text{Bit borrowed from 2nd LSB} \\
01 & \text{(Bit borrowed from 2nd LSB)} \\
11 & 0 \\
- & 1001 \\
0 & 0101
\end{array}
\]

**EXAMPLE 6.4**

Subtract 10000 - 101.

**SOLUTION**

\[
\begin{array}{c|c|c|c|c|c}
\text{10000 (original)} & \text{1111 (After borrowing)} & \text{10000} & \text{1111} & \text{10000} & \text{1111} \\
- & \text{101 (After borrowing from higher-order bits)} & - & 101 & \text{101} & \text{101} \\
\hline
\text{1011} & \text{1011} & \text{1011} & \text{1011} & \text{1011} & \text{1011}
\end{array}
\]
6.2 Representing Signed Binary Numbers

**KEY TERMS**

- **Sign bit**: A bit, usually the MSB, that indicates whether a signed binary number is positive or negative.
- **Magnitude bits**: The bits of a signed binary number that tell us how large the number is (i.e., its magnitude).
- **True-magnitude form**: A form of signed binary number whose magnitude is represented in true binary.
- **1's complement**: A form of signed binary notation in which negative numbers are created by complementing all bits of a number, including the sign bit.
- **2's complement**: A form of signed binary notation in which negative numbers are created by adding 1 to the 1’s complement form of the number.

**NOTE**

Positive numbers are the same in all three notations.

Binary arithmetic operations are performed by digital circuits that are designed for a fixed number of bits, since each bit has a physical location within a circuit. It is useful to have a way of representing binary numbers within this framework that accounts not only for the magnitude of the number, but for the sign as well.

This can be accomplished by designating one bit of a binary number, usually the most significant bit, as the **sign bit** and the rest as **magnitude bits**. When the number is negative, the sign bit is 1, and when the number is positive, the sign bit is 0.

There are several ways of writing the magnitude bits, each having its particular advantages. **True-magnitude form** represents the magnitude in straight binary form, which is relatively easy for a human operator to read. Complement forms, such as **1's complement** and **2's complement**, modify the magnitude so that it is more suited to digital circuitry.

**True-Magnitude Form**

In true-magnitude form, the magnitude of a number is translated into its true binary value. The sign is represented by the MSB, 0 for positive and 1 for negative.

**EXAMPLE 6.5**

Write the following numbers in 6-bit true-magnitude form:

- a. \(25_{10}\)
- b. \(-25_{10}\)
- c. \(12_{10}\)
- d. \(-12_{10}\)

**SOLUTION**

Translate the magnitudes of each number into 5-bit binary, padding with leading zeros as required, and set the sign bit to 0 for a positive number and 1 for a negative number.

- a. 011001
- b. 111001
- c. 001100
- d. 101100
1’s Complement Form

True-magnitude and 1’s complement forms of binary numbers are the same for positive numbers—the magnitude is represented by the true binary value and the sign bit is 0. We can generate a negative number in one of two ways:

1. Write the positive number of the same magnitude as the desired negative number. Complement each bit, including the sign bit; or
2. Subtract the \( n \)-bit positive number from a binary number consisting of \( n \) 1s.

EXAMPLE 6.6

Convert the following numbers to 8-bit 1’s complement form:

a. \( 57_{10} \)  
   b. \( -57_{10} \)  
   c. \( 72_{10} \)  
   d. \( -72_{10} \)

SOLUTION

Positive numbers are the same as numbers in true-magnitude form. Negative numbers are the bitwise complements of the corresponding positive number.

a. \( 57_{10} = 00111001 \)
b. \( -57_{10} = 11000110 \)
c. \( 72_{10} = 01001000 \)
d. \( -72_{10} = 10110111 \)

We can also generate an 8-bit 1’s complement negative number by subtracting its positive magnitude from 11111111 (eight 1s). For example, for part b:

\[
\begin{align*}
11111111 & \quad (1 \text{ of } 8) \\
-00111001 & \quad (-57_{10}) \\
11000110 & \quad (57_{10}) \\
\end{align*}
\]

2’s Complement Form

Positive numbers in 2’s complement form are the same as in true-magnitude and 1’s complement forms. We create a negative number by adding 1 to the 1’s complement form of the number.

EXAMPLE 6.7

Convert the following numbers to 8-bit 2’s complement form:

a. \( 57_{10} \)  
   b. \( -57_{10} \)  
   c. \( 72_{10} \)  
   d. \( -72_{10} \)

SOLUTION

a. \( 57 = 00111001 \)
b. \( -57 = 11000110 \) (1’s complement)
\[ \begin{array}{l}
\text{1} \\
11000111 \\
\text{(2’s complement)}
\end{array} \]
c. \( 72 = 01001000 \)
d. \( -72 = 10110111 \) (1’s complement)
\[ \begin{array}{l}
\text{1} \\
10111000 \\
\text{(2’s complement)}
\end{array} \]

A negative number in 2’s complement form can be made positive by 2’s complementing it again. Try it with the negative numbers in Example 6.7.
6.3 Signed Binary Arithmetic

**Signed Addition**

Signed addition is done in the same way as unsigned addition. The only difference is that both operands must have the same number of magnitude bits, and each has a sign bit.

**EXAMPLE 6.8**

Add $+30_{10}$ and $+75_{10}$. Write the operands and the sum as 8-bit signed binary numbers.

**SOLUTION**

\[
egin{array}{cc}
+30 & 00011110 \\
+75 & +01001011 \\
+105 & 01101001 \\
\end{array}
\]

(Magnitude bits)

(Sign bit)

**Subtraction**

The real advantage of complement notation becomes evident when we subtract signed binary numbers. In complement notation, we add a negative number instead of subtracting a positive number. We thus have only one kind of operation—addition—and can use the same circuitry for both addition and subtraction.

This idea does not work for true-magnitude numbers. In the complement forms, the magnitude bits change depending on the sign of the number. In true-magnitude form, the magnitude bits are the same regardless of the sign of the number.

Let us subtract $80_{10} - 65_{10} = 15_{10}$ using 1’s complement and 2’s complement addition. We will also show that the method of adding a negative number to perform subtraction is not valid for true-magnitude signed numbers.

**1’s Complement Method**

**KEY TERM**

**End-around carry** An operation in 1’s complement subtraction where the carry bit resulting from a sum of two 1’s complement numbers is added to that sum.

Add the 1’s complement values of 80 and $-65$. If the sum results in a carry beyond the sign bit, perform an end-around carry. That is, add the carry to the sum.

\[
\begin{array}{c}
80_{10} = 01010000 \\
65_{10} = 01000001 \\
-65_{10} = 10111110 & (1’s \ complement) \\
\end{array}
\]

\[
\begin{array}{c}
80 & 01010000 \\
-65 & +10111110 \\
\hline
1 & 00001110 \\
\hline
\end{array}
\]

(End-around carry)

\[
\begin{array}{c}
+15 & 00001111
\end{array}
\]
2’s Complement Method
Add the 2’s complement values of 80 and -65. If the sum results in a carry beyond the sign bit, discard it.

\[
\begin{align*}
80_{10} & = 01010000 \\
65_{10} & = 01000001 \\
-65_{10} & = 10111110 \quad \text{(1’s complement)} \\
+ & 1 \\
10111111 \quad \text{(2’s complement)} \\
\end{align*}
\]

\[
\begin{align*}
80 & \quad 01010000 \\
-65 & \quad 10111111 \\
+15 & \quad 100001111 \\
\hline
\end{align*}
\]

(Discard carry)

True-Magnitude Method

\[
\begin{align*}
80_{10} & = 01010000 \\
65_{10} & = 01000001 \\
-65_{10} & = 11000001 \\
\end{align*}
\]

\[
\begin{align*}
80 & \quad 01010000 \\
-65 & \quad 11000001 \\
? & \quad 10000001 \\
\end{align*}
\]

If we perform an end-around carry, the result is 00010010 = 18_{10}. If we discard the carry, the result is 00010001 = 17_{10}. Neither answer is correct. Thus, adding a negative true-magnitude number is not equivalent to subtraction.

Negative Sum or Difference
All examples to this point have given positive-valued results. When a 2’s complement addition or subtraction yields a negative sum or difference, we can’t just read the magnitude from the result, since a 2’s complement operation modifies the bits of a negative number. We must calculate the 2’s complement of the sum or difference, which will give us the positive number that has the same magnitude. That is, \((-x) = +x\).

EXAMPLE 6.9
Subtract 65_{10} - 80_{10} in 2’s complement form.

SOLUTION

\[
\begin{align*}
65_{10} & = 01000001 \\
80_{10} & = 01010000 \\
-80_{10} & = 10101111 \quad \text{(1’s complement)} \\
+ & 1 \\
10110000 \quad \text{(2’s complement)} \\
\end{align*}
\]

\[
\begin{align*}
65 & \quad 01000001 \\
-80 & \quad 10110000 \\
\hline
11110001 \\
\hline
00001110 \quad \text{(1’s complement)} \\
+ & 1 \\
00001111 \quad \text{(2’s complement)} \\
\end{align*}
\]

Take the 2’s complement of the difference to find the positive number with the same magnitude.
00001111 = +15_{10}. We generated this number by complementing 11110001. Thus, 11110001 = −15_{10}.

### Range of Signed Numbers

The largest positive number in 2’s complement notation is a 0 followed by \( n \) 1s for a number with \( n \) magnitude bits. For instance, the largest positive 4-bit number is 0111 = +7_{10}.

The negative number with the largest magnitude is not the 2’s complement of the largest positive number. We can find the largest negative number by extension of a sequence of 2’s complement numbers.

The 2’s complement form of \( -7_{10} \) is 1000 + 1 = 1001. The positive and negative numbers with the next largest magnitudes are 0110 (= +6_{10}) and 1010 (= −6_{10}). If we continue this process, we will get the list of numbers in Table 6.1.

- We have generated the 4-bit negative numbers from \( -1_{10} \) (1111) through \( -7_{10} \) (1001) by writing the 2’s complement forms of the positive numbers 1 through 7. Notice that these numbers count down in binary sequence. The next 4-bit number in the sequence (which is the only binary number we have left) is 1000. By extension, 1000 = −8_{10}. This number is its own 2’s complement. (Try it.) It exemplifies a general rule for the \( n \)-bit negative number with the largest magnitude.

### EXAMPLE 6.10

Write the largest positive and negative numbers for an 8-bit signed number in decimal and 2’s complement notation.

#### SOLUTION

\[
01111111 = +127 \quad (7 \text{ magnitude bits: } 2^7 - 1 = 127)
\]

\[
10000000 = -128 \quad (1 \text{ followed by seven } 0s: -2^7 = -128)
\]

### EXAMPLE 6.11

Write \( -16_{10} \)

a. As an 8-bit 2’s complement number

b. As a 5-bit 2’s complement number

(8-bit numbers are more common than 5-bit numbers in digital systems, but it is useful to see how we must write the same number differently with different numbers of bits.)

#### SOLUTION

a. An 8-bit number has 7 magnitude bits and 1 sign bit.

\[
+16 = 00010000
\]

\[
-16 = 11101111 \quad \text{(1’s complement)}
\]

\[
+1 = 00000001
\]

\[
11100000 \quad \text{(2’s complement)}
\]

b. A 5-bit number has 4 magnitude bits and 1 sign bit. Four magnitude bits are not enough to represent 16. However, a 1 followed by \( n \) 0s is equal to \( -2^n \). For a 1 and four 0s, \( -2^4 = -16 \). Thus, 10000 = −16_{10}.
The last five bits of the binary equivalent of $-16$ are the same in both the 5-bit and 8-bit numbers.

**NOTE**

The 8-bit number is padded with leading 1s. This same general pattern applies for any negative number with a power-of-2 magnitude. ($-2^n = n$ 0s preceding all 1s within the defined number size.)

### SECTION 6.3 REVIEW PROBLEM

6.5 Write $-32$ as an 8-bit 2’s complement number.
6.6 Write $-32$ as a 6-bit 2’s complement number.

### Sign Bit Overflow

**KEY TERM**

**Overflow** An erroneous carry into the sign bit of a signed binary number that results from a sum or difference larger than can be represented by the number of magnitude bits.

Signed addition of positive numbers is performed in the same way as unsigned addition. The only problem occurs when the number of bits in the sum of two numbers exceeds the number of magnitude bits and overflows into the sign bit. This causes the number to appear to be negative when it is not. For example, the sum $75 \, \text{and} \, 96$ causes an overflow in 8-bit signed addition. In unsigned addition the binary equivalent is:

| 1001011 |
| + 1100000 |
| 10101011 |

In signed addition, the sum is the same, but has a different meaning.

| 0 | 1001011 |
| + 0 | 1100000 |
| 1 | 0101011 |

(Sign bit) ______ (Magnitude bits)

The sign bit is 1, indicating a negative number, which cannot be true, since the sum of two positive numbers is always positive.

**NOTE**

A sum of positive signed binary numbers must not exceed $2^n - 1$ for numbers having $n$ magnitude bits. Otherwise, there will be an overflow into the sign bit.

### Overflow in Negative Sums

Overflow can also occur with large negative numbers. For example, the addition of $-80_{10}$ and $-65_{10}$ should produce the result:

$$-80_{10} + (-65_{10}) = -145_{10}$$
In 2’s complement notation, we get:

+80_10 = 01010000
-80_10 = 10101111 (1’s complement)
+ 1
10110000 (2’s complement)

+65_10 = 01000001
-65_10 = 10111110 (1’s complement)
+ 1
10111111 (2’s complement)

-80 + (-65) 10110000
? 1 01101111 (Incorrect magnitude = 111_10)
(Erroneous sign bit = 0)
(Discard carry)

This result shows a positive sum of two negative numbers—clearly incorrect. We can extend the statement we made earlier about permissible magnitudes of sums to include negative as well as positive numbers.

**Note**

A sum of signed binary numbers must be within the range of $-2^n \leq \text{sum} \leq 2^n - 1$ for numbers having $n$ magnitude bits. Otherwise, there will be an overflow into the sign bit.

For an 8-bit signed number in 2’s complement form, the permissible range of sums is $10000000 \leq \text{sum} \leq 01111111$. In decimal, this range is $-128 \leq \text{sum} \leq +127$.

**Note**

A sum of two positive numbers is always positive. A sum of two negative numbers is always negative. Any 2’s complement addition or subtraction operation that appears to contradict these rules has produced an overflow into the sign bit.

**Example 6.12**

Which of the following sums will produce a sign bit overflow in 8-bit 2’s complement notation? How can you tell?

a. $67_{10} + 33_{10}$
b. $67_{10} + 63_{10}$
c. $-96_{10} - 22_{10}$
d. $-96_{10} - 42_{10}$

**Solution**

A sign bit overflow is generated if the sum of two positive numbers appears to produce a negative result or the sum of two negative numbers appears to produce a positive result. In other words, overflow occurs if the operand sign bits are both 1 and the sum sign bit is 0 or vice versa. We know this will happen if an 8-bit sum is outside the range $(-128 \leq \text{sum} \leq +127)$.

a. $+67_{10} \ 01000011$ (no overflow; sum of positive numbers
$+33_{10} \ 00100001$ is positive.)
b. 
\[ +67_{10} = 01000011 \]  (Overflow; sum of positive numbers is negative.)
\[ +63_{10} = 00111111 \]
\[ 130_{10} = 10000010 \]  Sum > +127; out of range.)

c. 
\[ +96 = 01100000 \]
\[ -96 = 10011111 \]  (1’s complement)
\[ + 1 \]
\[ 10100000 \]  (2’s complement)
\[ +22 = 00010110 \]
\[ -22 = 11101001 \]  (1’s complement)
\[ + 1 \]
\[ 11101010 \]  (2’s complement)
\[ -96 = 10100000 \]
\[ -22 = 11101010 \]
\[ -118 = 11001010 \]
(No overflow; sum of two negative numbers is negative.)

d. 
\[ +96 = 01100000 \]
\[ -96 = 10011111 \]  (1’s complement)
\[ + 1 \]
\[ 10100000 \]  (2’s complement)
\[ +42 = 00101010 \]
\[ -42 = 11010101 \]  (1’s complement)
\[ + 1 \]
\[ 11010110 \]  (2’s complement)
\[ -96 = 10100000 \]
\[ -42 = 11010110 \]
\[ -138 = 10110111 \]
(Overflow; sum of two negative numbers is positive. Sum < −128; out of range.)

**NOTE**

The carry bit generated in 1’s and 2’s complement operations is not the same as an overflow bit. (See Example 6.12, parts c and d.) An overflow is a change in the sign bit, which leads us to believe that the number is opposite in sign from its true value. A carry is the result of an operation carrying beyond the physical limits of an \( n \)-bit number. It is similar to the idea of an odometer rolling over from 999999.9 to 1 000000.0. There are not enough places to hold the new number, so it goes back to the beginning and starts over.

6.4 Hexadecimal Arithmetic

(This section may be omitted without loss of continuity.)

The main reason to be familiar with addition and subtraction in the hexadecimal system is that it is useful for calculations related to microcomputer and memory systems.
Microcomputer systems often use binary numbers of 8, 16, 20, or 32 bits. Rather than write out all these bits, we use hex numbers as shorthand. Binary numbers having 8, 16, 20, or 32 bits can be represented by 2, 4, 5, or 8 hex digits, respectively.

**Hex Addition**

Hex addition is very much like decimal addition, except that we must remember how to deal with the hex digits A to F. A few sums are helpful:

\[
\begin{align*}
F + 1 &= 10 \\
F + F &= 1E \\
F + F + 1 &= 1F
\end{align*}
\]

The positional multipliers for the hexadecimal system are powers of 16. Thus, the most significant bit of the first sum is the 16's column. The equivalent sum in decimal is:

\[15_{10} + 1_{10} = 16_{10} = 10H\]

The second sum is the largest possible sum of two hex digits; the carry to the next position is 1. This shows that the sum of two hex digits will never produce a carry larger than 1. The second sum can be calculated as follows:

\[
\begin{align*}
FH + FH &= 15_{10} + 15_{10} \\
&= 30_{10} \\
&= 16_{10} + 14_{10} \\
&= 10H + EH \\
&= 1EH
\end{align*}
\]

The third sum shows that if there is a carry from a previous sum, the carry to the next bit will still be 1.

**NOTE**

It is useful to think of any digits larger than 9 as their decimal equivalents. For any digit greater than 15\(_{10}\) (FH), subtract 16\(_{10}\), convert the difference to its hex equivalent, and carry 1 to the next digit position.

**EXAMPLE 6.13** Add 6B3H + A9CH.

**SOLUTION**

<table>
<thead>
<tr>
<th>Hex</th>
<th>Decimal Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>6B3</td>
<td>(6) (11) (3)</td>
</tr>
<tr>
<td>+A9C</td>
<td>(10) (9) (12)</td>
</tr>
</tbody>
</table>

For sums greater than 15, subtract 16 and carry 1 to the next position:

<table>
<thead>
<tr>
<th>Hex</th>
<th>Decimal Equivalents</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Carry)11</td>
<td>(1) (1)</td>
</tr>
<tr>
<td>6B3</td>
<td>(6) (11) (3)</td>
</tr>
<tr>
<td>+ A9C</td>
<td>(10) (9) (12)</td>
</tr>
<tr>
<td>114F</td>
<td>(1) (1) (4) (15)</td>
</tr>
</tbody>
</table>

Sum: 6B3H + A9CH = 114FH.
Hex Subtraction

There are two ways to subtract hex numbers. The first reverses the addition process in the previous section. The second is a complement form of subtraction.

EXAMPLE 6.14

Subtract 6B3H − 49CH.

SOLUTION

\[
\begin{array}{c|c|c}
\text{Hex} & \text{Decimal Equivalent} \\
6B3 & (6) (11) (3) \\
- 49C & - (4) (9) (12) \\
\hline
6A3 & (6) (10) (16 + 3) \\
- 49C & - (4) (9) (12) \\
\hline
217 & (2) (1) (7)
\end{array}
\]

To subtract the least significant digits, we must borrow 10H (16_{10}) from the previous position. This leaves the subtraction looking like this:

The second subtraction method is a complement method, where, as in 2’s complement subtractions, we add a negative number to subtract a positive number.

Calculate the 15’s complement of a hex number by subtracting it from a number having the same number of digits, all Fs. Calculate the 16’s complement by adding 1 to this number. This is the negated value of the number.

EXAMPLE 6.15

Negate the hex number 15AC by calculating its 16’s complement.

SOLUTION

\[
\begin{array}{c|c|c}
\text{Hex} & \text{Decimal Equivalent} \\
\hline
FFFF & \\
- 15AC & (15’s complement) \\
\hline
EA53 + 1 & EA54 (16’s complement) \\
\hline
\end{array}
\]

The original value, 15AC, can be restored by calculating the 16’s complement of EA54. Try it.

EXAMPLE 6.16

Subtract 8B63 − 55D7 using the complement method.

SOLUTION

Find the 16’s complement of 55D7.

\[
\begin{array}{c|c|c}
\text{Hex} & \text{Decimal Equivalent} \\
\hline
FFFF & \\
- 55D7 & (15’s complement) \\
\hline
AA28 + 1 & AA29 (16’s complement) \\
\hline
\end{array}
\]
Therefore, \(-55D7 = AA29\).

\[
\begin{array}{c}
1 \\
8B63 \\
+ AA29 \\
1 358C \\
\hline \\
\text{(Discard carry)}
\end{array}
\]

Difference: \(8B63 - 55D7 = 358C\).

**SECTION 6.4 REVIEW PROBLEM**

6.7 Perform the following hexadecimal calculations:

a. \(A25F + 74A2\)

b. \(7380 - 5FFF\)

### 6.5 Numeric and Alphanumeric Codes

**BCD Codes**

**KEY TERM**

**Binary-coded decimal (BCD).** A code that represents each digit of a decimal number by a binary value.

BCD stands for binary-coded decimal. As the name implies, BCD is a system of writing decimal numbers with binary digits. There is more than one way to do this, as BCD is a code, not a positional number system. That is, the various positions of the bits do not necessarily represent increasing powers of a specified number base.

Two commonly used BCD codes are 8421 code, where the bits for each decimal digit are weighted, and Excess-3 code, where each decimal digit is represented by a binary number that is 3 larger than the true binary value of the digit.

#### 8421 Code

**KEY TERM**

**8421 code** A BCD code that represents each digit of a decimal number by its 4-bit true binary value.

The most straightforward BCD code is the 8421 code, also called Natural BCD. Each decimal digit is represented by its 4-bit true binary value. When we talk about BCD code, this is usually what we mean.

This code is called 8421 because these are the positional weights of each digit. Table 6.2 shows the decimal digits and their BCD equivalents.

8421 BCD is not a positional number system, because each decimal digit is encoded separately as a 4-bit number.

**EXAMPLE 6.17**

Write \(4987_{10}\) in both binary and 8421 BCD.

**SOLUTION** The binary value of \(4987_{10}\) can be calculated by repeated division by 2:

\[4987_{10} = 100110111011_2\]
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The BCD digits are the binary values of each decimal digit, encoded separately. We
can break bits into groups of 4 for easier reading. Note that the first and last BCD digits
each have a leading zero to make them 4 bits long.
498710  0100 1001 1000 0111BCD

❘❙❚

Excess-3 Code
KEY TERMS

Table 6.3 Decimal Digits and
Their 8421 and Excess-3
Equivalents
Decimal
Digit

8421

Excess-3

0
1
2
3
4
5
6
7
8
9

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001

0011
0100
0101
0110
0111
1000
1001
1010
1011
1100

Excess-3 Code A BCD code that represents each digit of a decimal number by a
binary number derived by adding 3 to its 4-bit true binary value.
9’s complement A way of writing decimal numbers where a number is made
negative by subtracting each of its digits from 9 (e.g., 726  999  726  273 in
9’s complement).
Self-complementing A code that automatically generates a negative equivalent
(e.g., 9’s complement for a decimal code) when all its bits are inverted.

Excess-3 code is a type of BCD code that is generated by adding 112 (310) to the
8421 BCD codes. Table 6.3 shows the Excess-3 codes and their 8421 and decimal
equivalents.
The advantage of this code is that it is self-complementing. If the bits of the Excess-3
digit are inverted, they yield the 9’s complement of the decimal equivalent.
We can generate the 9’s complement of an n-digit number by subtracting it from a
number made up of n 9s. Thus, the 9’s complement of 632 is 999  632  367.
The Excess-3 equivalent of 632 is 1001 0110 0101. If we invert all the bits, we get
0110 1001 1010. The decimal equivalent of this Excess-3 number is 367, the 9’s complement of 632.
This property is useful for performing decimal arithmetic digitally.

Gray Code
KEY TERM
Table 6.4 4-Bit Gray Code
Decimal

True
Binary

Gray
Code

0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

0000
0001
0011
0010
0110
0111
0101
0100
1100
1101
1111
1110
1010
1011
1001
1000

Gray code A binary code that progresses such that only one bit changes between
two successive codes.

Table 6.4 shows a 4-bit Gray code compared to decimal and binary values. Any two adjacent Gray codes differ by exactly one bit.
Gray code can be extended indefinitely if you understand the relationship between
the binary and Gray digits. Let us name the binary digits b3b2b1b0, with b3 as the
most significant bit, and the Gray code digits g3g2g1g0 for a 4-bit code. For a 4-bit
code:
g3
g2
g1
g0

 b3
 b3 䊝 b2
 b2 䊝 b1
 b1 䊝 b0

For an n-bit code, the MSBs are the same in Gray and binary (gn  bn). The other
Gray digits are generated by the Exclusive OR function of the binary digits in the same
position and the next most significant position.


Another way to generate a Gray code sequence is to recognize the inherent symmetry in the code. For example, a 2-bit Gray code sequence is given by:

00
01
11
10

To generate a 3-bit Gray code, write the 2-bit sequence, then write it again in reverse order.

00
01
11
10
10
11
01
00

Add an MSB of 0 to the first four codes and an MSB of 1 to the last four codes. The sequence followed by the last two bits of all codes is symmetrical about the center of the sequence.

000
001
011
010
110
111
101
100

We can apply a similar process to generate a 4-bit Gray code. Write the 3-bit sequence, then again in reverse order. Add an MSB of 0 to the first half of the table and an MSB of 1 to the second half. This procedure yields the code in Table 6.4.

**ASCII Code**

**KEY TERMS**

*Alphanumeric code*  A code used to represent letters of the alphabet and numerical characters.


*Case shift*  Changing letters from capitals (uppercase) to small letters (lowercase) or vice versa.

Digital systems and computers could operate perfectly well using only binary numbers. However, if there is any need for a human operator to understand the input and output data of a digital system, it is necessary to have a system of communication that is understandable to both a human operator and the digital circuit.

A code that represents letters (alphabetic characters) and numbers (numeric characters) as binary numbers is called an *alphanumeric code*. The most commonly used alphanumeric code is *ASCII* (“askey”), which stands for American Standard Code for Information Interchange. ASCII code represents letters, numbers, and other “typewriter characters” in 7 bits. In addition, ASCII has a repertoire of “control characters,” codes that
are used to send control instructions to and from devices such as video display terminals, printers, and modems.

Table 6.5 shows the ASCII code in both binary and hexadecimal forms. The code for any character consists of the bits in the column heading, then those in the row heading. For example, the ASCII code for “A” is $1000001_2$ or $41_{16}$. The code for “a” is $1100001_2$ or $61_{16}$. The codes for capital (uppercase) and lower case letters differ only by the second most significant bit, for all letters. Thus, we can make an alphabetic **case shift**, like using the Shift key on a typewriter or computer keyboard, by switching just one bit.

Numeric characters are listed in column 3, with the least significant digit of the ASCII code being the same as the represented number value. For example, the numeric character “0” is equivalent to $30_{16}$ in ASCII. The character “9” is represented as $39_{16}$.

The codes in columns 0 and 1 are control characters. They cannot be displayed on any kind of output device, such as a printer or video monitor, although they may be used to control the device. For instance, if the codes $0AH$ (Line Feed) and $0DH$ (Carriage Return)

<table>
<thead>
<tr>
<th>MSBs</th>
<th>000</th>
<th>001</th>
<th>010</th>
<th>011</th>
<th>100</th>
<th>101</th>
<th>110</th>
<th>111</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
<td>NUL</td>
<td>DLE</td>
<td>SP</td>
<td>0</td>
<td>@</td>
<td>P</td>
<td>'</td>
<td>p</td>
</tr>
<tr>
<td>0001</td>
<td>SOH</td>
<td>DC1</td>
<td>!</td>
<td>1</td>
<td>A</td>
<td>Q</td>
<td>a</td>
<td>q</td>
</tr>
<tr>
<td>0010</td>
<td>STX</td>
<td>DC2</td>
<td>&quot;</td>
<td>2</td>
<td>B</td>
<td>R</td>
<td>b</td>
<td>r</td>
</tr>
<tr>
<td>0011</td>
<td>ETX</td>
<td>DC3</td>
<td>#</td>
<td>3</td>
<td>C</td>
<td>S</td>
<td>c</td>
<td>s</td>
</tr>
<tr>
<td>0100</td>
<td>EOT</td>
<td>DC4</td>
<td>$</td>
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**Control Characters:**

NUL—Null
SOH—Start of Header
STX—Start Text
ETX—End Text
EOT—End of Transmission
ENQ—Enquiry
ACK—Acknowledge
BEL—Bell
BS—Backspace
HT—Horizontal Tabulation
LF—Line Feed
VT—Vertical Tabulation
FF—Form Feed
CR—Carriage Return
SO—Shift Out
SI—Shift In
SP—Space

DLE—Data Link Escape
DC1—Device Control 1
DC2—Device Control 2
DC3—Device Control 3
DC4—Device Control 4
NAK—No Acknowledgment
SYN—Synchronous Idle
ETB—End of Transmission Block
CAN—Cancel
EM—End of Medium
SUB—Substitute
ESC—Escape
FS—Form Separator
GS—Group Separator
RS—Record Separator
US—Unit Separator
DEL—Delete
are sent to a printer, the paper will advance by one line and the print head will return to the beginning of the line.

The displayable characters begin at 20H ("space") and continue to 7EH ("tilde"). Spaces are considered ASCII characters.

EXAMPLE 6.17
Encode the following string of characters into ASCII (hexadecimal form). Do not include quotation marks.

“Total system cost: $4,000,000. @ 10%”

SOLUTION
Each character, including spaces, is represented by two hex digits as follows:

```
54 6F 74 61 6C 20 73 79 73 74 65 6D 20 63 6F 73 74 3A 20
Total system cost: SP $4 , 0 0 0 , 0 0 . SP @ SP 1 0 %
```

SECTION 6.5 REVIEW PROBLEM
6.8 Decode the following sequence of hexadecimal ASCII codes.

```
54 72 75 65 20 6F 72 20 46 61 6C 73 65 3A 20 31
2F 34 2C 30 30 30 2E 20 40 20 31 2F 32
```

6.6 Binary Adders and Subtractors

Half and Full Adders

**KEY TERMS**

- **Half adder**: A circuit that will add two bits and produce a sum bit and a carry bit.
- **Full adder**: A circuit that will add a carry bit from another full or half adder and two operand bits to produce a sum bit and a carry bit.

There are only three possible sums of two 1-bit binary numbers:

- \(0 + 0 = 00\)
- \(0 + 1 = 01\)
- \(1 + 1 = 10\)

We can build a simple combinational logic circuit to produce the above sums. Let us designate the bits on the left side of the above equalities as inputs to the circuit and the bits on the right side as outputs. Let us call the LSB of the output the sum bit, symbolized by \(\Sigma\), and the MSB of the output the carry bit, designated \(C_{OUT}\).

Figure 6.1 shows the logic symbol of the circuit, which is called a **half adder**. Its truth table is given in Table 6.6. Since addition is subject to the commutative property, \((A + B = B + A)\), the second and third lines of the truth table are the same.

The Boolean functions of the two outputs, derived from the truth table, are:

\[
C_{OUT} = AB \\
\Sigma = \overline{AB} + \overline{AB} = A \oplus B
\]
The corresponding logic circuit is shown in Figure 6.2.

The half adder circuit cannot account for an input carry, that is, a carry from a lower-order 1-bit addition. A full adder, shown in Figure 6.3, can add two 1-bit numbers and accept a carry bit from a previous adder stage. Operation of the full adder is based on the following sums:

\[
\begin{align*}
0 + 0 + 0 &= 00 \\
0 + 0 + 1 &= 01 \\
0 + 1 + 1 &= 10 \\
1 + 1 + 1 &= 11
\end{align*}
\]

Designating the left side of the above equalities as circuit inputs \(A\), \(B\), and \(C_{IN}\) and the right side as outputs \(C_{OUT}\) and \(\Sigma\), we can make the truth table in Table 6.7. (The second and third of the above sums each account for three lines in the full adder truth table.)

The unsimplified Boolean expressions for the outputs are:

\[
\begin{align*}
C_{OUT} &= \overline{A} \overline{B} \overline{C_{IN}} + A \overline{B} C_{IN} + A B \overline{C_{IN}} + A B C_{IN} \\
\Sigma &= \overline{A} B \overline{C_{IN}} + \overline{A} B C_{IN} + A \overline{B} \overline{C_{IN}} + A B C_{IN}
\end{align*}
\]

There are a couple of ways to simplify these expressions.

### Karnaugh Map Method

Since we have expressions for \(\Sigma\) and \(C_{OUT}\) in sum-of-products form, let us try to use the Karnaugh maps in Figure 6.4 to simplify them. The expression for \(\Sigma\) doesn’t reduce at all. The simplified expression for \(C_{OUT}\) is:

\[
C_{OUT} = A B + A C_{IN} + B C_{IN}
\]

### Boolean Algebra Method

The simplest circuit for \(C_{OUT}\) and \(\Sigma\) involves the Exclusive OR function, which we cannot derive from K-map groupings. This can be shown by Boolean algebra, as follows:

\[
\begin{align*}
C_{OUT} &= \overline{A} B C_{IN} + A \overline{B} C_{IN} + A B \overline{C_{IN}} + A B C_{IN} \\
&= (\overline{A} B + A \overline{B}) C_{IN} + A B (\overline{C_{IN}} + C_{IN}) \\
&= (A \oplus B) C_{IN} + A B
\end{align*}
\]

The corresponding logic circuits for \(\Sigma\) and \(C_{OUT}\), shown in Figure 6.5, don’t give us much of a simplification.

### Boolean Algebra Method

The simplest circuit for \(C_{OUT}\) and \(\Sigma\) involves the Exclusive OR function, which we cannot derive from K-map groupings. This can be shown by Boolean algebra, as follows:

\[
\begin{align*}
C_{OUT} &= \overline{A} B C_{IN} + A \overline{B} C_{IN} + A B \overline{C_{IN}} + A B C_{IN} \\
&= (\overline{A} B + A \overline{B}) C_{IN} + A B (\overline{C_{IN}} + C_{IN}) \\
&= (A \oplus B) C_{IN} + A B
\end{align*}
\]
Figure 6.5 shows the logic circuit derived from these equations. If you refer back to the half adder circuit in Figure 6.2, you will see that the full adder can be constructed from two half adders and an OR gate, as shown in Figure 6.7.
EXAMPLE 6.18
Evaluate the Boolean expression for $\Sigma$ and $C_{OUT}$ of the full adder in Figure 6.8 for the following input values. What is the binary value of the outputs in each case?

a. $A = 0$, $B = 0$, $C_{IN} = 1$

b. $A = 1$, $B = 0$, $C_{IN} = 0$

c. $A = 1$, $B = 0$, $C_{IN} = 1$

d. $A = 1$, $B = 1$, $C_{IN} = 0$

SOLUTION

The output of a full adder for any set of inputs is simply given by $C_{OUT} \Sigma = A + B + C_{IN}$. For each of the stated sets of inputs:

a. $C_{OUT} \Sigma = A + B + C_{IN} = 0 + 0 + 1 = 01$

b. $C_{OUT} \Sigma = A + B + C_{IN} = 1 + 0 + 0 = 01$

c. $C_{OUT} \Sigma = A + B + C_{IN} = 1 + 0 + 1 = 10$

d. $C_{OUT} \Sigma = A + B + C_{IN} = 1 + 1 + 0 = 10$

We can verify each of these sums algebraically by plugging the specified inputs into the full adder Boolean equations:

$C_{OUT} = (A \oplus B) \cdot C_{IN} + A \cdot B$
$
\Sigma = (A \oplus B) \oplus C_{IN}$

a. $C_{OUT} = (0 \oplus 0) \cdot 1 + 0 \cdot 0$
$= 0 \cdot 1 + 0$
$= 0 + 0 = 0$

$\Sigma = (0 \oplus 0) \oplus 1$
$= 0 \oplus 1 = 1$

(Binary equivalent: $C_{OUT} \Sigma = 01$)

b. $C_{OUT} = (1 \oplus 0) \cdot 0 + 1 \cdot 0$
$= 1 \cdot 0 + 0$
$= 0 + 0 = 0$

$\Sigma = (1 \oplus 0) \oplus 0$
$= 1 \oplus 0 = 1$

(Binary equivalent: $C_{OUT} \Sigma = 01$)

c. $C_{OUT} = (1 \oplus 0) \cdot 1 + 1 \cdot 0$
$= 1 \cdot 1 + 0$
$= 1 + 0 = 1$

$\Sigma = (1 \oplus 0) \oplus 1$
$= 1 \oplus 1 = 0$

(Binary equivalent: $C_{OUT} \Sigma = 10$)
d. \[ C_{OUT} = (1 \oplus 1) \cdot 0 + 1 \cdot 1 \]
   \[ = 0 \cdot 0 + 1 \]
   \[ = 0 + 1 = 1 \]
\[ \Sigma = (1 \oplus 1) \oplus 0 \]
   \[ = 0 \oplus 0 = 0 \]  \hspace{1cm} \text{(Binary equivalent: } C_{OUT} \Sigma = 10)\]

In each case, the binary equivalent is the same as the number of HIGH inputs, regardless of which inputs they are.

**EXAMPLE 6.19**

Combine a half adder and a full adder to make a circuit that will add two 2-bit numbers. Check that the circuit will work by adding the following numbers and writing the binary equivalents of the inputs and outputs:

a. \( A_2 A_1 = 01, B_2 B_1 = 01 \)

b. \( A_2 A_1 = 11, B_2 B_1 = 10 \)

**SOLUTION**

The 2-bit adder is shown in Figure 6.9. The half adder combines \( A_1 \) and \( B_1 \); \( A_2, B_2, \) and \( C_1 \) are added in the full adder. The carry output, \( C_1 \), of the half adder is connected to the carry input of the full adder. (A half adder can be used only in the LSB of a multiple-bit addition.)

**FIGURE 6.9**

Example 6.19
2-Bit Adder

**Sums:**

a. \( 01 + 01 = 010 \)

\[ A_1 = 1, B_1 = 1 \hspace{1cm} C_1 = 1, \Sigma_1 = 0 \]

\[ A_2 = 0, B_2 = 0, C_1 = 1 \hspace{1cm} C_2 = 0, \Sigma_2 = 1 \]

(Binary equivalent: \( A_2 A_1 + B_2 B_1 = C_2 \Sigma_2 \Sigma_1 = 010 \))

b. \( 11 + 10 = 101 \)

\[ A_1 = 1, B_1 = 0 \hspace{1cm} C_1 = 0, \Sigma_1 = 1 \]

\[ A_2 = 1, B_2 = 1, C_1 = 0 \hspace{1cm} C_2 = 1, \Sigma_2 = 0 \]

(Binary equivalent: \( A_2 A_1 + B_2 B_1 = C_2 \Sigma_2 \Sigma_1 = 101 \))
Parallel Binary Adder/Subtractor

**KEY TERMS**

**Parallel binary adder** A circuit, consisting of \( n \) full adders, that will add two \( n \)-bit binary numbers. The output consists of \( n \) sum bits and a carry bit.

**Ripple carry** A method of passing carry bits from one stage of a parallel adder to the next by connecting \( C_{OUT} \) of one full adder to \( C_{IN} \) of the following stage.

**Cascade** To connect an output of one device to an input of another, often for the purpose of expanding the number of bits available for a particular function.

As Example 6.19 implies, a binary adder can be expanded to any number of bits by using a full adder for each bit addition and connecting their carry inputs and outputs in cascade. Figure 6.10 shows four full adders connected as a 4-bit parallel binary adder.

The first stage (LSB) can be either a full adder with its carry input forced to logic 0 or a half adder, since there is no previous stage to provide a carry. The addition is done one bit at a time, with the carry from each adder propagating to the next stage.

![Figure 6.10](http://www.electronictech.com)

**EXAMPLE 6.20**

Verify the summing operation of the circuit in Figure 6.10 by calculating the output for the following sets of inputs:

a. \( A_4 A_3 A_2 A_1 = 0101, B_4 B_3 B_2 B_1 = 1001 \)

b. \( A_4 A_3 A_2 A_1 = 1111, B_4 B_3 B_2 B_1 = 0001 \)

**SOLUTION** At each stage, \( A + B + C_{IN} = C_{OUT} \Sigma \).

a. \( 0101 + 1001 = 1110 \) 
\( (5_{10} + 9_{10} = 14_{10}) \)
\( A_1 = 1, B_1 = 1, C_0 = 0; C_1 = 1, \Sigma_1 = 0 \)
\( A_2 = 0, B_2 = 0, C_1 = 1; C_2 = 0, \Sigma_2 = 1 \)
\( A_3 = 1, B_3 = 0, C_2 = 0; C_3 = 0, \Sigma_3 = 1 \)
\( A_4 = 0, B_4 = 1, C_3 = 0; C_4 = 0, \Sigma_4 = 1 \)

(Binary equivalent: \( C_4 \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 = 01110 \))
b. \(1111 + 0001 = 10000\)  
\((15_{10} + 1_{10} = 16_{10})\)

\[
\begin{align*}
A_1 &= 1, B_1 = 1, C_0 = 0; C_1 = 1, \Sigma_1 = 0 \\
A_2 &= 1, B_2 = 0, C_1 = 1; C_2 = 1, \Sigma_2 = 0 \\
A_3 &= 1, B_3 = 0, C_2 = 1; C_3 = 1, \Sigma_3 = 0 \\
A_4 &= 1, B_4 = 0, C_3 = 1; C_4 = 1, \Sigma_4 = 0 \\
\end{align*}
\]
(Binary equivalent: \(C_4 \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 = 10000\))

The internal carries in the parallel binary adder in Figure 6.10 are achieved by a system called ripple carry. The carry output of one full adder cascades directly to the carry input of the next. Every time a carry bit changes, it “ripples” through some or all of the following stages. A sum is not complete until the carry from another stage has arrived. The equivalent circuit of a 4-bit ripple carry is shown in Figure 6.11.

A potential problem with this design is that the adder circuitry does not switch instantaneously. A carry propagating through a ripple adder adds delays to the summation time and, more importantly, can introduce unwanted intermediate states.

Examine the sum \((1111 + 0001 = 10000)\). For a parallel adder having a ripple carry, the output goes through the following series of changes as the carry bit propagates through the circuit:

\[
\begin{align*}
C_4 \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 &= 01111 \\
01110 \\
01100 \\
01000 \\
10000 \\
\end{align*}
\]

If the output of the full adder is being used to drive another circuit, these unwanted intermediate states may cause erroneous operation of the load circuit.

**Fast Carry**

**KEY TERM**

**Fast carry (or look-ahead carry)** A gate network that generates a carry bit directly from all incoming operand bits, independent of the operation of each full adder stage.

An alternative carry circuit is called **fast carry** or **look-ahead carry**. The idea behind fast carry is that the circuit will examine all the \(A\) and \(B\) bits simultaneously and produce an output carry that uses fewer levels of gating than a ripple carry circuit. Also, since there is
a carry bit gate network for each internal stage, the propagation delay is the same for each full adder, regardless of the input operands.

The algebraic relation between operand bits and fast carry output is presented below, without proof. It can be developed from the fast carry circuit of Figure 6.12 by tracing the logic of the gates in the circuit.

\[
C_4 = A_4 B_4 + A_3 B_3 (A_4 + B_4) + A_2 B_2 (A_4 + B_4)(A_3 + B_3) \\
+ A_1 B_1 (A_4 + B_4)(A_3 + B_3)(A_2 + B_2) \\
+ C_0 (A_4 + B_4)(A_3 + B_3)(A_2 + B_2)(A_1 + B_1)
\]
We can make some intuitive sense of the above expression by examining it a term at a time. The first term says if the MSBs of both operands are 1, there will be a carry (e.g., 1000 + 1000 = 1000; carry generated).

The second term says if both second bits are 1 AND at least one MSB is 1, there will be a carry (e.g., 0100 + 1100 = 10000, or 1100 + 1100 = 11000; carry generated in either case). This pattern can be followed logically through all the terms.

The internal carry bits are generated by similar circuits that drive the carry input of each full adder stage in the parallel adder. In general, we can generate each internal carry by expanding the following expression:

\[ C_n = A_n B_n + C_{n-1} (A_n + B_n) \]

The algebraic expressions for the remaining carry bits are:

\[ C_1 = A_1 B_1 + C_0 (A_1 + B_1) \]
\[ C_2 = A_2 B_2 + A_1 B_1 (A_2 + B_2) + C_0 (A_2 + B_2)(A_1 + B_1) \]
\[ C_3 = A_3 B_3 + A_2 B_2 (A_3 + B_3) + A_1 B_1 (A_3 + B_3)(A_2 + B_2) + C_0 (A_3 + B_3)(A_2 + B_2)(A_1 + B_1) \]

**SECTION 6.6A REVIEW PROBLEM**

6.9 Refer to the logic diagrams for the ripple carry and fast carry circuits (Figures 6.11 and 6.12). How many gates must a carry bit propagate through in each device if the effect of the carry input ripples through to the \( \Sigma_4 \) bit? (See Figure 6.32 on page 273 and Figure 6.33 on page 273.)

**Using VHDL Components to Implement a Parallel Adder**

**KEY TERMS**

- **Hierarchy** A group of design entities associated in a series of levels or layers in which complete designs form portions of another, more general design entity. The more general design is considered to be the higher level of the hierarchy.
- **Component** A complete VHDL design entity that can be used as a part of a higher-level file in a hierarchical design.
- **Port** An input or output of a VHDL design entity or component.
- **Component declaration statement** A statement that defines the input and output port names of a component used in a VHDL design entity.
- **Instantiate** To use an instance of a component.
- **Component instantiation statement** A statement that maps port names of a VHDL component to the port names, internal signals, or variables of a higher-level VHDL design entity.

VHDL designs can be created using a hierarchy of design entities. Certain functions, such as full adders, decoders, and so on, can be created once and used in many designs or multiple times in a single design.

We can create a parallel adder in VHDL by using multiple instances of a full adder component in the top-level file of a VHDL design hierarchy. Figure 6.13 shows a graphical illustration of this concept. Each full adder shown is an instance of a component written in VHDL, as shown in the following.

```vhdl
-- full_add.vhd
-- Full adder: adds two bits, a and b, plus input carry
-- to yield sum bit and output carry.
```
ENTITY full_add IS
PORT (
    a, b, c_in : IN BIT;
    c_out, sum : OUT BIT);
END full_add;

ARCHITECTURE adder OF full_add IS
BEGIN
    c_out <= ((a xor b) and c_in) or (a and b);
    sum <= (a xor b) xor c_in;
END adder;

FIGURE 6.13
4-bit Parallel Adder with Ripple Carry

We can create the same design as in Figure 6.13 using VHDL only. To make this hierarchical design we require:

1. A separate component file for a full adder (full_add.vhd), saved in a folder where the compiler can find it (i.e., on a library path)
2. A component declaration statement in the top-level file of the design hierarchy
3. A component instantiation statement for each instance of the full adder component

The general form of a design entity using components is:

ENTITY entity_name IS
    PORT ( input and output definitions);
END entity_name;

ARCHITECTURE arch_name OF entity_name IS
    component declaration(s);
    signal declaration(s);
BEGIN

Component instantiation(s);
Other statements;
END arch_name;

The VHDL file for a 4-bit parallel adder using full adder components is shown next.

-- add4par.vhd
-- 4-bit parallel adder, using 4 instances
-- of the component full_add

ENTITY add4par IS
   PORT(
       c0 : IN BIT;
       a, b : IN BIT_VECTOR (4 downto 1);
       c4 : OUT BIT;
       sum : OUT BIT_VECTOR (4 downto 1));
END add4par;

ARCHITECTURE adder OF add4par IS

   --- Component declaration
   COMPONENT full_add
       PORT (  
           a, b, c_in : IN BIT;
           c_out, sum : OUT BIT);
   END COMPONENT;

   --- Define a signal for internal carry bits
   SIGNAL c : BIT_VECTOR (3 downto 1);

BEGIN

   --- Four Component Instantiation Statements
   adder1: full_add
       PORT MAP (  
           a => a(1),
           b => b(1),
           c_in => c0,
           c_out => c(1),
           sum => sum (1));

   adder2: full_add
       PORT MAP (  
           a => a(2),
           b => b(2),
           c_in => c(1),
           c_out => c(2),
           sum => sum (2));

   adder3: full_add
       PORT MAP (  
           a => a(3),
           b => b(3),
           c_in => c(2),
           c_out => c(3),
           sum => sum (3));

   adder4: full_add
       PORT MAP (  
           a => a(4),
           b => b(4),
           c_in => c(3),
           c_out => c4,
           sum => sum (4));

END adder;
The component declaration statement defines the ports of the component with the same names as in the `full_add.vhd`. Note that the form of the component declaration statement is almost the same as that of the component’s entity declaration. In effect, we are re-defining the component entity in the top-level file of the design hierarchy.

The component instantiation statement is of the following form:

```vhdl
_instance_name: _component_name
GENERIC MAP (_parameter_name => _parameter_value ,
               _parameter_name => _parameter_value)
PORT MAP (_component_port => _connect_port,
           _component_port => _connect_port);
```

In the generic map, a generalized parameter name can be mapped to a specific value when the component is instantiated. For example, a parameter name can be given a value that specifies the number of component output bits. We will not use this feature in our present examples.

In the port map, component ports are the names of the ports used in the component file and connect ports are the names of the ports, variables, or signals used in the higher-level design entity. For example, the component ports of the full adder component are `a`, `b`, `c in`, `c out`, and `sum`. The connect ports for the instance `adder1` are `a(1)`, `b(1)`, `c0`, `c(1)`, and `sum1`. The ripple carry from `adder1` to `adder2` is achieved by mapping the port `c in` of `adder2` to `c(1)`, which is also mapped to the port `c out` of `adder1`.

We can write the component instantiation statements more efficiently if we decide to use all ports of the component in the order they are defined. In this case, we can simply list the connect ports in the port map in the correct order, as follows:

```vhdl
adder1: full_add PORT MAP (a(1), b(1), c0, c(1), sum(1));
adder2: full_add PORT MAP (a(2), b(2), c(1), c(2), sum(2));
adder3: full_add PORT MAP (a(3), b(3), c(2), c(3), sum(3));
adder4: full_add PORT MAP (a(4), b(4), c(3), c4, sum(4));
```

If we only wish to use some of the component ports or use them in a different order than the order in which they were originally defined, we must use the previous form of port map (i.e., `a => a(1)`, etc.).

**GENERATE Statements**

**KEY TERM**

**GENERATE statement** A VHDL construct that is used to create repetitive portions of hardware.

The four component instantiation statements shown previously can be written in a more general form:

```vhdl
adder(i): full_add PORT MAP (a(i), b(i), c(i-1), c(i), sum(i));
```

A statement that can be written in this indexed form can be implemented using a `GENERATE statement`, which has the form:

```vhdl
label:
FOR index IN range GENERATE
    statements;
END GENERATE;
```

The VHDL code that follows shows how to use the statement to create a 4-bit adder.

```vhdl
-- add4gen.vhd
-- 4-bit parallel adder, using a generate statement
-- and components
```
ENTITY add4gen IS
  PORT (
    c0 : IN BIT;
    a, b : IN BIT_VECTOR (4 downto 1);
    c4 : OUT BIT;
    sum : OUT BIT_VECTOR (4 downto 1));
END add4gen;

ARCHITECTURE adder OF add4gen IS
  -- Component declaration
  COMPONENT full_add
    PORT (a, b, c_in : IN BIT;
          c_out, sum : OUT BIT);
  END COMPONENT;

  -- Define a signal for internal carry bits
  SIGNAL c : BIT_VECTOR (4 downto 0);

BEGIN
  c(0) <= c0;
  adders:
    FOR i IN 1 to 4 GENERATE
      adder: full_add PORT MAP (a(i),b(i),c(i-1),c(i),sum(i));
    END GENERATE;

  c4 <= c(4);
END adder;

The GENERATE statement will create hardware that corresponds to the range of the
index variable, i. In this case i goes from 1 to 4, so the statement instantiates four instances
of the full adder. Since we have an input carry, an output carry and three internal carries,
we must use a 5-bit signal (BIT_VECTOR (4 downto 0)) if we are to include all carry
bits in indexed form. The input carry, c0, defined in the entity declaration, is assigned to the
vector element c(0). Similarly, the output, c4, is assigned the value of the element c(4).

It is easy to expand the adder width by changing the range of the FOR GENERATE
statement. For example, to make an 8-bit adder, we change the vectors to have a width of
eight bits. The required VHDL code, shown next, requires the same number of lines of
code as the 4-bit adder.

-- add8gen.vhd
-- 8-bit parallel adder, using a generate statement
-- and components

ENTITY add8gen IS
  PORT (c0 : IN BIT;
        a, b : IN BIT_VECTOR (8 downto 1);
        c8 : OUT BIT;
        sum : OUT BIT_VECTOR (8 downto 1));
END add8gen;

ARCHITECTURE adder OF add8gen IS
  -- Component declaration
  COMPONENT full_add
    PORT (a, b, c_in : IN BIT;
          c_out, sum : OUT BIT);
  END COMPONENT;

  -- Define a signal for internal carry bits
  SIGNAL c : BIT_VECTOR (8 downto 0);

BEGIN
  c(0) <= c0;
  adders:
    FOR i IN 1 to 8 GENERATE
      adder: full_add PORT MAP (a(i),b(i),c(i-1),c(i),sum(i));
    END GENERATE;

  c8 <= c(8);
END adder;
--- Define a signal for internal carry bits
SIGNAL c : BIT_VECTOR (8 downto 0);
BEGIN
  c(0) <= c0;
  adders:
  FOR i IN 1 to 8 GENERATE
    adder: full_add PORT MAP (a(i), b(i), c(i-1), c(i),
    sum(i));
  END GENERATE;
  c8 <= c(8);
END adder;

2’s Complement Subtractor
Recall the technique for subtracting binary numbers in 2’s complement notation. For example, to find the difference 0101 \( \text{11002} \) 0011 by 2’s complement subtraction:

1. Find the 2’s complement of 0011:

\[
\begin{align*}
0011 & \quad (1’s \ complement) \\
1100 & \quad (2’s \ complement)
\end{align*}
\]

2. Add the 2’s complement of the subtrahend to the minuend:

\[
\begin{align*}
0101 & \quad (+5) \\
+ 1101 & \quad (-3) \\
\_1 & \quad 0010 & \quad (+2)
\end{align*}
\]

We can easily build a circuit to perform 2’s complement subtraction, using a parallel binary adder and an inverter for each bit of one of the operands. The circuit shown in Figure 6.14 performs the operation \((A - B)\).

![Figure 6.14](image)

2’s Complement Subtractor

The four inverters generate the 1’s complement of \(B\). The parallel adder generates the 2’s complement by adding the carry bit (held at logic 1) to the 1’s complement at the \(B\) inputs. Algebraically, this is expressed as:

\[
A - B = A + (-B) = A + \overline{B} + 1
\]

where \(\overline{B}\) is the 1’s complement of \(B\), and \((\overline{B} + 1)\) is the 2’s complement of \(B\).
EXAMPLE 6.21 Verify the operation of the 2’s complement subtractor in Figure 6.14 by subtracting:

a. $1001 - 0011$ (unsigned)
b. $0100 - 0111$ (signed)

SOLUTION Let $\bar{B}$ be the 1’s complement of $B$.

a. Inverter inputs ($B$): 0011
   Inverter outputs ($\bar{B}$): 1100
   Sum ($A + \bar{B} + 1$): 1001 (+9)
                         1100 (+3) + 1
                         1 0110 (+6)
   (Discard carry)

b. Inverter inputs ($B$): 0111
   Inverter outputs ($\bar{B}$): 1000
   Sum ($A + \bar{B} + 1$): 0100 (+4)
                         1000 (+7) + 1
   Negative result: 1101 (-3)
   1’s complement of 1101: 0010 + 1
   2’s complement of 1101: 0011 (+3)

Parallel Binary Adder/Subtractor

Figure 6.15 shows a parallel binary adder configured as a programmable adder/subtractor. The Exclusive OR gates work as programmable inverters to pass $B$ to the parallel adder in either true or complement form, as shown in Figure 6.16.

FIGURE 6.15
2’s Complement Adder/Subtractor
The ADD/SUB input is tied to the XOR inverter/buffers and to the carry input of the parallel adder. When ADD/SUB = 1, B is complemented and the 1 from the carry input is added to the complement sum. The effect is to subtract \((A - B)\). When ADD/SUB = 0, the \(B\) inputs are presented to the adder in true form and the carry input is 0. This produces an output equivalent to \((A + B)\).

This circuit can add or subtract 4-bit signed or unsigned binary numbers.

**EXAMPLE 6.22**

Write a VHDL file to implement the 4-bit adder/subtractor shown in Figure 6.15. Also create a simulation file to test a representative selection of addition and subtraction operations.

**SOLUTION**

The VHDL file is as follows:

```
-- addsub4g.vhd
ENTITY addsub4g IS
  PORT (  
    sub : IN BIT;  
    a, b : IN BIT_VECTOR (4 downto 1);  
    c4 : OUT BIT;  
    sum : OUT BIT_VECTOR (4 downto 1));
END addsub4g;
ARCHITECTURE adder OF addsub4g IS
  -- Component declaration
  COMPONENT full_add
    PORT (  
      a, b, c_in : IN BIT;  
      c_out, sum : OUT BIT);  
  END COMPONENT;
  -- Define a signal for internal carry bits
  SIGNAL c : BIT_VECTOR (4 downto 0);  
  SIGNAL b_comp : BIT_VECTOR (4 downto 1);  
BEGIN
  -- add/subtract select to carry input (sub=1 for subtract)
  c(0) <= sub;
  adders:
  FOR i IN 1 to 4 GENERATE
    -- invert b for subtract (b(i) xor 1),
    -- do not invert for add (b(i) xor 0)
    b_comp(i) <= b(i) xor sub;
    adder: full_add PORT MAP (a(i), b_comp(i), c(i-1), c(i),
        sum(i));
  END GENERATE;
  c4 <= c(4);
END adder;
```
The VHDL code for the adder/subtractor is the same as that for the 4-bit adder created using a GENERATE statement, except that there is an input to select the add or subtract function. This input (sub) is tied to c(0) and to a set of XOR functions that invert b for subtraction. Input b is transferred through the XOR functions without inversion for the add function.

Figure 6.17 shows the simulation for the adder/subtractor. Table 6.8 shows the operations included in the simulation in both hexadecimal and binary form. Note that the sums are interpreted as unsigned operations and the differences are interpreted as signed operations. Any sum or difference can be interpreted either way, but this will sometimes result in a sign bit overflow. (e.g., the sums 8 + 8 = 10 and F + 1 = 10 both indicate an overflow if they are interpreted as signed additions.)

### Table 6.8 Add/Subtract Results

<table>
<thead>
<tr>
<th>Hexadecimal Sum/Difference</th>
<th>Binary Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 + 1 = 8</td>
<td>0111 + 0001 = 0 1000 (Unsigned)</td>
</tr>
<tr>
<td>8 + 8 = 10</td>
<td>1000 + 1000 = 1 0000 (Unsigned)</td>
</tr>
<tr>
<td>A + 1 = B</td>
<td>1010 + 0001 = 0 1011 (Unsigned)</td>
</tr>
<tr>
<td>F + 0 = F</td>
<td>1111 + 0000 = 0 1111 (Unsigned)</td>
</tr>
<tr>
<td>F + 1 = 10</td>
<td>1111 + 0001 = 1 0000 (Unsigned)</td>
</tr>
<tr>
<td>0 − 1 = F</td>
<td>0000 − 0001 = 1111 (Signed: −1)</td>
</tr>
<tr>
<td>0 − 8 = 8</td>
<td>0000 − 1000 = 1000 (Signed: −8)</td>
</tr>
<tr>
<td>0 − A = 6</td>
<td>0000 − 1010 = 0110 (Signed: 0 − (−6) = +6)</td>
</tr>
<tr>
<td>0 − F = 1</td>
<td>0000 − 1111 = 0001 (Signed: 0 − (−1) = +1)</td>
</tr>
</tbody>
</table>

Note that the simulation in Figure 6.17 shows some intermediate states on the sum waveform in between steady state values. Examine the transition from the sum F + 0 = F to the sum F + 1 = 10 by using the Zoom In function in the Simulator window. Briefly explain how the intermediate states arise in this transition.

**SOLUTION** Figures 6.18 and 6.19 show the transition from F + 0 = F to F + 1 = 10. The transition on the sum waveform is from F to E to 0 or in binary from 1111 to 1110 to
0000. This transition is the result of a change from 0 to 1 on the b1 input of the adder/subtractor.

Figure 6.18 shows the interval from F to E (the time difference between the vertical line marking 36 ns and the arrow cursor, shown in the box labeled Interval) as 7.4 ns. This is the delay from b1 to sum1.

Figure 6.19 shows the interval from F to 0 on the sum waveform, given as 12.6 ns. This interval represents the time required for sum2, sum3, and sum4 to change after a change on b1.

Overflow Detection

We will examine two methods for detecting overflow in a binary adder/subtractor: one that requires access to the sign bits of the operands and result and another that requires access to the internal carry bits of the circuit.

Recall from Example 6.12 the condition for detecting a sign bit overflow in a sum of two binary numbers.

\[
\text{If the sign bits of both operands are the same and the sign bit of the sum is different from the operand sign bits, an overflow has occurred.}
\]

This implies that overflow is not possible if the sign bits of the operands are different from each other. This is true because the sum of two opposite-sign numbers will always be smaller in magnitude than the larger of the two operands.

Here are two examples:

1. \((+15) + (-7) = (+8)\); +8 has a smaller magnitude than +15.
2. \((-13) + (+9) = (-4)\); -4 has a smaller magnitude than -13.

No carry into the sign bit will be generated in either case.

An 8-bit parallel binary adder will add two signed binary numbers as follows:

\[
\begin{align*}
S_A & = A_7 A_6 A_5 A_4 A_3 A_2 A_1 \\
S_B & = B_7 B_6 B_5 B_4 B_3 B_2 B_1 \\
S & = \Sigma_7 \Sigma_6 \Sigma_5 \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1
\end{align*}
\]

\[(S_A = \text{Sign bit of } A) \quad (S_B = \text{Sign bit of } B) \quad (S = \text{Sign bit of sum})\]

From our condition for overflow detection, we can make a truth table for an overflow variable, \(V\), in terms of \(S_A\), \(S_B\), and \(S\). Let us specify that \(V = 1\) when there is an overflow condition. This condition occurs when \(S_A = S_B \neq S\). Table 6.9 shows the truth table for the overflow detector function.

<table>
<thead>
<tr>
<th>(S_A)</th>
<th>(S_B)</th>
<th>(S)</th>
<th>(V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
The SOP Boolean expression for the overflow detector is:

\[ V = S_A S_B \overline{S_C} + \overline{S_A} S_B S_C \]

Figure 6.20 shows a logic circuit that will detect a sign bit overflow in a parallel binary adder. The inputs \( S_A, S_B, \) and \( S_C \) are the MSBs of the adder \( A \) and \( B \) inputs and \( \Sigma \) outputs, respectively.

**EXAMPLE 6.24**

Combine two instances of the 4-bit counter shown in Figure 6.15 and other logic to make an 8-bit adder/subtractor that includes a circuit to detect sign bit overflow.

**SOLUTION** Figure 6.21 represents the 8-bit adder/subtractor with an overflow detector of the type shown in Figure 6.20.
A second method of overflow detection generates an overflow indication by examining the carry bits into and out of the MSB of a 2’s complement adder/subtractor.

Consider the following 8-bit 2’s complement sums. We will use our previous knowledge of overflow to see whether overflow occurs and then compare the carry bits into and out of the MSB.

a. $\text{80H} + \text{80H}$
   - Carry into MSB = 0
   - Carry out of MSB = 1
   - (Sign bit overflow; $V = 1$)

b. $\text{7FH} + \text{01H}$
   - Carry into MSB = 1
   - Carry out of MSB = 0
   - (Sign bit overflow; $V = 1$)

c. $\text{7FH} + \text{80H}$
   - Carry into MSB = 0
   - Carry out of MSB = 0
   - (No sign bit overflow; $V = 0$)

d. $\text{7FH} + \text{C0H}$
   - Carry into MSB = 1
   - Carry out of MSB = 1
   - (No sign bit overflow; $V = 0$)

The above examples suggest that a 2’s complement sum has overflowed if there is a carry into or out of the MSB, but not both. For an 8-bit adder/subtractor, we can write the Boolean equation for this condition as $V = C_8 \oplus C_7$. More generally, for an $n$-bit adder/subtractor, $V = C_n \oplus C_{n-1}$.

Figure 6.22 shows a circuit that can implement the overflow detection from the carry into and out of the MSB of an 8-bit adder.

![Figure 6.22](image-url)
SECTION 6.6B REVIEW PROBLEM

6.10 What is the permissible range of values of a sum or difference, \( x \), in a 12-bit parallel binary adder if it is written as:

a. A signed binary number?

b. An unsigned binary number?

6.7 BCD Adders

(This section may be omitted without loss of continuity.)

**KEY TERM**

**BCD adder** A parallel adder whose output is in groups of 4 bits, each group representing a BCD digit.

It is sometimes convenient to have the output of an adder circuit available as a BCD number, particularly if the result is to be displayed numerically. The problem is that most parallel adders have binary outputs, and 6 of the 16 possible 4-bit binary sums—1010 to 1111—are not within the range of the BCD code.

BCD numbers range from 0000 to 1001, or 0 to 9 in decimal. The unsigned binary sum of any two BCD numbers plus an input carry can range from 00000 \((= 0000 + 0000 + 0)\) to 10011 \((= 1001 + 1001 + 1 = 19_{10})\).

For any sum up to 1001, the BCD and binary values are the same. Any sum greater than 1001 must be modified, since it requires a second BCD digit. For example, the binary value of \(19_{10}\) is 10011. The BCD value of \(19_{10}\) is \(0001\ 1001_{BCD}\). (The most significant digit of a sum of two BCD digits and a carry will never be larger than 1, since the largest such sum is \(19_{10}\).)

Table 6.10 shows the complete list of possible binary sums of two BCD digits \((A\) and \(B)\) and a carry \((C)\), their decimal equivalents, and their corrected BCD values. The MSD of the BCD sum is shown only as a carry bit, with leading zeros suppressed.

**Table 6.10** Binary Sums of Two BCD Digits and a Carry Bit

<table>
<thead>
<tr>
<th>Binary Sum ((A + B + C))</th>
<th>Decimal</th>
<th>Corrected BCD ((\text{Carry + BCD}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000</td>
<td>0</td>
<td>0 + 0000</td>
</tr>
<tr>
<td>00001</td>
<td>1</td>
<td>0 + 0001</td>
</tr>
<tr>
<td>00010</td>
<td>2</td>
<td>0 + 0010</td>
</tr>
<tr>
<td>00011</td>
<td>3</td>
<td>0 + 0011</td>
</tr>
<tr>
<td>00100</td>
<td>4</td>
<td>0 + 0100</td>
</tr>
<tr>
<td>00101</td>
<td>5</td>
<td>0 + 0101</td>
</tr>
<tr>
<td>00110</td>
<td>6</td>
<td>0 + 0110</td>
</tr>
<tr>
<td>00111</td>
<td>7</td>
<td>0 + 0111</td>
</tr>
<tr>
<td>01000</td>
<td>8</td>
<td>0 + 1000</td>
</tr>
<tr>
<td>01001</td>
<td>9</td>
<td>0 + 1001</td>
</tr>
<tr>
<td>01010</td>
<td>10</td>
<td>1 + 0000</td>
</tr>
<tr>
<td>01011</td>
<td>11</td>
<td>1 + 0001</td>
</tr>
<tr>
<td>01100</td>
<td>12</td>
<td>1 + 0010</td>
</tr>
<tr>
<td>01101</td>
<td>13</td>
<td>1 + 0011</td>
</tr>
<tr>
<td>01110</td>
<td>14</td>
<td>1 + 0100</td>
</tr>
<tr>
<td>01111</td>
<td>15</td>
<td>1 + 0101</td>
</tr>
<tr>
<td>10000</td>
<td>16</td>
<td>1 + 0110</td>
</tr>
<tr>
<td>10001</td>
<td>17</td>
<td>1 + 0111</td>
</tr>
<tr>
<td>10010</td>
<td>18</td>
<td>1 + 1000</td>
</tr>
<tr>
<td>10011</td>
<td>19</td>
<td>1 + 1001</td>
</tr>
</tbody>
</table>
Figure 6.23 shows how we can add two BCD digits and get a corrected output. The BCD adder circuit consists of a standard 4-bit parallel adder to get the binary sum and a code converter to translate it into BCD.

The Binary-to-BCD code converter operates on the binary inputs as follows:

1. A carry output is generated if the binary sum is in the range 01010 ≤ sum ≤ 10011 (BCD equivalent: 1 0000 ≤ sum ≤ 1 1001).
2. If the binary sum is less than 01001, the output is the same as the input.
3. If the sum is in the range 01010 ≤ sum ≤ 10011, the four LSBs of the input must be corrected to a BCD value. This can be done by adding 0110 to the four LSBs of the input and discarding any resulting carry. We add 0110₂ (6₁₀) because we must account for six unused codes.

Let’s look at how each of these requirements can be implemented by a digital circuit.

**Carry Output**

The carry output will be automatically 0 for any uncorrected sum from 00000 to 01001 and automatically 1 for any sum from 10000 to 10011. Thus, if the binary adder’s carry output, which we will call \( C_4 \), is 1, the BCD adder’s carry output, \( C_4 \), will also be 1.

Any sum falling between these ranges, that is, between 01010 and 01111, must have its MSB modified. This modifying condition is a function, designated \( C_4'' \), of the binary adder’s sum outputs when its carry output is 0. This function can be simplified by a Karnaugh map, as shown in Figure 6.24, resulting in the following Boolean expression.

\[
C_4'' = \Sigma_4' \Sigma_3' + \Sigma_4' \Sigma_2'
\]

The BCD carry output \( C_4 \) is given by:

\[
C_4 = C_4' + C_4''
  = C_4' + \Sigma_4' \Sigma_3' + \Sigma_4' \Sigma_2'
\]

The BCD carry circuit is shown in Figure 6.25.
Sum Correction

The four LSBs of the binary adder output need to be corrected if the sum is 01010 or greater and need not be corrected if the binary sum is 01001 or less. This condition is indicated by the BCD carry. Let us designate the binary sum outputs as \( \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 \) and the BCD sum outputs as \( \Sigma_4' \Sigma_3' \Sigma_2' \Sigma_1' \) and the BCD sum outputs as \( \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 \):

- If \( C_4 = 0 \), \( \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 = \Sigma_4' \Sigma_3' \Sigma_2' \Sigma_1' + 0000 \);
- If \( C_4 = 1 \), \( \Sigma_4 \Sigma_3 \Sigma_2 \Sigma_1 = \Sigma_4' \Sigma_3' \Sigma_2' \Sigma_1' + 0110 \).

Figure 6.26 shows a BCD adder, complete with a binary adder, BCD carry, and sum correction. A second parallel adder is used for sum correction. The \( B \) inputs are the uncorrected binary sum inputs. The \( A \) inputs are either 0000 or 0110, depending on the value of the BCD carry.
EXAMPLE 6.25

Write a VHDL file for a BCD adder, using two parallel adder components, such as in the logic diagram in Figure 6.26.

SOLUTION

```vhdl
-- bcd_add.vhd
-- BCD adder, using 2 instances of the component add4par

ENTITY bcd_add IS
  PORT (
    c0 : IN BIT;
    a, b : IN BIT_VECTOR (4 downto 1);
    c4 : OUT BIT;
    sum : OUT BIT_VECTOR (4 downto 1));
END bcd_add;

ARCHITECTURE adder OF bcd_add IS
  -- Component declaration
  COMPONENT add4par
    PORT (
      c0 : IN BIT;
      a, b : IN BIT_VECTOR (4 downto 1);
      c4 : OUT BIT;
      sum : OUT BIT_VECTOR (4 downto 1));
  END COMPONENT;
  SIGNAL c4_bin : BIT;
  SIGNAL sum_bin : BIT_VECTOR (4 downto 1);
  SIGNAL a_bcd : BIT_VECTOR (4 downto 1);
  SIGNAL b_bcd : BIT_VECTOR (4 downto 1);
  SIGNAL c0_bcd : BIT;
  BEGIN
    -- Instantiate 4-bit adder (binary sum)
    add_bin: add4par
      PORT MAP ( c0 => c0,
                 a => a,
                 b => b,
                 c4 => c4_bin,
                 sum => sum_bin);
    -- Instantiate 4-bit adder (binary-BCD converter)
    converter: add4par
      PORT MAP ( c0 => c0_bcd,
                 a => a_bcd,
                 b => b_bcd,
                 sum => sum);
    -- Connect components
    c0_bcd <= "0";
    b_bcd <= sum_bin;
    a_bcd(4) <= "0";
    a_bcd(3) <= c4_bin or (sum_bin(4) and sum_bin(3))
                 or (sum_bin(4) and sum_bin(2));
    a_bcd(2) <= c4_bin or (sum_bin(4) and sum_bin(3))
                 or (sum_bin(4) and sum_bin(2));
    a_bcd(1) <= "0";
    c4 <= c4_bin or (sum_bin(4) and sum_bin(3))
            or (sum_bin(4) and sum_bin(2));
  END adder;
```

Multiple-Digit BCD Adders

Several BCD adders can be cascaded to add multidigit BCD numbers. Figure 6.27 shows a 4½-digit BCD adder. The carry output of the most significant digit is considered to be a half-digit since it can only be 0 or 1. The output range of the 4½-digit BCD adder is 00000 to 19999.

![Diagram of 4½-Digit BCD Adder](image)

BCD adders are cascaded by connecting the code converter carry output of one stage to the binary adder carry input of the next most significant stage. Each BCD output digit represents a decade, designated as the units, tens, hundreds, thousands, and ten thousands digits.

### SECTION 6.7 REVIEW PROBLEM

6.11 What is the maximum BCD sum of two 3-digit numbers with no carry input? How many digits are required to display this result on a numerical output?

### 6.8 Carry Generation in MAX+PLUS II

**KEY TERMS**

- **Speed grade** A specification that indicates the internal delay time that can be expected of a CPLD.
- **Expander buffer** A MAX+PLUS II primitive that supplies an inverted product term for general use within a CPLD.

The VHDL adder circuits implemented in Section 6.6 were all defined using a ripple carry format. Is it necessary to design a fast carry circuit when compiling one of these adder designs in MAX+PLUS II? Probably not.

Recall that the design strategy behind the fast carry circuit was to flatten the gate network, that is, to replace a long network (many gates for the carry bit to pass through) with a wide one (fewer levels of gating). Also recall that any combinational logic function can be implemented as a sum-of-products (SOP) network, which inherently is a very flat network form.
The internal circuit of a MAX7000S CPLD is a programmable SOP network. In order to program such a device, the MAX+PLUS II compiler must analyze the design entity, break it into product terms and reassemble it as an SOP network. (This is an oversimplification. Sometimes SOP outputs are fed back into the circuit to be reused by other parts of the circuit, thus lengthening the logic path.)

MAX+PLUS II allows us to choose a style of logic synthesis that balances circuit speed and chip area occupied by the programmed circuit. The styles can be user-defined or we can use one of three predefined synthesis styles called Normal, Fast, and WYSIWYG (What You See Is What You Get). Each one of these styles is optimized for speed, area, or a compromise. The Normal and Fast styles disassemble the design entity and reassemble it after optimizing the logic according to the style rules. The WYSIWYG style allows us (rather than the compiler) to largely define the logic synthesis without altering our design format by very much.

To choose a synthesis style, select Global Project Logic Synthesis, from the MAX+PLUS II Assign menu, as shown in Figure 6.28. A drop-down menu in the resulting dialog box, shown in Figure 6.29, allows us to select one of the three Altera-defined synthesis styles.

NOTE
To use the WYSIWYG style, you must also check the box that says Multi-Level Synthesis for MAX 5000/7000 Devices.

We can calculate the circuit delays for an adder by running the compiled design through the MAX+PLUS II Timing Analyzer. Figure 6.30 shows an example of such an analysis for the parallel adder add4par.vhd with a Normal synthesis style and an EPM7128SLC84-7 as the selected device.
The values in the Destination columns are the delays from logic level changes on the inputs specified by the Source rows. For example, a change on input a1 reaches the output sum1 in 7.5 ns and sum4 in 12.5 ns. Most of the entries in the c4 column have two values (7.5 ns and 11.5 ns), indicating that the delay to output carry is the same from all input bits (i.e., a fast carry). The actual delay to c4 will depend on the logic level change that takes place on the source line and thus which logic path is taken. The delay time of 7.5 ns is about the lowest value possible in the EPM7128SLC84-7 device. (The “/H1102” tells us that the chip has a speed grade of minus 7, meaning an internal delay of about 7 ns.)

Figure 6.31 shows the timing analysis of the same adder with a WYSIWYG synthesis style. (The synthesis style is the only design change.) In this analysis, the delay from an input bit to c4 varies from 7.5 ns (from a4 or b4) to as much as 16.5 ns (for a1, b1, a2, or b2). Since the lower-order bits result in a longer delay to the carry bit, we can infer that the compiler has not synthesized a fast carry circuit.
We can examine the actual equations from the MAX+PLUS II report file to confirm our assessment. The synthesized equations for \( c4 \) are given below.

**WYSIWYG Synthesis:**

---

--- Node name is `'c4' = [full_add:adder4] :12`
--- Equation name is `'c4'`, type is output
\[
c4 = \text{LCELL}(_{\text{EQ001}} \text{ $ GND});
\]
\[
_{\text{EQ001}} = \text{a3} \& \text{b3} \& \text{b4}
# \text{b4} \& \text{LC113} \& \text{LC114}
# \text{a3} \& \text{a4} \& \text{b3}
# \text{a4} \& \text{LC113} \& \text{LC114}
# \text{a4} \& \text{b4};
\]

--- Node name is `[full_add:adder2]:12`
--- Equation name is `'LC113'`, type is buried
\[
_{\text{LC113}} = \text{LCELL}(_{\text{EQ008}} \text{ $ GND});
\]
\[
_{\text{EQ008}} = \text{a2} \& \text{b2}
# \text{c0} \& _{\text{X007}}
# \text{b2} \& \text{c0} \& _{\text{X007}}
# \text{a1} \& \text{b1} \& _{\text{X002}};
\]
\[
_{\text{X007}} = \text{EXP}(!\text{a1} \& !\text{b1});
_{\text{X002}} = \text{EXP}(!\text{a2} \& !\text{b2});
\]

--- Node name is `[full_add:adder3]:9`
--- Equation name is `'LC114'`, type is buried
\[
_{\text{LC114}} = \text{LCELL}(\text{b3} \text{ $ a3});
\]

**Normal synthesis:**

--- Node name is `'c4`
--- Equation name is `'c4'`, location is LC123, type is output.
\[
c4 = \text{LCELL}(\text{EQ001} \text{ $ VCC});
\]
\[
_{\text{EQ001}} = !\text{a1} \& _{\text{X001}} \& _{\text{X002}} \& _{\text{X003}} \& _{\text{X004}}
# !\text{b1} \& !\text{c0} \& _{\text{X001}} \& _{\text{X003}} \& _{\text{X004}}
# !\text{a2} \& !\text{b2} \& _{\text{X003}} \& _{\text{X004}}
# !\text{a3} \& !\text{b3} \& _{\text{X004}}
# !\text{a4} \& !\text{b4};
\]
\[
_{\text{X001}} = \text{EXP}(\text{a2} \& \text{b2});
_{\text{X002}} = \text{EXP}(\text{b1} \& \text{c0});
_{\text{X003}} = \text{EXP}(\text{a3} \& \text{b3});
_{\text{X004}} = \text{EXP}(\text{a4} \& \text{b4});
\]

The function EXP(signal) is for a MAX+PLUS II primitive called an **expander buffer**, which represents a **shared logic expander** in the CPLD. There will be more detail about this type of buffer in Chapter 8, but for now, just be aware that this type of buffer supplies inverted product terms for general use within the CPLD.

The Normal synthesis mode generates a sum-of-products equation that uses a number of expanders, but only one logic cell (i.e., one SOP output), indicated as LC123. The WYSIWYG synthesis uses an unnumbered output logic cell, which in turn uses two other logic cell outputs (LC113 and LC114) as inputs. Thus, in the Normal synthesis mode, the input signals propagate through one logic cell, and in the WYSIWYG mode, the input signals go through two layers of logic cells, increasing the path length, and thus the delay.

What can we conclude? If MAX+PLUS II is allowed to synthesize a design for a full adder in the defined Normal style, it will optimize the design equations to produce as flat a network as possible. Thus we do not need to explicitly design an adder circuit to have a fast carry function.
1. Addition combines an addend \((x)\) and an augend \((y)\) to get a sum \((z = x + y)\).

2. Binary addition is based on four sums:
   \[
   \begin{align*}
   0 + 0 &= 0 \\
   0 + 1 &= 1 \\
   1 + 1 &= 10 \\
   1 + 1 + 1 &= 11
   \end{align*}
   \]

3. A sum of two bits generates a sum bit and a carry bit. (For the first two sums above, the carry bit is 0; the last two sums have a carry of 1. The last sum includes a carry from a lower-order bit.)

4. Subtraction combines a minuend \((x)\) and a subtrahend \((y)\) to get a difference \((z = x - y)\).

5. Binary subtraction is based on the following four differences:
   \[
   \begin{align*}
   0 - 0 &= 0 \\
   1 - 0 &= 1 \\
   1 - 1 &= 0 \\
   10 - 1 &= 1
   \end{align*}
   \]

6. If the subtrahend bit is larger than the minuend bit, as in the fourth difference above, a 1 must be borrowed from the next higher-order bit.

7. Binary addition or subtraction can be unsigned, where the magnitudes of the operands and result are presumed to be positive, or signed, where the operands and result can be positive or negative. The sign is indicated by a sign bit.

8. The sign bit (usually MSB) of a binary number indicates that the number is positive if it is 0 and negative if it is 1.

9. Signed binary numbers can be written in true-magnitude, 1’s complement, or 2’s complement form. True magnitude has the same binary value for positive and negative numbers, with only the sign bit changed. A 1’s complement negative number is generated by inverting all bits of the positive number of the same magnitude. A 2’s complement negative number is generated by adding 1 to the equivalent 1’s complement number. Positive numbers are the same in all three forms.

10. 1’s complement or 2’s complement binary numbers are used in signed addition or subtraction. Subtraction is performed by adding a negative number in complement form to another number in complement form (i.e., \(x - y = x + (-y)\)). This technique does not work for true-magnitude form.

11. A negative sum or difference in 2’s complement subtraction must be converted to a positive form to read its magnitude (i.e., \((-x) = +x\)).

12. A signed binary number, \(x\), with \(n\) bits has a valid range of \(-2^n \leq x \leq +2^n - 1\).

13. A negative number with a power-of-2 magnitude (i.e., \(-2^n\)) is written in 2’s complement form as \(n\) 0s preceded by all 1s to fill the defined size of the number (e.g., in 8-bit 2’s complement form, \(-128 = 10000000\) (1 followed by seven 0s; \(128 = 2^7\)); in 8-bit 2’s complement form, \(-8 = 1111100\) (all 1s, followed by three 0s; \(8 = 2^3\)).

14. If a sum or difference falls outside the permissible range of magnitudes for a 2’s complement number, it generates an overflow into the sign bit of the number. The result is that the sum of two positive numbers appears to be negative (e.g., \(01111111 + 00000010 = 10000001\); \(127 + 2 = 129\)) or the sum of two negative numbers appears to be positive (e.g., \(11111111 + 10000000 = 01111111\), where the carry beyond the 8th place is discarded: \(-1 + (-128) = -129\).

15. When adding two hexadecimal digits, any digit sum greater than 15 (F) can be converted to a hexadecimal value by subtracting 16 and carrying a 1 to the next digit position.

16. Hexadecimal numbers can be subtracted conventionally or by a complement method. To get the 16’s complement of a number, obtain the 15’s complement by subtracting the number from all Fs and adding 1 to the result.

17. Binary numbers can be used in nonpositional codes to represent numbers or alphanumeric characters.

18. Binary coded decimal (BCD) codes represent decimal numbers as a series of 4-bit groups of numbers. Natural BCD or 8421 code does this as a positionally weighted code for each digit (e.g., 158 = 0001 0101 1000 (NBCD)). Other codes, such as Excess-3, are not positionally weighted.

19. Gray code is a binary code that has a difference of one bit between adjacent codes. It can be generated by a set of XOR functions or by recognizing the symmetry inherent in the code. In any Gray code sequence, the MSB is 0 for the first half of the sequence and 1 for the second half. The remaining bits are symmetrical about the halfway point of the sequence.

20. ASCII code represents alphanumeric characters and computer control codes as a 7-bit group of binary numbers. Alpha characters are listed in uppercase in columns 4 and 5 of the ASCII table. Lowercase alpha characters are in columns 6 and 7. Numeric characters are in column 3.

21. A half adder combines two bits to generate a sum and a carry. It can be represented by the following truth table:

<table>
<thead>
<tr>
<th>(A)</th>
<th>(B)</th>
<th>(C_{OUT})</th>
<th>(\Sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

22. From the half adder truth table, we can derive two equations:
   \[
   \begin{align*}
   C_{OUT} &= AB \\
   \Sigma &= A \oplus B
   \end{align*}
   \]

23. A full adder can accept an input carry from a lower-order adder and combine the input carry with two operands to generate
a sum and output carry. Its operation can be summarized in the following truth table:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$C_{IN}$</th>
<th>$C_{OUT}$</th>
<th>$\Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

24. The following Boolean equations for a full adder can be derived from the truth table and Boolean algebra:

\[
C_{OUT} = (A \oplus B) \cdot C_{IN} + AB
\]

\[
\Sigma = (A \oplus B) \oplus C_{IN}
\]

25. Two half adders can be combined to make a full adder. Operands $A$ and $B$ go to the first half adder. The sum output of the first half adder and the carry input go to the inputs of the second half adder. The carry outputs of both half adders are combined in an OR gate.

26. Multiple full adders can be cascaded to make a parallel binary adder. Operands $A_1$ and $B_1$ are applied to the first full adder. Carry bit $C_1$ is grounded. $A_2$ and $B_2$ go to the second adder stage, and so on. The carry output of one stage is cascaded to the carry input of the following stage. This connection is called ripple carry.

27. Ripple carry has the disadvantage of increasing the time required to generate an output result as more stages are added. Fast carry, or look-ahead carry, examines all adder inputs simultaneously and generates each internal and output carry with a separate circuit. This makes the carry circuit wider, but flatter, thus reducing the delay time of the circuit.

28. A parallel adder can be implemented in VHDL by creating a design entity for a full adder, then using multiple instances of the full adder as components in the parallel adder.

29. To use a component in a VHDL design hierarchy, we require a parallel adder can be implemented in VHDL by creating a component instantiation statement for every instance of the design entity that uses the component.

30. The general form of a design entity using components is:

```
ENTITY entity_name IS
    PORT ( input and output definitions);
END entity_name;
ARCHITECTURE arch_name OF entity_name IS
    component declaration (s);
    signal declaration(s);
BEGIN
    Component instantiation(s);
    Other statements;
END arch_name;
```

31. The port map of a component maps the port names defined in a component to the port, signal, or variable names defined in the design entity that uses the component.

32. If all ports of a component are to be used in the same order as in the component definition in the original component design entity, the port map can simply contain the user names in the same order. For example:

```
adder1: full_add PORT MAP (a(1), b(1), c0, c(1), sum(1));
```

33. If only a portion of the component ports are to be used or they are not used in the same sequence as they are declared, the port map must be more explicit. For example:

```
adder1: full_add
    PORT MAP ( b => b(1),
               a => a(1),
               c_in => c0,
               sum => sum(1)),
      c_out => c(1);
```

34. A GENERATE statement can be used to instantiate multiple instances of a component. The GENERATE statement has the form:

```
label:
    FOR index IN range GENERATE
        statements;
    END GENERATE;
```

35. MAXPLUS II will synthesize an adder to minimize carry delays without much intervention.

36. A parallel binary adder can be made into a 2’s complement subtractor by inverting one set of inputs and tying the input carry to a logic HIGH.

37. A parallel binary adder can be made into a 2’s complement adder/subtractor by using a set of XOR gates as programmable inverters and connecting the XOR control line to the carry input of the adder.

38. One method of detecting a sign bit overflow in a 2’s complement adder/subtractor is to compare the sign bits of the operands to the sign bit of the result. If the sign bits of the operands are the same as each other, but different from the sign bit of the result, there has been an overflow. The Boolean equation for this detector is given by $V = S_A \cdot S_B + S_A \cdot S_B \cdot S_0$.

39. Another method of overflow detection compares the carry out of the MSB of the adder/subtractor to the carry into the MSB. An overflow occurs if there is a carry out of or into the MSB, but not both. The Boolean equation for this detector is given by $V = C_n \oplus C_{n-1}$, for an $n$-bit adder/subtractor.

40. A BCD adder adds two binary coded decimal (BCD) digits and generates a BCD digit and a carry bit.

41. Since BCD is a 4-bit code, BCD addition can be done with a 4-bit binary adder and a code converter. The code converter can be synthesized from another 4-bit binary adder and a circuit to generate a carry.
Glossary

1's complement A form of signed binary notation in which negative numbers are created by complementing all bits of a number, including the sign bit.

10's complement A way of writing decimal numbers where a negative number is generated by adding 1 to its 9's complement.

2's complement A form of signed binary notation in which negative numbers are created by adding 1 to the 1's complement form of the number.

8421 Code (or NBCD; natural binary coded decimal) A BCD code that represents each digit of a decimal number by its 4-bit true binary value.

9's complement A way of writing decimal numbers where a number is made negative by subtracting each of its digits from 9 (e.g., \(-726 = 999 - 726 = 273\) in 9's complement).

Addend The number in an addition operation that is added to another.

Alphanumeric code A code used to represent letters of the alphabet and numerical characters.


Augend The number in an addition operation to which another number is added.

BCD Binary coded decimal. A code that represents each digit of a decimal number by a 4-bit binary value.

BCD adder A parallel adder whose output is in groups of 4 bits, each group representing a BCD digit.

Borrow A digit brought back from a more significant position when the subtrahend digit is larger than the minuend digit.

Carry A digit which is “carried over” to the next most significant position when the sum of two single digits is too large to be expressed as a single digit.

Carry bit A bit that holds the value of a carry (0 or 1) resulting from the sum of two binary numbers.

Cascade To connect an output of one device to a input of another, often for the purpose of expanding the number of bits available for a particular function.

Case shift Changing letters from capitals (UPPERCASE) to small letters (lowercase) or vice versa.

Component A complete VHDL design entity that can be used as a part of a higher-level file in a hierarchical design.

Component declaration statement A statement that defines the input and output port names of a component used in a VHDL design entity.

Component instantiation statement A statement that maps port names of a VHDL component to the port names, internal signals, or variables of a higher-level VHDL design entity.

Difference The result of a subtraction operation.

End-around carry An operation in 1’s complement subtraction where the carry bit resulting from a sum of two 1’s complement numbers is added to that sum.

Excess-3 code A BCD code that represents each digit of a decimal number by a binary number derived by adding 3 to its 4-bit true binary value. Excess-3 code has the advantage of being “self-complementing.”

Expander buffer A MAX+PLUS II primitive that supplies an inverted product term for general use within a CPLD.

Fast carry (or look-ahead carry) A gate network which generates a carry bit directly from all incoming operand bits, independent of the operation of each full adder stage.

Full adder A circuit that will add a carry bit from another full or half adder and two operand bits to produce a sum bit and a carry bit.

GENERATE statement A VHDL construct that is used to create repetitive portions of hardware.

Gray code A binary code which progresses such that only one bit changes between two successive codes.

Half adder A circuit that will add two bits and produce a sum bit and a carry bit.

Hierarchy A group of design entities associated in a series of levels or layers in which complete designs form portions of another, more general design entity. The more general design is considered to be the higher level of the hierarchy.

Instantiate To use an instance of a component.

Magnitude bits The part of a signed binary number that tell us how large the number is (i.e., its magnitude).

Minuend The number in a subtraction operation from which another number is subtracted.

Operand A number upon which an arithmetic function operates (e.g., in the expression \(x + y = z\), \(x\) and \(y\) are the operands).

Overflow An erroneous carry into the sign bit of a signed binary number which results from a sum larger than can be represented by the number of magnitude bits.

Parallel binary adder A circuit, consisting of \(n\) full adders, which will add two \(n\)-bit binary numbers. The output consists of \(n\) sum bits and a carry bit.

Port An input or output of a VHDL design entity or component.

Ripple carry A method of passing carry bits from one stage of a parallel adder to the next by connecting \(C_{\text{OUT}}\) of one full adder to \(C_{\text{IN}}\) of the following stage.

Self-complementing A code that automatically generates a negative-equivalent (e.g., 9's complement for a decimal code) when all its bits are inverted.

Sign bit A bit, usually the MSB, that indicates whether a signed binary number is positive or negative.

Signed binary arithmetic Arithmetic operations performed using signed binary numbers.

Signed binary number A binary number of fixed length whose sign is represented by one bit, usually the most significant bit, and whose magnitude is represented by the remaining bits.
Speed grade A specification that indicates the internal delay time that can be expected of a CPLD.

Subtrahend The number in a subtraction operation that is subtracted from another number.

Sum The result of an addition operation.

Sum bit (single-bit addition) The least significant bit of the sum of two 1-bit binary numbers.

True-magnitude form A form of signed binary number whose magnitude is represented in true binary.

Unsigned binary arithmetic Arithmetic operations performed using unsigned binary numbers.

Unsigned binary number A binary number whose sign is not indicated by a sign bit. A positive sign is assumed unless explicitly stated otherwise.

PROBLEMS

Section 6.1 Digital Arithmetic

6.1 Add the following unsigned binary numbers.
   - a. 10101 + 1010
   - b. 10101 + 1011
   - c. 1111 + 1111
   - d. 11100 + 1110
   - e. 11001 + 10011
   - f. 111011 + 101001

6.2 Subtract the following unsigned binary numbers.
   - a. 1100 - 100
   - b. 10001 - 1001
   - c. 10110 - 1100
   - d. 1110 - 1010
   - e. 10110 - 1001
   - f. 10001 - 1111
   - g. 100010 - 10111
   - h. 1100011 - 100111

Section 6.2 Representing Signed Binary Numbers

6.3 Write the following decimal numbers in 8-bit true-magnitude, 1’s complement, and 2’s complement forms.
   - a. -110
   - b. 67
   - c. -54
   - d. -93
   - e. 0
   - f. -1
   - g. 127
   - h. -127

Section 6.3 Signed Binary Arithmetic

6.4 Perform the following arithmetic operations in the true-magnitude (addition only), 1’s complement, and 2’s complement systems. Use 8-bit numbers consisting of a sign bit and 7 magnitude bits. (The numbers shown are in the decimal system.) Convert the results back to decimal to prove the correctness of each operation. Also demonstrate that the idea of adding a negative number to perform subtraction is not valid for the true-magnitude form.
   - a. 37 + 25
   - b. 85 + 40
   - c. 95 - 63
   - d. 63 - 95
   - e. -23 - 50
   - f. 120 - 73
   - g. 73 - 120

6.5 What are the largest positive and negative numbers, expressed in 2’s complement notation, that can be represented by an 8-bit signed binary number?

6.6 Perform the following signed binary operations, using 2’s complement notation where required. State whether or not sign bit overflow occurs. Give the signed decimal equivalent values of the sums in which overflow does not occur.
   - a. 01101 + 00110
   - b. 01101 + 10110
   - c. 01110 - 01001
   - d. 11110 + 00010
   - e. 11110 - 00010

6.7 Without doing any binary complement arithmetic, indicate which of the following operations will result in 2’s complement overflow. (Assume 8-bit representation consisting of a sign bit and 7 magnitude bits.) Explain the reasons for each choice.
   - a. -109 + 36
   - b. 109 + 36
   - c. 65 + 72
   - d. -110 - 29
   - e. 117 + 11
   - f. 117 - 11

6.8 Explain how you can know, by examining sign or magnitude bits of the numbers involved, when overflow has occurred in 2’s complement addition or subtraction.

Section 6.4 Hexadecimal Arithmetic

6.9 Add the following hexadecimal numbers.
   - a. 27H + 16H
   - b. 87H + 99H
   - c. A55H + C5H
   - d. C7FH + 380H
   - e. 1FFFH + A80H
Section 6.5 Numeric and Alphanumeric Codes

6.11 Convert the following decimal numbers to true binary, 8421 BCD code, and Excess-3 code.
   a. 709
   b. 1889
   c. 2395
   d. 1259
   e. 3972
   f. 7730

6.12 Make a table showing the equivalent Gray codes corresponding to the range from 010 to 3110.

6.13 Write your name in ASCII code.

6.14 Encode the following text into ASCII code: “10% off purchases over $50. (Monday only)”

6.15 Decode the following string of ASCII code:
   57 41 52 4E 49 4E 47 21 20 54 68 69 73 20 31 10 20 6F 66 20 6D 65 6D 6F 72 79 2E
   57 41 52 4E 49 4E 47 21 20 54 68 69 73 20 31 10 20 6F 66 20 6D 65 6D 6F 72 79 2E

Section 6.6 Binary Adders and Subtractors

6.16 Write the truth table for a half adder, and from the table derive the Boolean expressions for both Carry (C) (carry output) and Sum (Σ) (sum output) in terms of inputs A and B. Draw the half adder circuit.

6.17 Write the truth table for a full adder, and from the table derive the simplest possible Boolean expressions for COUT and Σ in terms of A, B, and CIN.

6.18 From the equations in Problems 6.16 and 6.17, draw a circuit showing a full adder constructed from two half adders.

6.19 Evaluate the Boolean expression for Σ and COUT of the full adder in Figure 6.7 for the following input values. What is the binary value of the outputs in each case?
   a. A = 0, B = 0, CIN = 0
   b. A = 0, B = 1, CIN = 0
   c. A = 0, B = 1, CIN = 1
   d. A = 1, B = 1, CIN = 1

6.20 Verify the summing operation of the circuit in Figure 6.10, as follows. Determine the output of each full adder based on the inputs shown below. Calculate each sum manually and compare it to the 5-bit output (C4, C3, C2, C1) of the parallel adder circuit.

6.21 Briefly describe the differences in the underlying design strategies of the ripple carry adder and the fast carry adder (i.e., what makes the fast carry faster than the ripple carry?). What is the main limitation for the fast carry circuit?

6.22 Write the general form of the fast carry equation. Use it to generate Boolean expression for C1, C2, and C3 for a fast carry adder.

6.23 The following equation describes the carry output function for a parallel binary adder:

   \[ C_{OUT} = A_4 B_4 + A_3 B_3 (A_4 + B_4) + A_2 B_2 (A_4 + B_4 + A_3 B_3) + A_1 B_1 (A_4 + B_4 + A_3 B_3 + A_2 B_2) + C_{IN} (A_4 + B_4 + A_3 B_3 + A_2 B_2 + A_1 B_1) \]

   Briefly explain how to interpret the third term of this equation.

6.24 Write a VHDL file for an 8-bit parallel adder, using eight instances of a full adder component.

6.25 Create a simulation for the 8-bit adder of Problem 6.24, showing a representative sample of sums. How many different sums would be required to show all possible combinations of inputs?

6.26 Write a VHDL file that creates a 12-bit adder using a GENERATE statement.

6.27 Create a simulation file for the 12-bit adder of Problem 6.26 showing only one transition, as follows. Set input a to 000 from 0 to 500 ns, then 001 from 500 ns to 1 μs. Set input b to FFF from 0 to 1 μs. From the simulation, determine the internal delays from a1 to each of the sum bits. Confirm your observations with the delay matrix from a timing analysis.

6.28 Use MAX+PLUSII to create a Graphic Design File for a 2’s complement subtractor based on a 4-bit parallel binary adder. Explain how the circuit generates the 2’s complement of B for the subtraction A − B.

6.29 Use MAX+PLUSII to create a Graphic Design File for a 2’s complement adder/subtractor based on a 4-bit parallel binary adder. Explain how the circuit is programmed to add or subtract and how it produces the 2’s complement of B for the subtraction A − B.

6.30 Use MAX+PLUSII to draw a circuit that will detect an overflow condition in a 4-bit 2’s complement adder/subtractor. The detector output should go HIGH upon overflow detection. Draw the circuit truth table, explain what all input and output variables are, and show any Boolean equations you need to complete the circuit design.

6.31 Modify the 4-bit adder/subtractor drawn in Figure 6.15 to include an overflow detection circuit.
6.32 Create a simulation for the 4-bit adder/subtractor with overflow detection (Problem 6.31), using the following representative hexadecimal input values: \( F + 1 = 10 \) (carry, but no overflow); \( 7 + 1 = 8 \) (overflow, but no carry); \( 8 + 8 = 10 \) (carry and overflow); \( 0 - 1 = F \) (result = \(-1\)).

6.33 Modify the VHDL file for the 4-bit parallel binary adder/subtractor (addsub4g.vhd) to include an overflow detection circuit. Use two different methods.

6.34 What is the permissible range of values that a sum or difference, \( x \), can have in a 16-bit parallel binary adder if it is written as:
   a. A signed binary number
   b. An unsigned binary number

Section 6.7 BCD Adders

6.35 What is the maximum BCD sum of two 3-digit BCD numbers plus an input carry? How many digits are needed to display the result?

6.36 What is the maximum BCD sum of two 4-digit BCD numbers plus an input carry? How many digits are needed to display the result?

ANSWERS TO SECTION REVIEW PROBLEMS

Section 6.1a
6.1 101000; 6.2 100000.

Section 6.1b
6.3 11; 6.4 1

Section 6.3
6.5 11100000; 6.6 100000.

Section 6.4
6.7a 11701H 6.7b 1281H

Section 6.5
6.8 “True or False: \( 1/4 < 1/2 \)”

Section 6.6a
6.9 Figures 6.32 and 6.33 show the propagation paths for the carry bits.
   Fast carry: 3 gates
   Ripple carry: 8 gates

Section 6.6b
6.10a Signed: \( -2048 \leq x \leq +2047 \) (11 magnitude bits, 1 sign bit)
6.10b Unsigned: \( 0 \leq x \leq +4095 \) (12 magnitude bits, no sign bit; positive implied)

Section 6.7
6.11 Maximum BCD sum = 1001 1001 1001 + 1001 1001 1001 = 1 1001 1001 1000 0BCD = 1998_{10}. This sum requires a 3\( \frac{3}{2} \)-digit numerical display.
FIGURE 6.32
Fast Carry from A4/B4 to C4.

FIGURE 6.33
Ripple Carry from C0 to C4
CHAPTER 7

Introduction to Sequential Logic

OUTLINE

7.1 Latches
7.2 NAND/NOR Latches
7.3 Gated Latches
7.4 Edge-Triggered D Flip-Flops
7.5 Edge-Triggered JK Flip-Flops
7.6 Edge-Triggered T Flip-Flops
7.7 Timing Parameters

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

• Explain the difference between combinational and sequential circuits.
• Define the set and reset functions of an SR latch.
• Draw circuits, function tables, and timing diagrams of NAND and NOR latches.
• Explain the effect of each possible input combination to a NAND and a NOR latch, including set, reset, and no change functions, as well as the ambiguous or forbidden input condition.
• Design circuit applications that employ NAND and NOR latches.
• Describe the use of the ENABLE input of a gated SR or D latch as an enable/inhibit function and as a synchronizing function.
• Outline the problems involved with using a level-sensitive ENABLE input on a gated SR or D latch.
• Explain the concept of edge-triggering and why it is an improvement over level-sensitive enabling.
• Draw circuits, function tables, and timing diagrams of edge-triggered D, JK, and T flip-flops.
• Describe the toggle function of a JK flip-flop and a T flip-flop.
• Describe the operation of the asynchronous preset and clear functions of D, JK, and T flip-flops and be able to draw timing diagrams showing their functions.
• Use MAX+PLUS II to create simple circuits and simulations with D latches and D, JK, and T flip-flops.
• Create simple flip-flop designs using VHDL.

The digital circuits studied to this point have all been combinational circuits, that is, circuits whose outputs are functions only of their present inputs. A particular set of input states will always produce the same output state in a combinational circuit.
This chapter will introduce a new category of digital circuitry: the sequential circuit. The output of a sequential circuit is a function both of the present input conditions and the previous conditions of the inputs and/or outputs. The output depends on the sequence in which the inputs are applied.

We will begin our study of sequential circuits by examining the two most basic sequential circuit elements: the latch and the flip-flop, both of which are part of the general class of circuits called bistable multivibrators. These are similar devices, each being used to store a single bit of information indefinitely. The difference between a latch and a flip-flop is the condition under which the stored bit is allowed to change.

Latches and flip-flops are also used as integral parts of more complex devices, such as programmable logic devices (PLDs), usually when an input or output state must be stored.

### 7.1 Latches

**KEY TERMS**

**Sequential circuit** A digital circuit whose output depends not only on the present combination of inputs, but also on the history of the circuit.

**Latch** A sequential circuit with two inputs called **SET** and **RESET**, which make the latch store a logic 0 (reset) or 1 (set) until actively changed.

**SET**
1. The stored HIGH state of a latch circuit.
2. A latch input that makes the latch store a logic 1.

**RESET**
1. The stored LOW state of a latch circuit.
2. A latch input that makes the latch store a logic 0.

All the circuits we have seen up to this point have been combinational circuits. That is, their present outputs depend only on their present inputs. The output state of a combinational circuit results only from a combination of input logic states.

The other major class of digital circuits is the **sequential circuit**. The present outputs of a sequential circuit depend not only on its present inputs, but also on its past input states.

The simplest sequential circuit is the SR **latch**, whose logic symbol is shown in Figure 7.1a. The latch has two inputs, **SET** (S) and **RESET** (R), and two complementary outputs, **Q** and **\( \bar{Q} \)**. If the latch is operating normally, the outputs are always in opposite logic states.

![SR Latch (Active HIGH Inputs)](image)

**FIGURE 7.1**
SR Latch (Active HIGH Inputs)

The latch operates like a momentary-contact pushbutton with START and STOP functions, shown in Figure 7.2. A momentary-contact switch operates only when it is held down. When released, a spring returns the switch to its rest position.

Suppose the switch in Figure 7.2 is used to control a motor starter. When you push the START button, the motor begins to run. Releasing the START switch does not turn the motor off; that can be done only by pressing the STOP button. If the motor is running,
pressing the START button again has no effect, except continuing to let the motor run. If the motor is not running, pressing the STOP switch has no effect, since the motor is already stopped.

There is a conflict if we press both switches simultaneously. In such a case we are trying to start and stop the motor at the same time. We will come back to this point later.

The latch SET input is like the START button in Figure 7.2. The RESET input is like the STOP button.

**NOTE**

By definition:

A latch is set when \( Q = 1 \) and \( \bar{Q} = 0 \).

A latch is reset when \( Q = 0 \) and \( \bar{Q} = 1 \).

The latch in Figure 7.1 has active-HIGH SET and RESET inputs. To set the latch, make \( R = 0 \) and make \( S = 1 \). This makes \( Q = 1 \) until the latch is actively reset, as shown in the timing diagram in Figure 7.1b. To activate the reset function, make \( S = 0 \) and make \( R = 1 \). The latch is now reset (\( Q = 0 \)) until the set function is next activated.

Combinational circuits produce an output by combining inputs. In sequential circuits, it is more accurate to think in terms of activating functions. In the latch described, \( S \) and \( R \) are not combined by a Boolean function to produce a particular result at the output. Rather, the set function is activated by making \( S = 1 \), and the reset function is activated by making \( R = 1 \), much as we would activate the START or STOP function of a motor starter by pressing the appropriate pushbutton.

The timing diagram in Figure 7.1b shows that the inputs need not remain active after the set or reset functions have been selected. In fact, the \( S \) or \( R \) input must be inactive before the opposite function can be applied, in order to avoid conflict between the two functions.

### EXAMPLE 7.1

Latches can have active-HIGH or active-LOW inputs, but in each case \( Q = 1 \) after the set function is applied and \( Q = 0 \) after reset. For each latch shown in Figure 7.3, complete the timing diagram shown. \( Q \) is initially LOW in both cases. (The state of \( Q \) before the first active SET or RESET is unknown unless specified, since the present state depends on previous history of the circuit.)
**SOLUTION** The $Q$ and $\overline{Q}$ waveforms are shown in Figure 7.3. Note that the outputs respond only to the first set or reset command in a sequence of several pulses.

**EXAMPLE 7.2**

Figure 7.4 shows a latching HOLD circuit for an electronic telephone. When HIGH, the $HOLD$ output allows you to replace the handset without disconnecting a call in progress.

**FIGURE 7.4**
Example 7.2
Latching HOLD Button

The two-position switch is the telephone's hook switch (the switch the handset pushes down when you hang up), shown in the off-hook (in-use) position. The normally closed pushbutton is a momentary-contact switch used as a $HOLD$ button. The circuit is such that the $HOLD$ button does not need to be held down to keep the $HOLD$ active. The latch "remembers" that the switch was pressed, until told to "forget" by the reset function.

Describe the sequence of events that will place a caller on hold and return the call from hold. Also draw timing diagrams showing the waveforms at the $HOLD$ input, hook switch inputs, $S$ input, and $HOLD$ output for one hold-and-return sequence. ($HOLD$ output = 1 means the call is on hold.)

**SOLUTION** To place a call on hold, we must set the latch. We can do so if we press and hold the $HOLD$ switch, then the hook switch. This combines two HIGHs—one from the $HOLD$ switch and one from the on-hook position of the hook switch—into the AND gate, making $S = 1$ and $R = 0$. Note the sequence of events: press $HOLD$, hang up, release $HOLD$. The $S$ input is HIGH only as long as the $HOLD$ button is pressed. The handset can be kept on-hook and the $HOLD$ button released. The latch stays set, as $S = R = 0$ (neither SET nor RESET active) as long as the handset is on-hook.

To restore a call, lift the handset. This places the hook switch into the off-hook position and now $S = 0$ and $R = 1$, which resets the latch and turns off the $HOLD$ condition.

Figure 7.5 shows the timing diagram for the sequence described.

**FIGURE 7.5**
Example 7.2
HOLD Timing Diagram
### SECTION 7.1 REVIEW PROBLEM

7.1 A latch with active-HIGH $S$ and $R$ inputs is initially set. $R$ is pulsed HIGH three times, with $S = 0$. Describe how the latch responds.

### 7.2 NAND/NOR Latches

An SR latch is easy to build with logic gates. Figure 7.6 shows two such circuits, one made from NOR gates and one from NANDs. The NAND gates in the second circuit are drawn in DeMorgan equivalent form.

**FIGURE 7.6**
SR Latch Circuits

The two circuits both have the following three features:

1. OR-shaped gates
2. Logic level inversion between the gate input and output
3. Feedback from the output of one gate to an input of the opposite gate

During our examination of the NAND and NOR latches, we will discover why these features are important.

A significant difference between the NAND and NOR latches is the placement of \textit{SET} and \textit{RESET} inputs with respect to the $Q$ and $\bar{Q}$ outputs. Once we define which output is $Q$ and which is $\bar{Q}$, the locations of the \textit{SET} and \textit{RESET} inputs are automatically defined.

In a NOR latch, the gates have active-HIGH inputs and active-LOW outputs. When the input to the $Q$ gate is HIGH, $\bar{Q} = 0$, since either input HIGH makes the output LOW. Therefore, this input must be the \textit{RESET} input. By default, the other is the \textit{SET} input.

In a NAND latch, the gate inputs are active LOW (in DeMorgan equivalent form) and the outputs are active HIGH. A LOW input on the $Q$ gate makes $\bar{Q} = 1$. This, therefore, is the \textit{SET} input, and the other gate input is \textit{RESET}.

Since the NAND and NOR latch circuits have two binary inputs, there are four possible input states. Table 7.1 summarizes the action of each latch for each input combination. The functions are the same for each circuit, but they are activated by opposite logic levels.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$R$</th>
<th>Action (NOR Latch)</th>
<th>$\bar{S}$</th>
<th>$\bar{R}$</th>
<th>Action (NAND Latch)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Neither \textit{SET} nor \textit{RESET} active; output does not change from previous state</td>
<td>0</td>
<td>0</td>
<td>Both \textit{SET} and \textit{RESET} active; forbidden condition</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>\textit{RESET} input active</td>
<td>0</td>
<td>1</td>
<td>\textit{SET} input active</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>\textit{SET} input active</td>
<td>1</td>
<td>0</td>
<td>\textit{RESET} input active</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Both \textit{SET} and \textit{RESET} active; forbidden condition</td>
<td>1</td>
<td>1</td>
<td>Neither \textit{SET} nor \textit{RESET} active; output does not change from previous state</td>
</tr>
</tbody>
</table>
We will examine the NAND latch circuit for each of the input conditions in Table 7.1. The analysis of a NOR latch is similar and will be left as an exercise.

**NAND Latch Operation**

Figure 7.7 shows a NAND latch in its two possible stable states. In each case the inputs $S$ and $R$ are both HIGH (inactive).

![Figure 7.7 NAND Latch Stable States](image)

**FIGURE 7.7**
NAND Latch Stable States

Figure 7.7a shows the latch in its SET condition ($Q = 1$). The feedback connections from each gate output to the input of the opposite gate keep the latch in a stable condition. The upper gate has a LOW on the “inner” input. Since, for a NAND gate, either input LOW makes the output HIGH, this makes $Q = 1$. This HIGH value is fed to the gate on the other side of the latch. The lower gate has both inputs HIGH, thus keeping its output LOW. The LOW at $Q$ feeds back to the upper gate, forming a closed loop of consistent logic levels. There is no tendency for the outputs to change under these conditions.

Figure 7.7b shows a similar state for the latch in a RESET condition ($Q = 0$). As with the SET state, the stability of the latch depends on the feedback connections. The logic values of the latch gate inputs are the same as before, except that the LOW input is on the lower gate, not the upper gate as in the SET condition.

Figure 7.8 shows a NAND latch as a Graphic Design File created with MAX+PLUS II. The inputs are labeled nS and nR and one output as nQ as we cannot enter input names with bars over them. (BOR2 = “Bubbled OR, 2-inputs”.)

![Figure 7.8 Graphic Design File representation of a NAND Latch](image)

**FIGURE 7.8**
Graphic Design File representation of a NAND Latch.

**NOTE**

The documentation for MAX+PLUS II recommends that you do not create your own latch circuits or similar cross-coupled structures. Rather, you should use primitives such as **LATCH**, or components such as **lpm_latch**, which can be used in **gdf** or **vhd** files. We will use the design in Figure 7.8 only to illustrate the function of a NAND latch and to generate some timing data with the MAX+PLUS II simulator.

In order to make MAX+PLUS II synthesize this circuit as we have drawn it in Figure 7.8, we must select **Global Project Logic Synthesis** from the **Assign Menu** (Figure 7.9).
In the resulting dialog box (Figure 7.10), we must choose the WYSIWYG (What You See Is What You Get) synthesis style and check the box that says **Multi-Level Synthesis for MAX5000/7000 Devices.**

When we compile the graphic file, MAX+PLUS II synthesizes the following equations, which we can read in the project report file:

```
** EQUATIONS **
nR      : INPUT;
nS      : INPUT;

-- Node name is 'nQ' = ':3'
-- Equation name is 'nQ', type is output
nQ      = LCELL( _EQ001 $ GND);
_EQ001 = !nR
  # !Q;

-- Node name is 'Q' = ':2'
-- Equation name is 'Q', type is output
Q       = LCELL( _EQ002 $ GND);
_EQ002 = !nS
  # !nQ;
```

We can rewrite the synthesized latch equations as:

\[ Q = \overline{nS} + nQ \]
\[ nQ = \overline{nR} + Q \]
When we run the MAX+PLUS II Timing Analyzer, we get the delay matrix shown in Figure 7.11. The delays are symmetrical for this circuit. The delay from nS to Q (7.5 ns) is through one gate; from nS to nQ (12.5 ns) is through two gates. These values are the same for the path from nR to nQ (7.5 ns; one gate) and from nR to Q (12.5 ns; two gates). We can see these changes on simulation waveforms for the SET and RESET functions.

**FIGURE 7.11**
NAND Latch Delay Matrix (WYSIWYG Synthesis)

Figures 7.12 and 7.13 show the transition of a NAND latch from the RESET to the SET condition. In Figure 7.12a, the latch is stable in the RESET condition ($Q = 0$) at time $t < 0$ (i.e., before a SET pulse is applied to the latch). At time $t = 0$, the $\bar{S}$ input goes LOW (Figure 7.12b) and 7.5 ns later, the output $Q$ goes HIGH (Figure 7.12c). This applies a HIGH to the lower gate in the latch and at $t = 12.5$ ns (Figure 7.12d), the $\bar{Q}$ output goes LOW, closing the loop. The latch is now in a new stable configuration and the $\bar{S}$ input can go back HIGH, as shown in Figure 7.12e.

**FIGURE 7.12**
RESET-to-SET transition
The waveforms in Figure 7.13 also show this transition. The simulation window has a 2.5 ns grid, so three grid spaces are equivalent to 7.5 ns and five grid spaces to 12.5 ns. The waveforms show Q going HIGH 7.5 ns after nS goes LOW, followed by nQ going LOW at 12.5 ns after nS.

Figures 7.14 and 7.15 show the same thing for the RESET function. The latch is in a stable \textit{SET} condition at time $t < 0$ (Figure 7.14a). Input $R$ goes LOW at $t = 0$ (Figure 7.14b). At time $t = 7.5$ ns, $Q$ goes HIGH, which is transferred to the upper gate in the latch circuit Figure 7.14c). Since both inputs of the upper gate are now HIGH, $Q$ goes LOW at time $t = 12.5$ ns (Figure 7.14d). At this point the latch is stable in the \textit{RESET} condition and the input $R$ can return to the HIGH (inactive) state, as shown in Figure 7.14e. Figure 7.15 shows the simulation waveforms for this transition.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{nand_latch_set_reset_transition.png}
\caption{SET-to-RESET Transition}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{nand_latch_reset_simulation.png}
\caption{NAND latch \textit{RESET} function simulation}
\end{figure}
Note that the latch is not stable in its new condition until the new logic levels have propagated through both gates. Figure 7.16 shows the result of a \textit{RESET} pulse that only lasts for 7.5 ns. This pulse is too short to allow both gates to change states. The outputs both oscillate, since the changing logic levels never “catch up” as they move around the latch. This is due to the fact that both paths (nS-Q-nQ and nR-nQ-Q) are the same length. If one path were slightly longer, the logic level controlled by the longer path would dominate and the latch would stabilize in one state or the other.

**FIGURE 7.16**
NAND latch oscillation due to a \textit{RESET} pulse that is too short

![Image of NAND latch oscillation](image)

\[ \bar{S} = 0 \quad Q = 1 \]
\[ \bar{R} = 0 \quad Q = 1 \]

**FIGURE 7.17**
NAND Latch Forbidden State

**FIGURE 7.18**
SET goes HIGH before \textit{RESET}

**FIGURE 7.19**
RESET goes HIGH before \textit{SET}

Figure 7.17 shows a NAND latch with \( \bar{S} = \bar{R} = 0 \). This implies that both \textit{SET} and \textit{RESET} functions are active. Since a NAND gate requires at least one input LOW to make the output HIGH, both outputs respond by going HIGH. This condition is not unstable in and of itself, but instability can result when the inputs change.

There are three possible results when the outputs go back to the HIGH state.

1. The \textit{SET} input goes HIGH before the \textit{RESET} input. In this case the latch resets, as \textit{RESET} is the last input active. This is shown in the simulation in Figure 7.18.

2. The \textit{RESET} input goes HIGH before \textit{SET}. In this case, the latch sets, as shown in Figure 7.19.
3. The SET and RESET inputs go HIGH at the same time. This is an unstable case. Figure 7.20 shows how the latch will oscillate under this condition. When the inputs $S$ and $R$ are both LOW (Figure 7.20a), both latch outputs are HIGH. When $S$ and $R$ go HIGH (Figure 7.20b), all gate inputs are HIGH. This makes both outputs LOW (Figure 7.20c). The LOWs transfer across the latch to the opposite gates and, after a delay, make both outputs HIGH (Figure 7.20d). At this point, oscillations will be sustained until the latch is SET or RESET. The simulation waveforms in Figure 7.21 show the oscillatory condition of the latch outputs.

![Figure 7.20](image)

**FIGURE 7.20**
NAND Latch Forbidden State Transition

In practice, the oscillatory condition of Figure 7.21 is unlikely to be sustained for very long. One of the two gates is likely to be slightly faster than the other, which will allow one state or the other to dominate.

![Figure 7.21](image)

**FIGURE 7.21**
SET and RESET go HIGH simultaneously

The operation of the NAND latch can be summarized in a function table, shown in Table 7.2. The notation $Q_{t+1}$ indicates that the column shows the value of $Q$ after the specified input is applied. $Q_t$ indicates the present state of the $Q$ input.* Thus, the entry for the no change state indicates that after the inputs $S = 0, R = 0$ are applied, the next state of the output is the same as its present state.

*Many sources (such as data sheets) use the notation $Q_0$ to refer to the previous state of $Q$. We will use the notation indicated ($Q_t$ for present state and $Q_{t+1}$ for next state) so as to be able to reserve $Q_0$ for the least significant bit of a circuit requiring multiple $Q$ outputs.
Table 7.3 shows the function table for the NOR latch.

**Practical Synthesis in MAX+PLUS II**

The NAND latch shown previously (Figure 7.8) was synthesized in MAX+PLUS II, using the WYSIWYG synthesis style. We did this so as to be able to use the MAX+PLUS II simulation tool to get waveforms for a standard NAND latch. However, if we allow MAX+PLUS II to synthesize the latch circuit in the Normal synthesis style, the software will choose a more stable configuration, shown in Figure 7.22.

The equations for the circuit in Figure 7.22 from the MAX+PLUS II report file are given as:

```
** EQUATIONS **
nR       : INPUT;
nS       : INPUT;

-- Node name is 'nQ'
-- Equation name is 'nQ', location is LC117, type is output.
nQ       = LCELL( _EQ001 $ VCC);
_EQ001   =  nR &  Q;

-- Node name is 'Q' = 'Q~ = (~2~1'
-- Equation name is 'Q', location is LC115, type is output.
Q        = LCELL( _EQ002 $ !ns);
_EQ002   =  nR &  nS & Q;
```

We can rewrite the above equations as:

\[
Q = (nR \cdot nS \cdot Q) \oplus \overline{nS} \\
nQ = (nR \cdot Q) \oplus 1 = nR \cdot \overline{Q}
\]

Without going into a detailed analysis, we will just note that the latching occurs through a combination of the XOR gate at Q and the feedback from the Q output to the 3-input AND. The lower AND/XOR structure simply serves to invert the Q output to provide a complementary value at nQ.

This configuration is more stable because both SET and RESET functions go through the same path (the 3-input AND gate). Delay is the same from nS to Q and from nR to Q. In the WYSIWYG version, the path is equal from nS to nQ and from nR to Q, but not from nS to Q and nR to Q. The SET and RESET pulses thus go through different paths in the
WYSIWYG synthesis, resulting in unequal delays from the inputs to the Q output, which can lead to instability.

**Latch as a Switch Debouncer**

Pushbutton or toggle switches are sometimes used to generate pulses for digital circuit inputs, as illustrated in Figure 7.23. However, when a switch is operated and contact is made on a new terminal, the contact, being mechanical, will bounce a few times before settling into the new position. Figure 7.23d shows the effect of contact bounce on the waveform for a pushbutton switch. The contact bounce is shown only on the terminal where contact is being made, not broken.

Contact bounce can be a serious problem, particularly when a switch is used as an input to a digital circuit that responds to individual pulses. If the circuit expects to receive one pulse, but gets several from a bouncy switch, it will behave unpredictably.

A latch can be used as a switch debouncer, as shown in Figure 7.24a. When the pushbutton is in the position shown, the latch is set, since $S = 0$ and $R = 1$. (Recall that the NAND latch inputs are active LOW.) When the pushbutton is pressed, the $R$ contact
bounces a few times, as shown in Figure 7.24b. However, on the first bounce, the latch is reset. Any further bounces are ignored, since the resulting input state is either $S = R = 1$ (no change) or $S = 1, R = 0$ (reset).

Similarly, when the pushbutton is released, the $S$ input bounces a few times, setting the latch on the first bounce. The latch ignores any further bounces, since they either do not change the latch output ($S = R = 1$) or set it again ($S = 0, R = 1$). The resulting waveforms at $Q$ and $\bar{Q}$ are free of contact bounce and can be used reliably as inputs to digital sequential circuits.

**EXAMPLE 7.3**

A NOR latch can be used as a switch debouncer, but not in the same way as a NAND latch. Figure 7.25 shows two NOR latch circuits, only one of which works as a switch debouncer. Draw a timing diagram for each circuit, showing $R$, $S$, $Q$, and $\bar{Q}$, to prove that the circuit in Figure 7.25b eliminates switch contact bounce but the circuit in Figure 7.25a does not.

**FIGURE 7.25**
Example 7.3
NOR Latch Circuits

**SOLUTION**

Figure 7.26 shows the timing diagrams of the two NOR latch circuits. In the circuit in Figure 7.25a, contact bounce causes the latch to oscillate in and out of the forbidden state of the latch ($S = R = 1$). This causes one of the two outputs to bounce for each contact closure. (Use the function table of the NOR latch to examine each part of the timing diagram to see that this is so.)

By making the resistors pull down rather than pull up, as in Figure 7.25b, the latch oscillates in and out of the no change state ($S = R = 0$) as a result of contact bounce. The first...
bounce on the SET terminal sets the latch, and other oscillations are disregarded. The first bounce on the RESET input resets the latch, and further pulses on this input are ignored.

The principle illustrated here is that a closed switch must present the active input level to the latch, since switch bounce is only a problem on contact closure. Thus, a closed switch must make the input of a NOR latch HIGH or the input of a NAND latch LOW to debounce the switch waveform.

The NOR latch is seldom used in practice as a switch debouncer. The pull-down resistors need to be about 500 Ω or less to guarantee a logic LOW at the input of a TTL NOR gate. In such a case, a constant current of about 10 mA flows through the resistor connected to the normally closed portion of the switch. This value is unacceptably high in most circuits, as it draws too much idle current from the power supply. For this reason, the NAND latch, which uses higher-value pull-up resistors (about 1 kΩ or larger) and therefore draws less idle current, is preferred for a switch debouncer.

SECTION 7.2 REVIEW PROBLEM

7.2 Why is the input state \( S = R = 1 \) considered forbidden in the NOR latch? Why is the same state in the NAND latch the no change condition?

7.3 Gated Latches

Gated SR latch

An SR latch whose ability to change states is controlled by an extra input called the ENABLE input.

Steering gates

Logic gates, controlled by the ENABLE input of a gated latch, that steer a SET or RESET pulse to the correct input of an SR latch circuit.

Transparent latch (gated D latch)

A latch whose output follows its data input when its ENABLE input is active.

Gated SR Latch

It is not always desirable to allow a latch to change states at random times. The circuit shown in Figure 7.27, called a gated SR latch, regulates the times when a latch is allowed to change state.

The gated SR latch has two distinct subcircuits. One pair of gates is connected as an SR latch. A second pair, called the steering gates, can be enabled or inhibited by a control signal, called ENABLE, allowing one or the other of these gates to pass a SET or RESET signal to the latch gates.

The ENABLE input can be used in two principal ways: (1) as an ON/OFF signal, and (2) as a synchronizing signal.

Figure 7.27b shows the ENABLE input functioning as an ON/OFF signal. When ENABLE = 1, the circuit acts as an active-HIGH latch. The upper gate converts a HIGH at \( S \) to a LOW at \( S \), setting the latch. The lower gate converts a HIGH at \( R \) to a LOW at \( R \), thus resetting the latch.

When ENABLE = 0, the steering gates are inhibited and do not allow SET or RESET signals to reach the latch gate inputs. In this condition, the latch outputs cannot change.
Figure 7.27c shows the ENABLE input as a synchronizing signal. A periodic pulse waveform is present on the ENABLE line. The S and R inputs are free to change at random, but the latch outputs will change only when the ENABLE input is active. Since the ENABLE pulses are equally spaced in time, changes to the latch output can occur only at fixed intervals. The outputs can change out of synchronization if S or R change when ENABLE is HIGH. We can minimize this possibility by making the ENABLE pulses as short as possible.

Table 7.4 represents the function table for a gated SR latch.

<table>
<thead>
<tr>
<th>EN</th>
<th>S</th>
<th>R</th>
<th>(Q_{t+1})</th>
<th>(\bar{Q}_{t+1})</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>(\bar{Q})</td>
<td>(\bar{Q})</td>
<td>No change</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>Reset</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Set</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Forbidden</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
<td>X</td>
<td>(Q)</td>
<td>(\bar{Q})</td>
<td>Inhibited</td>
</tr>
</tbody>
</table>
Example 7.4

Figure 7.28 shows two gated latches with the same $S$ and $R$ input waveforms but different $ENABLE$ waveforms. $EN_1$ has a 50% duty cycle. $EN_2$ has a duty cycle of 16.67%.

Draw the output waveforms, $Q_1$ and $Q_2$. Describe how the length of the $ENABLE$ pulse affects the output of each latch, assuming that the intent of each circuit is to synchronize the output changes to the beginning of the $ENABLE$ pulse.

SOLUTION

Figure 7.28b shows the completed timing diagram. The longer $ENABLE$ pulse at latch 1 allows the output to switch too soon during pulses 1 and 4. (“Too soon” means before the beginning of the next $ENABLE$ pulse.) In each of these cases, the $S$ and $R$ inputs change while the $ENABLE$ input is HIGH. This premature switching is eliminated in latch 2 because the $S$ and $R$ inputs change after the shorter $ENABLE$ pulse is finished. A shorter pulse gives less chance for synchronization error, since the time for possible output changes is minimized.

Transparent Latch (Gated D Latch)

Figure 7.29 shows the equivalent circuit of a gated D (“data”) latch, or transparent latch. This circuit has two modes. When the $ENABLE$ input is HIGH, the latch is transparent because the output $Q$ goes to the level of the data input, $D$. (We say, “$Q$ follows $D$.”) When the $ENABLE$ input is LOW, the latch stores the data that was present at $D$ when $ENABLE$ was last HIGH. In this way, the latch acts as a simple memory circuit.
The latch in Figure 7.29 is a modification of the gated SR latch, configured so that the $S$ and $R$ inputs are always opposite. Under these conditions, the states $S = R = 0$ (no change) and $S = R = 1$ (forbidden) can never occur. However, the equivalent of the no change state happens when the ENABLE input is LOW, when the latch steering gates are inhibited.

Figure 7.30 shows the operation of the transparent latch in the inhibit (no change), set, and reset states. When the latch is inhibited, the steering gates block any LOW pulses to the latch gates; the latch does not change states, regardless of the logic level at $D$.

If $EN = 1$, $Q$ follows $D$. When $D = 1$, the upper steering gate transmits a LOW to the SET input of the latch and $Q = 1$. When $D = 0$, the lower steering gate transmits a LOW to the RESET input of the output latch and $Q = 0$.

Table 7.5 shows the function table for a transparent latch.

<table>
<thead>
<tr>
<th>$EN$</th>
<th>$D$</th>
<th>$Q_{t+1}$</th>
<th>$\overline{Q}_{t+1}$</th>
<th>Function</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>X</td>
<td>$Q_t$</td>
<td>$\overline{Q}_t$</td>
<td>No Change</td>
<td>Store</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>Reset</td>
<td>Transparent</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Set</td>
<td>Transparent</td>
</tr>
</tbody>
</table>
Implementing D Latches in MAX+PLUS II

A D latch can be implemented in MAX+PLUS II as a primitive in a Graphic Design File or in a VHDL design entity. It can also be created with a behavioral or structural description in a VHDL file.

Figure 7.31 shows a D latch primitive in a MAX+PLUS II Graphic Design File. Figure 7.32 shows a simulation of the latch. From 0 to 500 ns, ENABLE is HIGH and the latch is in the transparent mode (Q follows D). When ENABLE goes LOW, the last value of D (0) is stored until ENABLE goes high again, just before 800 ns. When ENABLE goes LOW again, a new value of D (1) is stored until the end of the simulation.

---

In VHDL, a PROCESS statement is concurrent, but the statements inside the PROCESS are sequential. In other words, anything described by a PROCESS acts like a separate component in a design entity. However, the interior of the component so described acts as a sequential circuit. Since the behavior of a D latch is sequential, its description can be created inside a PROCESS. (You can pull a latch out of a bin of parts and connect it in a circuit, but the inside of the part is sequential.) The following VHDL code describes a D latch.

```vhdl
ENTITY d_lch IS
PORT(
    d, ena : IN BIT;
    q : OUT BIT);
END d_lch;
ARCHITECTURE a OF d_lch IS
BEGIN
    PROCESS (d, ena)
    BEGIN
        IF (ena = '1') THEN
            q <= d;
        END IF;
    END PROCESS;
END a;
```

---
Another method, recommended by the MAX+PLUS II documentation, is to instantiate a LATCH primitive in a VHDL file. The primitive is contained in the `altera` library, in a package called `maxplus2`. The component declaration for this primitive is:

```
COMPONENT LATCH
  PORT (d : IN STD_LOGIC;
        ena : IN STD_LOGIC;
        q : OUT STD_LOGIC);
END COMPONENT;
```

Since the component declaration is in the `maxplus2` package, you do not have to declare it in the file in which you are using it. A VHDL file that uses the latch primitive is listed next. The component declaration uses STD LOGIC types, so we must include the type definitions in the `ieee` library (`std_logic_1164` package).

```
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;
ENTITY lch_prim IS
  PORT(
    d_in, enable : IN   STD_LOGIC;
    q_out         : OUT  STD_LOGIC);
END lch_prim;
ARCHITECTURE a OF lch_prim IS
BEGIN
  -- Instantiate a latch from a MAX+PLUS II primitive
  latch_primitive: latch
  PORT MAP (d => d_in,
            ena => enable,
            q => q_out);
END a;
```

More information about MAX+PLUS II primitives can be found in MAX+PLUS II Help. In the Help menu, select Primitives. By clicking on the name of a particular primitive, you can determine whether it can be instantiated in a VHDL file and what its component declaration is, if available.

### Example 7.5

A system for monitoring automobile traffic is set up at an intersection, with four sensors, placed as shown in Figure 7.33. Each sensor monitors traffic for a particular direction. When a car travels over a sensor, it produces a logic HIGH. The status of the sensor system
is captured for later analysis by a set of D latches, as shown in Figure 7.34. A timing pulse enables the latches once every five seconds and thus stores the system status as a “snap-shot” of the traffic pattern.

Figure 7.35 shows the timing diagram of a typical traffic pattern at the intersection. The $D$ inputs show the cars passing through the intersection in the various lanes. Complete this timing diagram by drawing the $Q$ outputs of the latches.

How should we interpret the $Q$ output waveforms?
SOLUTION  Figure 7.35 shows the completed timing diagram. The ENABLE input synchronizes the random sensor pattern to a 5-second standard interval. A HIGH on any Q output indicates a car over a sensor at the beginning of the interval. For example, at the beginning of the first interval, there is a car in the northbound lane (Q1) and one in the southbound lane (Q2). Similar interpretations can be made for each interval.

Multi-bit Latches in VHDL

**KEY TERMS**

**Library of Parameterized Modules (LPM)**  A standardized set of components for which certain properties can be specified when the component is instantiated.  
**Parameter (in an LPM component)**  A property of a component that can be specified when the component is instantiated.  
**Generic map**  A VHDL construct that maps one or more parameters of a component to a value for that instance of the component.  
**Port map**  A VHDL construct that maps the name of a port in a component to the name of a port, variable, or signal in a design entity that uses the component.

We can easily use VHDL to implement latches with multiple D inputs and Q outputs, but with a common ENABLE line, as in Figure 7.34. Three approaches are:

1. Use a behavioral description, as we did earlier for a single latch (d_lch.vhd). Use STD_LOGIC_VECTOR types for D and Q, rather than STD_LOGIC.
2. Altera recommends using a latch primitive or predefined component, rather than creating your own latch structures. We can use multiple LATCH primitives, instantiated by a GENERATE statement, as we did for multiple instances of a full adder in Chapter 6.
3. Use a latch component from the **Library of Parameterized Modules (LPM)**. These components are specified in the lpm_components package in the lpm library.

   Certain properties of an LPM component, such as the number of inputs or outputs, can be specified when the component is instantiated. These properties are referred to as parameters, and are listed in a generic map. For example, to make the latch output and input four bits wide, we set the parameter called LPM_WIDTH to a value of 4. The various parameters of an LPM component can be found in the LPM Quick Reference on the CD that accompanies this book or in the MAX+PLUS II Help menu under Megafunctions/LPM.

   An input or output of an LPM component is called a port. A port map is used to make a correspondence between the port names in the component declaration and the port names used in the file containing the component. Since LPM components are declared in a separate package, we must refer to the MAX+PLUS II Help or the LPM Quick Reference to determine the port names for a component. LPM components are instantiated the same as any other component.

   The three VHDL files that follow each specify a 4-bit latch with common enable, each using one of the above methods.

**Behavioral Description:**

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
```
ENTITY ltch4bhv IS
PORT(d : IN STD_LOGIC_VECTOR (3 downto 0);
     enable : IN STD_LOGIC;
     q : OUT STD_LOGIC_VECTOR (3 downto 0));
END ltch4bhv;

ARCHITECTURE a OF ltch4bhv IS
BEGIN
  PROCESS (enable, d)
  BEGIN
    IF (enable = '1') THEN
      q <= d;
    END IF;
  END PROCESS;
END a;

4 LATCH Primitives and a GENERATE Statement:

-- ltch4prm.vhd
-- D latch with active-HIGH level-sensitive enable
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;

ENTITY ltch4prm IS
PORT(d_in : IN STD_LOGIC_VECTOR (3 downto 0);
     enable : IN STD_LOGIC;
     q_out : OUT STD_LOGIC_VECTOR (3 downto 0));
END ltch4prm;

ARCHITECTURE a OF ltch4prm IS
BEGIN
  -- Instantiate a latch from a MAX+PLUS II primitive
  latch4:
  FOR i IN 3 downto 0 GENERATE
    latch_primitive: latch
    PORT MAP (d => d_in (i), ena => enable, q => q_out (i));
  END GENERATE;
END a;

LPM Latch:

-- ltch4lpm.vhd
-- 4-BIT D latch with active-HIGH level-sensitive enable
-- Uses a latch component from the Library of Parameterized
-- Modules (LPM)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY ltch4lpm IS
PORT(d_in  : IN STD_LOGIC_VECTOR (3 downto 0);
     enable : IN STD_LOGIC;
     q_out : OUT STD_LOGIC_VECTOR (3 downto 0));
END ltch4lpm;
ARCHITECTURE a OF lch4pm IS
BEGIN
-- Instantiate latch from an LPM component
latch4: lpm_latch
GENERIC MAP (LPM_WIDTH => 4)
PORT MAP (data => d_in,
gate => enable,
q => q_out);
END a;

All three files can be tested with the same simulation, shown in Figure 7.36. The inputs, \( d_{in} \), represent a 4-bit group of signals, as do the outputs, \( q_{out} \). An increasing count, from 5 to C (0101 to 1100) is applied to \( d_{in} \). This count contains both states (0 and 1) for each input bit. For each applied input state, the output bus, \( q_{out} \), does not change until the enable line goes HIGH.

FIGURE 7.36
Simulation of a 4-bit D Latch

SECTION 7.3 REVIEW PROBLEM

7.3 Write the VHDL code for a 16-bit latch with common active-HIGH enable, using MAX+PLUS II latch primitives.

7.4 Edge-Triggered D Flip-Flops

KEY TERMS

- **Edge**  The HIGH-to-LOW (negative edge) or LOW-to-HIGH (positive edge) transition of a pulse waveform.
- **CLOCK**  An enabling input to a sequential circuit that is sensitive to the positive- or negative-going edge of a waveform.
- **Edge-triggered**  Enabled by the positive or negative edge of a digital waveform.
- **Edge-sensitive**  Edge-triggered.
- **Level-sensitive**  Enabled by a logic HIGH or LOW level.
- **Flip-flop**  A sequential circuit based on a latch whose output changes when its \( CLOCK \) input receives an edge.

In Example 7.4, we saw how a shorter pulse width at the \( ENABLE \) input of a gated latch increased the chance of the output being synchronized to the \( ENABLE \) pulse waveform. This is because a shorter \( ENABLE \) pulse gives less chance for the \( SET \) and \( RESET \) inputs to change during the time the latch is enabled.

A logical extension of this idea is to enable the latch for such a small time that the width of the \( ENABLE \) pulse is almost zero. The best approximation we can make to this is to allow changes to the circuit output only when an enabling, or \( CLOCK \), input receives the edge of an input waveform. An edge is the part of a waveform that is in transition from
LOW to HIGH (positive edge) or HIGH to LOW (negative edge), as shown in Figure 7.37. We can say that a device enabled by an edge is edge-triggered or edge-sensitive.

Since the CLOCK input enables a circuit only while in transition, we can refer to it as a “dynamic” input. This is in contrast to the ENABLE input of a gated latch, which is level-sensitive or “static,” and will enable a circuit for the entire time it is at its active level.

Latches vs. Flip-Flops

**KEY TERM**

**Edge detector** A circuit in an edge-triggered flip-flop that converts the active edge of a CLOCK input to an active-level pulse at the internal latch’s SET and RESET inputs.

A gated latch with a clock input is called a flip-flop. Although the distinction is not always understood, we will define a latch as a circuit with a level-sensitive enable (e.g., gated D latch) or no enable (e.g., NAND latch) and a flip-flop as a circuit with an edge-triggered clock (e.g., D flip-flop). A NAND or NOR latch is sometimes called an SR flip-flop. By our definition this is not correct, since neither of these circuits has a clock input. (An SR flip-flop would be like the gated SR latch of Figure 7.27 with a clock instead of an enable input.)

The symbol for the D, or data, flip-flop is shown in Figure 7.38. The D flip-flop has the same behavior as a gated D latch, except that the outputs change only on the positive edge of the clock waveform, as opposed to the HIGH state of the enable input. The triangle on the CLK (clock) input of the flip-flop indicates that the device is edge-triggered.

Table 7.6 shows the function table of a positive edge-triggered D flip-flop.

<table>
<thead>
<tr>
<th>CLK</th>
<th>D</th>
<th>$Q_t$</th>
<th>$Q_{t+1}$</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>Reset</td>
</tr>
<tr>
<td>↑</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Set</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
<td>$Q_t$</td>
<td>$Q_{t+1}$</td>
<td>Inhibited</td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>$Q_t$</td>
<td>$Q_{t+1}$</td>
<td>Inhibited</td>
</tr>
<tr>
<td>↓</td>
<td>X</td>
<td>$Q_t$</td>
<td>$Q_{t+1}$</td>
<td>Inhibited</td>
</tr>
</tbody>
</table>

Figure 7.39 shows the equivalent circuit of a positive edge-triggered D flip-flop. The circuit is the same as the transparent latch of Figure 7.29, except that the enable input (called CLK in the flip-flop) passes through an edge detector, a circuit that converts a positive edge to a brief positive-going pulse. (A negative edge detector converts a negative edge to a positive-going pulse.)
Figure 7.40 shows a circuit that acts as a simplified positive edge detector. Edge detection depends on the fact that a gate output does not switch immediately when its input switches. There is a delay of about 3 to 10 ns from input change to output change, called propagation delay.

When input \( x \), shown in the timing diagram of Figure 7.40, goes from LOW to HIGH, the inverter output, \( \bar{x} \), goes from HIGH to LOW after a short delay. This delay causes both \( x \) and \( \bar{x} \) to be HIGH for a short time, producing a high-going pulse at the circuit output immediately following the positive edge at \( x \).

When \( x \) returns to LOW, \( \bar{x} \) goes HIGH after a delay. However, there is no time in this sequence when both AND inputs are HIGH. Therefore, the circuit output stays LOW after the negative edge of the input waveform.

Figure 7.41 shows how the D flip-flop circuit operates. When \( D = 0 \) and the edge detector senses a positive edge at the \( CLK \) input, the output of the lower NAND gate steers a low-going pulse to the \( \text{RESET} \) input of the latch, thus storing a 0 at \( Q \). When \( D = 1 \), the upper NAND gate is enabled. The edge detector sends a high-going pulse to the upper steering gate, which transmits a low-going \( \text{SET} \) pulse to the output latch. This action stores a 1 at \( Q \).
**EXAMPLE 7.6**

Figure 7.42 shows a MAX+PLUS II Graphic Design File with a D latch and a D flip-flop connected to the same data input and clock. Create a MAX+PLUS II simulation that illustrates the difference between the latch (level-sensitive enable) and the flip-flop (edge-triggered clock).

**FIGURE 7.42**  
D Latch and D Flip-Flop

**SOLUTION**  
The simulation, shown in Figure 7.43, has a 200 ns grid. Several points on the waveform indicate the similarities and differences between the latch and flip-flop operation.

**FIGURE 7.43**  
Simulation showing the Difference between D Latch and D Flip Flop

1. At 1.2 μs, D goes HIGH. The latch output (Q_latch) and the flip-flop output (Q_flip_flop) both go HIGH at 1.4 μs, since the beginning of the enable HIGH state and the positive edge of the CLK both correspond to this time.
2. D goes LOW at 2 μs. Both Q outputs go LOW at 2.8 μs since the positive edge of the CLK and its HIGH level occur at the same time.
3. The D input goes HIGH at 4.4 μs, in the middle of a CLK pulse. Since the CLK line is HIGH, Q_latch changes immediately. Q_flip_flop does not change until the next positive edge, at 6 μs.
4. D goes LOW at 7.8 μs. Q_latch also changes at this time, since CLK is HIGH. Q_flip_flop changes on the next positive edge, at 9.2 μs.
Note that the latch output is in an unknown state until the first \( CLK \) pulse, whereas the flip-flop output is LOW, even before the first \( CLK \) pulse. This is because Altera CPLDs have power-on reset circuitry that ensures that flip-flop outputs in a CPLD are LOW immediately after power is applied to the device. The MAX+PLUS II simulator accounts for this condition.

**EXAMPLE 7.7**

Two positive edge-triggered D flip-flops are connected as shown in Figure 7.44a. Inputs \( D_0 \) and \( CLK \) are shown in the timing diagram. Complete the timing diagram by drawing the waveforms for \( Q_0 \) and \( Q_1 \), assuming that both flip-flops are initially reset.

**FIGURE 7.44**

Example 7.7
Circuit and Timing Diagram

**SOLUTION** Figure 7.44b shows the output waveforms. \( Q_0 \) follows \( D_0 \) at each point where the clock input has a positive edge. One result of this is that the HIGH pulse on \( D_0 \) between clock pulses 5 and 6 is ignored, since \( D_0 = 0 \) on positive edges 5 and 6.

Since \( D_1 = Q_0 \) and \( Q_1 \) follows \( D_1 \), the waveform at \( Q_1 \) is the same as at \( Q_0 \), but delayed by one clock cycle. If \( Q_0 \) changes due to \( CLK \), we assume that the value of \( D_1 \) is the same as \( Q_0 \) just before the clock pulse. This is because delays within the circuitry of the flip-flops ensure that their outputs will not change for several nanoseconds after an applied clock pulse. Therefore, the level at \( D_1 \) remains constant long enough for it to be clocked into the second flip-flop.

The data entering the circuit at \( D_0 \) are moved, or shifted, from one flip-flop to the next. This type of data movement, called “serial shifting,” is frequently used in data communication and digital arithmetic circuits.

**SECTION 7.4 REVIEW PROBLEM**

7.4 Which part of a D flip-flop accounts for the difference in operation between a D flip-flop and a D latch? How does it work?
Edge-Triggered JK Flip-Flops

A versatile and widely used sequential circuit is the JK flip-flop.

Figure 7.45 shows the logic symbols of a positive- and a negative-edge triggered JK flip-flop. J acts as a SET input and K acts as a RESET input, with the output changing on the active clock edge in response to J and K. When J and K are both HIGH, the flip-flop will toggle between opposite logic states with each applied clock pulse. The function tables of the devices in Figure 7.45 are shown in Table 7.7.

Figure 7.46 shows the simplified circuit of a negative-edge triggered JK flip-flop. The circuit is like that of a gated SR latch with an edge detector (an SR flip-flop), except that there are two extra feedback lines from the latch outputs to the steering gate inputs. This extra feedback is responsible for the flip-flop's toggling action.

Figure 7.47 illustrates how the additional two lines cause the flip-flop to toggle. The cross-feedback from $Q$ to $K$ and from $Q$ to $J$ enables one, but not both, of the steering gates. The edge detector just after the CLK input produces a short positive-going pulse upon detecting a negative edge on the CLK waveform. The enabled steering gate complements and transmits this pulse to the latch, activating either the set or reset function. This in turn changes the latch state and enables the opposite steering gate.

Since all inputs of the steering gates must be HIGH to enable one of the latch functions, J and K must both be HIGH to sustain a repeated toggling action. Under these conditions, $Q$ and $\overline{Q}$ alternately enable one of the steering gates.

**Table 7.7** Function Tables for Edge-Triggered JK Flip-Flops

| CLK | J  | K  | $Q_{t+1}$ | $\overline{Q}_{t+1}$ | Function | CLK | J  | K  | $Q_{t+1}$ | $\overline{Q}_{t+1}$ | Function |
|-----|----|----|-----------|---------------------|----------|-----|----|-----------|---------------------|----------|
| ↑   | 0  | 0  | $Q$, $\overline{Q}$ | No change | ↓   | 0  | 0  | $Q$, $\overline{Q}$ | No change |
| ↑   | 0  | 1  | 0, 1      | Reset       | ↓   | 0  | 1  | 0, 1      | Reset       |
| ↑   | 1  | 0  | 1, 0      | Set         | ↓   | 1  | 0  | 1, 0      | Set         |
| ↑   | 1  | 1  | $\overline{Q}$, $Q$ | Toggle     | ↓   | 1  | 1  | $\overline{Q}$, $Q$ | Toggle     |
| 0   | X  | X  | $Q$, $\overline{Q}$ | Inhibited  | 0   | X  | X  | $Q$, $\overline{Q}$ | Inhibited  |
| 1   | X  | X  | $Q$, $\overline{Q}$ | Inhibited  | 1   | X  | X  | $Q$, $\overline{Q}$ | Inhibited  |
| ↓   | X  | X  | $Q$, $\overline{Q}$ | Inhibited  | ↑   | X  | X  | $Q$, $\overline{Q}$ | Inhibited  |

Positive Edge-Triggered Negative Edge-Triggered

**Figure 7.45**
Edge-Triggered JK Flip-Flops

**Figure 7.46**
JK Flip-Flop Circuit (Simplified)
EXAMPLE 7.8

The \( J \), \( K \), and \( CLK \) inputs of a negative edge-triggered JK flip-flop are as shown in the timing diagram in Figure 7.48. Complete the timing diagram by drawing the waveforms for \( Q \) and \( Q' \). Indicate which function (no change, set, reset, or toggle) is performed at each clock pulse. The flip-flop is initially reset.

![Timing Diagram](image)

**FIGURE 7.47**
Toggle Action of a JK Flip-Flop

**FIGURE 7.48**
Example 7.8
Timing Diagram (Negative-Edge-Triggered JK Flip-Flop)

**SOLUTION**
The completed timing diagram is shown in Figure 7.48. The outputs change only on the negative edges of the \( CLK \) waveform. Note that the same output sometimes results from different inputs. For example, the function at clock pulse 4 is reset and the function at pulses 5 and 6 is no change, but the \( Q \) waveform is LOW in each case.

EXAMPLE 7.9

The toggle function of a JK flip-flop is often used to generate a desired output sequence from a series of flip-flops. The circuit shown in Figure 7.49 is configured so that all flip-flops are permanently in toggle mode.

Assume that all flip-flops are initially reset. Draw a timing diagram showing the \( CLK \), \( Q_0 \), \( Q_1 \), and \( Q_2 \) waveforms when eight clock pulses are applied. Make a table showing each
combination of $Q_2$, $Q_1$, and $Q_0$. What pattern do the outputs form over the period shown on the timing diagram?

**SOLUTION** The circuit timing diagram is shown in Figure 7.50. All flip-flops are in toggle mode. Each time a negative clock edge is applied to the flip-flop $CLK$ input, the $Q$ output will change to the opposite state.

For flip-flop 0, this happens with every clock pulse, since it is clocked directly by the $CLK$ waveform. Each of the other flip-flops is clocked by the $Q$ output waveform of the previous stage. Flip-flop 1 is clocked by the negative edge of the $Q_0$ waveform. Flip-flop 2 toggles when $Q_1$ goes from HIGH to LOW.

Table 7.8 shows the flip-flop outputs after each clock pulse. The outputs form a 3-bit number that counts from 000 to 111 in binary sequence, then returns to 000 and repeats. This flip-flop circuit is called a 3-bit asynchronous counter.

**Synchronous versus Asynchronous Circuits**

<table>
<thead>
<tr>
<th>Clock Pulse</th>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The asynchronous counter in Figure 7.49 has the advantage of being simple to construct and analyze. However, because it is asynchronous (that is, not synchronized to a single clock), it is seldom used in modern digital designs. The main problem with this and other asynchronous circuits is that their outputs do not change at the same time, due to delays in the flip-flops. This yields intermediate states that are not part of the desired output sequence.
Figure 7.51 shows a simulation of a circuit similar to that in Figure 7.49. The outputs are shown separately, and also as a group labeled Q[2..0] that shows the combined binary value of the outputs.

Figure 7.52 shows a detail of the simulation at the point where the output goes from 7 to 0 (111 to 000). At 300 ns, the circuit output is 111. A negative clock edge, applied to flip-flop 0, makes Q₀ toggle after a short delay. The output is now 110 (=6₁₀). The resulting negative edge on Q₀ clocks flip-flop 1, making it toggle, and yields a new output of 100 (=4₁₀). The negative edge on Q₁ clocks flip-flop 2, making the output equal to 000 after a short delay.

Thus, the output goes through two short intermediate states that are not in the desired output sequence. Instead of going directly from 111 to 000, as in Figure 7.50, the output goes in the sequence 111–110–100–000. We see in Figure 7.51 that the counter output goes through one or more intermediate transitions after each negative edge of the Q₀ waveform. In other words, intermediate states arise whenever a change propagates through more than one flip-flop. This happens because the flip-flops are clocked from different sources.
Figure 7.53 shows the circuit of a 3-bit synchronous counter. Unlike the circuit in Figure 7.49, the flip-flops in this circuit are clocked from a common source. Therefore, flip-flop delays do not add up through the circuit, and all the outputs change at the same time. Figure 7.54 shows a simulation of the circuit of Figure 7.53. Note that the outputs progress in a binary sequence, and there are no intermediate states.

The circuit works as follows:

1. Flip-flop 0 is configured for toggle mode ($J_0K_0 = \text{11}$). Since the flip-flops in Figure 7.53 are positive edge-triggered, $Q_0$ toggles on each positive clock edge.

2. $Q_0$ is connected to inputs $J_1$ and $K_1$. Since these inputs are tied together, only two states are possible: no change ($JK = \text{00}$) or toggle ($JK = \text{11}$). If $Q_0 = 1$, $Q_1$ toggles. Otherwise, it does not change. This results in a $Q_1$ waveform that toggles at half the rate of $Q_0$.

3. $J_2$ and $K_2$ are both tied to the output of an AND gate. The AND gate output is HIGH if both $Q_1$ and $Q_0$ are HIGH. This makes $Q_2$ toggle, since $J_2K_2 = \text{11}$. In all other cases, there is no change on $Q_2$. The result of this is that $Q_2$ toggles every fourth clock pulse, the only times when $Q_1$ and $Q_0$ are both HIGH.

**Asynchronous Inputs (Preset and Clear)**

- **KEY TERMS**

  - **Synchronous inputs**: The inputs of a flip-flop that do not affect the flip-flop’s Q outputs unless a clock pulse is applied. Examples include $D$, $J$, and $K$ inputs.

  - **Asynchronous inputs**: The inputs of a flip-flop that change the flip-flop’s Q outputs immediately, without waiting for a pulse at the CLK input. Examples include preset and clear inputs.

  - **Preset**: An asynchronous set function.

  - **Clear**: An asynchronous reset function.

The $D$, $J$, and $K$ inputs of the flip-flops examined so far are called synchronous inputs. This is because any effect they have on the flip-flop outputs is synchronized to the $CLK$ input.

Another class of input is also provided on many flip-flops. These inputs, called asynchronous inputs, do not need to wait for a clock pulse to make a change at the output. The two functions usually provided are preset, an asynchronous set function, and clear, an asynchronous reset function. These functions are generally active LOW, and are abbreviated $PRE$ and $CLR$.

Figure 7.55 shows a modification to the JK flip-flop of Figure 7.46. The $PRE$ and $CLR$ inputs have direct access to the latch gates of the flip-flop and thus are not affected by the...
CLK input. They act exactly the same as the SET and RESET inputs of an SR latch and will override any synchronous input functions currently active.

EXAMPLE 7.10

The waveforms for the CLK, J, K, PRE, and CLR inputs of a negative edge-triggered JK flip-flop are shown in the timing diagram of Figure 7.56. Complete the diagram by drawing the waveform for output Q.

SOLUTION

The Q waveform is shown in Figure 7.56. The asynchronous inputs cause an immediate change in Q, whereas the synchronous inputs must wait for the next negative clock edge. If asynchronous and synchronous inputs are simultaneously active, the asynchronous inputs have priority. This occurs in two places: pulse 3 (K, PRE) and pulse 10 (J, CLR).

The diagram shows the synchronous functions (no change, reset, set, and toggle) at each clock pulse and the asynchronous functions (preset and clear) at the corresponding transition points.

The function table of a negative edge-triggered JK flip-flop with preset and clear functions is shown in Table 7.9.
If preset and clear functions are not used, they should be disabled by connecting them to logic HIGH (for active-LOW inputs). This prevents them from being activated inadvertently by circuit noise. The synchronous functions of some flip-flops will not operate properly unless \( P_E \) and \( C_L \) are HIGH. In MAX+PLUS II, the asynchronous inputs of all flip-flop primitives are set to a default level of HIGH.

Table 7.9  Function Table of a Negative Edge-Triggered JK Flip-Flop with Preset and Clear Functions

<table>
<thead>
<tr>
<th>( PRE )</th>
<th>( CLR )</th>
<th>( CLK )</th>
<th>( J )</th>
<th>( K )</th>
<th>( Q_{t+1} )</th>
<th>( Q_{t+1} )</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>↓</td>
<td>0</td>
<td>0</td>
<td>( Q_t )</td>
<td>( \bar{Q}_t )</td>
<td>No change</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>↓</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>Reset</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>↓</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Set</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>↓</td>
<td>1</td>
<td>1</td>
<td>( Q_t )</td>
<td>( Q_t )</td>
<td>Toggle</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>1</td>
<td>0</td>
<td>Preset</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>0</td>
<td>1</td>
<td>Clear</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>1</td>
<td>1</td>
<td>Forbidden</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>X</td>
<td>X</td>
<td>( Q_t )</td>
<td>( \bar{Q}_t )</td>
<td>Inhibited</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>X</td>
<td>X</td>
<td>( Q_t )</td>
<td>( Q_t )</td>
<td>Inhibited</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>↑</td>
<td>X</td>
<td>X</td>
<td>( Q_t )</td>
<td>( Q_t )</td>
<td>Inhibited</td>
</tr>
</tbody>
</table>

\( X = \text{Don’t care} \)  \( \downarrow = \text{HIGH-to-LOW transition} \)
\( \downarrow = \text{LOW-to-HIGH transition} \)

Using Asynchronous Reset in a Synchronous Circuit

**KEY TERM**

**Master Reset**  An asynchronous reset input used to set a sequential circuit to a known initial state.

Figure 7.57 shows an application of asynchronous clear inputs in a 3-bit synchronous counter. An input called \( \text{RESET} \) is tied to the asynchronous \( CLR \) inputs of all flip-flops. The counter output is set to 000 when the \( \text{RESET} \) line goes LOW.

**FIGURE 7.57**
Synchronous Counter with Asynchronous Reset
Figure 7.58 shows a set of simulation waveforms that illustrate the asynchronous clear function. When \( \text{RESET} \) is HIGH, the count proceeds normally. The positive clock edge at 440 ns drives the counter to state 011. The reset pulse at 460 ns sets the counter to 000 as soon as it goes LOW. On the next clock edge, the count proceeds from 000.

![Simulation of Synchronous Counter with Asynchronous Reset](image)

**FIGURE 7.58**
Simulation of Synchronous Counter with Asynchronous Reset

The function that sets all flip-flops in a circuit to a known initial state is sometimes called **Master Reset**.

---

**SECTION 7.5 REVIEW PROBLEM**

7.5 What is the main difference between synchronous and asynchronous circuits, such as the two counters in Figures 7.49 and 7.53? What disadvantage is there to an asynchronous circuit?

---

**7.6 Edge-Triggered T Flip-Flops**

A flip-flop whose output toggles between HIGH and LOW states on each applied clock pulse when a synchronous input, called \( T \), is active.

In the section on the JK flip-flop, we saw how that device can be set to toggle between HIGH and LOW output states. Other types of flip-flops can perform this function, as well. For example, Figure 7.59 shows a D flip-flop configured for toggle operation. Since \( Q \) follows \( D \) and \( D = \overline{Q} \) in this circuit, then the flip-flop output must change to its opposite state with each clock pulse. Figure 7.60 shows a MAX+PLUS II simulation of this circuit.

![D Flip-Flop Configured for Toggle Function](image)

**FIGURE 7.59**
D Flip-Flop Configured for Toggle Function
It is seldom useful for flip-flops in synchronous circuits to be permanently configured in toggle mode. What made the JK flip-flops suitable elements for the synchronous counter in Figure 7.53 was the fact that sometimes they toggled and sometimes they didn’t, depending on the current point in the output sequence of the counter. Figure 7.61 shows a D flip-flop configured for a switchable toggle function.

The XOR gate acts as an inverter when the $T$ input is HIGH and as a noninverting buffer when $T$ is LOW. Thus, when $T$ is LOW, the $Q$ output is circulated back to the $D$ input of the flip-flop and the current value of $Q$ is reloaded on the next clock pulse. When $T$ is HIGH, the circuit acts like that of Figure 7.59 and toggles.

A T flip-flop has this equivalent function. Figure 7.62 shows the symbol of a T flip-flop in a MAX+PLUS II Graphic Design File. A MAX+PLUS II simulation in Figure 7.63 shows the operation of this device. The $Q$ output toggles on each clock pulse when...
$T$ is HIGH; otherwise $Q$ retains its last value. A function table for the T flip-flop is shown in Table 7.10.

### Table 7.10 Function Table for a T Flip-Flop

<table>
<thead>
<tr>
<th>$CLK$</th>
<th>$T$</th>
<th>$Q_{t-1}$</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>0</td>
<td>$Q_t$</td>
<td>No Change</td>
</tr>
<tr>
<td>↑</td>
<td>1</td>
<td>$\bar{Q}_t$</td>
<td>Toggle</td>
</tr>
<tr>
<td>0</td>
<td>X</td>
<td>$Q_t$</td>
<td>Inhibited</td>
</tr>
<tr>
<td>1</td>
<td>X</td>
<td>$Q_t$</td>
<td>Inhibited</td>
</tr>
<tr>
<td>↓</td>
<td>X</td>
<td>$Q_t$</td>
<td>Inhibited</td>
</tr>
</tbody>
</table>

#### SECTION 7.6 REVIEW PROBLEM

7.6 Draw a circuit showing how the JK flip-flops in Figure 7.53 can be replaced by T flip-flops.

### 7.7 Timing Parameters

**KEY TERMS**

- **Setup time** ($t_{su}$) The time required for the synchronous inputs of a flip-flop to be stable before a $CLK$ pulse is applied.
- **Hold time** ($t_h$) The time that the synchronous inputs of a flip-flop must remain stable after the active $CLK$ transition is finished.
- **Pulse width** ($t_w$) Minimum time required for an active-level pulse applied to a $CLK$, $CLR$, or $PRE$ input, as measured from the midpoint of the leading edge of the pulse to the midpoint of the trailing edge.
- **Recovery time** ($t_{rec}$) Minimum time from the midpoint of the trailing edge of a $CLR$ or $PRE$ pulse to the midpoint of an active $CLK$ edge.
- **Propagation delay** The time required for the output of a digital circuit to change states after a change at one or more of its inputs.

Flip-flops are electrical devices with inherent internal switching delays. As such, they have specific requirements for the timing of the input and output waveforms in order for them to operate reliably. We will examine the basic timing requirements for two small scale integration (SSI) devices: the 74LS107A JK flip-flop (LSTTL family) and the 74HC107 JK flip-flop (high-speed CMOS family). Figure 7.64 shows some of the basic timing requirements of a JK flip-flop.

Figure 7.64a illustrates the definitions of **setup time** ($t_{su}$), **hold time** ($t_h$), and **pulse width** ($t_w$). The notation used for the “$J$ or $K$” waveform indicates that the $J$ or $K$ input could be at either logic level and makes a transition to the opposite level at some point. The setup time is measured from the midpoint of the $J$ or $K$ transition to the midpoint of the active $CLK$ edge. The logic level on the $J$ or $K$ input must be steady for at least this time for the flip-flop to operate correctly. Setup time for both LSTTL and high-speed CMOS flip-flops is about 20 ns.

Similarly, the hold time is measured from the midpoint of the $CLK$ transition to the midpoint of the next $J$ or $K$ transition. The $J$ or $K$ level must be held steady for at least this time to ensure dependable operation. Hold time is 0 for LSTTL and 3 ns for a high-speed CMOS flip-flop.

The pulse width, $t_w$, shows how long the $CLK$ needs to be held LOW after an active $CLK$ edge. Although the LOW level does not itself latch data into the flip-flop, internal logic levels must reach a steady state before the device can accept a new clock pulse. This
minimum pulse width allows the necessary time for these internal transitions. The data sheet for a 74HC107 flip-flop (high-speed CMOS) gives the clock pulse width as 16 ns; a data sheet for a 74LS107A device gives the value as 20 ns.

Figure 7.64b shows the pulse width required at the \( \overline{C} \) input, the propagation delay from \( \overline{C} \) to \( Q \) and \( Q' \), and the recovery time that must be allowed from the end of a \( \overline{C} \) pulse to the beginning of a \( CLK \) pulse. These times also apply to a pulse on the \( PRE \) input of a flip-flop.

Propagation delay is the result of internal electrical delays, primarily the charging and discharging of internal capacitances of the gate transistor junctions. The practical result of this is that a pulse at the \( \overline{C} \) input makes \( Q \) go LOW, but not immediately; there is a delay of several nanoseconds between input pulse and output response.

Propagation delay is defined by the direction of the output transition. The delay at \( Q \), which goes from HIGH to LOW, is called \( t_{pHL} \). The delay at \( Q' \), which goes from LOW to HIGH when cleared, is called \( t_{pLH} \). Values for propagation delay from \( \overline{C} \) to \( Q \) or \( Q' \) are about 20 ns for LSTTL and 31 ns for high-speed CMOS.

The recovery time, \( t_{rec} \), allows the internal logic levels of the flip-flop to reach a steady state after a \( \overline{C} \) pulse. When the internal levels are stable, the device is ready to accept an active \( CLK \) edge. The recovery time for high-speed CMOS is 20 ns and 25 ns for an LSTTL device. (The LSTTL data sheet treats this parameter as a species of setup time; it is shown as setup time after the \( \overline{C} \) is inactive. Same thing.)

Finally, Figure 7.64c shows the propagation delay from \( CLK \) to \( Q \). This is the time from the midpoint of an active \( CLK \) edge to the midpoint of a transition at \( Q \) caused by that \( CLK \) edge. The parameters are defined, as before, by the direction of the output transition. Propagation delays \( t_{pHL} \) and \( t_{pLH} \) are 20 ns, maximum, for a 74LS107A device and 25 ns for a 74HC107 flip-flop.
The timing restrictions of a flip-flop imply that there is a maximum $CLK$ frequency beyond which the device will not operate reliably. Data sheets give these values as about 30 MHz for both LSTTL and high-speed CMOS devices.

Table 7.11 summarizes the timing parameters of a 74LS107A flip-flop and a 74HC107 device. The values for the latter device are for $V_{cc} = 4.5$ V and a temperature range of $-55^\circ$C to $25^\circ$C; they increase with a higher temperature range or a lower supply voltage.

### Table 7.11  Timing Parameters of an LSTTL and a High-Speed CMOS Flip-Flop

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>74LS107A</th>
<th>74HC107</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{ss}$</td>
<td>Setup time</td>
<td>20 ns</td>
<td>20 ns</td>
</tr>
<tr>
<td>$t_h$</td>
<td>Hold time</td>
<td>0 ns</td>
<td>3 ns</td>
</tr>
<tr>
<td>$t_w$</td>
<td>$CLR$ pulse width</td>
<td>25 ns</td>
<td>16 ns</td>
</tr>
<tr>
<td>$t_{rec}$</td>
<td>Recovery time</td>
<td>25 ns</td>
<td>20 ns</td>
</tr>
<tr>
<td>$t_{pHL}$ (from $CLR$)</td>
<td>Propagation delay</td>
<td>20 ns</td>
<td>31 ns</td>
</tr>
<tr>
<td>$t_{pLH}$ (from $CLK$)</td>
<td>Propagation delay</td>
<td>20 ns</td>
<td>25 ns</td>
</tr>
<tr>
<td>$f_{max}$</td>
<td>Maximum frequency</td>
<td>30 MHz</td>
<td>30 MHz</td>
</tr>
</tbody>
</table>

#### EXAMPLE 7.11

The timing diagrams in Figure 7.65 represent some of the timing parameters of a JK flip-flop. From these diagrams, determine the setup and hold times and the propagation delays from $CLK$ and $CLR$ to $Q$ and $\overline{Q}$.

![Timing Diagrams](image)

**FIGURE 7.65**

Example 7.11
Timing Parameters

**SOLUTION**

The values are as follows:

- Setup time $= 15$ ns
- Hold time $= 5$ ns
- Propagation delays (from $CLK$): 25 ns ($t_{pLH}$ and $t_{pHL}$)
- (from $CLR$): 20 ns ($t_{pLH}$ and $t_{pHL}$)
### SECTION 7.7 REVIEW PROBLEM

7.7 An active edge on the clock input of a JK flip-flop makes \( Q \) go from HIGH-to-LOW. Name the timing parameter that measures the delay between the input and output change. Write the symbol for the parameter.

---

**SUMMARY**

1. A combinational circuit combines inputs to generate a particular output logic level that is always the same, regardless of the order in which the inputs are applied. A sequential circuit might generate different outputs for the same inputs, depending on the sequence in which the inputs were applied.

2. An SR latch is a sequential circuit with SET (\( S \)) and RESET (\( R \)) inputs and complementary outputs (\( Q \) and \( \bar{Q} \)). By definition, a latch is set when \( Q = 1 \) and reset when \( Q = 0 \).

3. A latch sets when its \( S \) input activates. When \( S \) returns to the inactive state, the latch remains in the set condition until explicitly reset by activating its \( R \) input.

4. A latch can have active-HIGH inputs (designated \( S \) and \( R \)) or active-LOW inputs (designated \( S \) and \( R \)).

5. Two basic SR latch circuits are the NAND latch and the NOR latch, each consisting of two gates with cross-coupled feedback. In the NAND form, we draw the gates in their DeMorgan equivalent form so that each circuit has OR-shaped gates, inversion from input to output, and feedback to the opposite gate.

6. A NOR latch has active-HIGH inputs. It is described by the following function table:

<table>
<thead>
<tr>
<th>( S )</th>
<th>( R )</th>
<th>( Q_{t+1} )</th>
<th>( \bar{Q}_{t+1} )</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( Q_t )</td>
<td>( \bar{Q}_t )</td>
<td>No change</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>Reset</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Set</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>Forbidden</td>
</tr>
</tbody>
</table>

7. A NAND latch has active-LOW inputs and is described by the following function table:

<table>
<thead>
<tr>
<th>( \bar{S} )</th>
<th>( \bar{R} )</th>
<th>( Q_{t+1} )</th>
<th>( \bar{Q}_{t+1} )</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Forbidden</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Set</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>Reset</td>
</tr>
<tr>
<td>1</td>
<td>( \bar{Q}_t )</td>
<td>( \bar{Q}_t )</td>
<td>( \bar{Q}_t )</td>
<td>No change</td>
</tr>
</tbody>
</table>

8. A NAND latch can be used as a switch debouncer for a switch with a grounded common terminal, a normally open, and a normally closed contact. When the switch operates, one contact closes, resetting the latch on the first bounce. Further bounces are ignored. When the switch returns to its normal position, it sets the latch on the first bounce and further bounces are ignored.

9. A gated SR latch controls the times when a latch can switch. The circuit consists of a pair of latch gates and a pair of steering gates. The steering gates are enabled or inhibited by a control signal called ENABLE. When the steering gates are enabled, they can direct a set or reset pulse to the latch gates. When inhibited, the steering gates block any set or reset pulses to the latch gates so the latch output cannot change.

10. A gated D ("data") latch can be constructed by connecting opposite logic levels to the \( S \) and \( R \) inputs of an SR latch. Since \( S \) and \( R \) are always opposite, the D latch has no forbidden state. The no change state is provided by the inhibit property of the ENABLE input.

11. In a gated D latch (or transparent latch), \( Q \) follows \( D \) when ENABLE is active. This is the transparent mode of the latch. When ENABLE is inactive, the latch stores the last value of \( D \).

12. A D latch can be described in VHDL by an IF statement within a PROCESS. The PROCESS statement in VHDL is concurrent, but the statements inside the PROCESS are sequential.

13. A D latch can also be implemented in VHDL by instantiating a LATCH primitive as a component in a VHDL design entity or by instantiating a component called lpm_latch from the Library of Parameterized Modules (LPM).

14. An LPM component is a standard component with certain properties, called parameters, that can be specified when the component is instantiated. The inputs and outputs of an LPM component are called ports. Parameter values are assigned in the generic map of a component instantiation statement. Component port names are associated with user port names in the port map of a component instantiation statement.

15. A flip-flop is like a gated latch that responds to the edge of a pulse applied to an enable input called CLOCK. A flip-flop output will change only when the input makes a transition from LOW to HIGH (for a positive edge-triggered device) or HIGH to LOW (for a negative edge-triggered device).

16. In a positive edge-triggered D flip-flop, \( Q \) follows \( D \) when there is a positive edge on the clock input.

17. D flip-flops are used primarily for data storage and transfer.

18. A JK flip-flop has two synchronous inputs, called \( J \) and \( K \). \( J \) acts as an active-HIGH set input. \( K \) acts as an active-HIGH reset function. When both inputs are asserted, the flip-flop toggles between 0 and 1 with each applied clock pulse.

19. The toggle function in a JK flip-flop is implemented with additional cross-coupled feedback from the latch gate outputs to the steering gate inputs.

20. A chain of JK flip-flops can implement an asynchronous binary counter if the \( Q \) of each flip-flop is connected to the
clock input of the next. Although this is an easy way to create a counter, it is seldom used because internal flip-flop delays result in unwanted intermediate states in the count sequence.

21. JK flip-flops can be combined with a network of logic gates to make a synchronous binary counter. The gates are connected in such a way that each flip-flop toggles when all previous bits are HIGH; otherwise the flip-flops are in a no change state. Although more complex than an asynchronous counter, a synchronous counter is free of unwanted intermediate states.

22. Many flip-flops are provided with asynchronous preset (set) and clear (reset) functions. Since these functions are connected directly to the latch gates of a flip-flop, they act immediately, without waiting for the clock. In most cases, these functions are active-LOW.

23. Asynchronous inputs, such as preset and clear, are usually designed so that they will override the synchronous inputs, such as D or JK.

24. Unused asynchronous inputs should be disabled by tying them to a logic HIGH (for an active-LOW input). Flip-flop primitives in MAX+PLUS II automatically have their asynchronous inputs connected to HIGH unless otherwise specified by a design entry file.

25. The outputs of a T (toggle) flip-flop toggle with each clock pulse when the T input is HIGH and do not change when T is LOW.

26. Several important timing parameters for a flip-flop include: setup and hold time, propagation delay, minimum pulse width, and recovery time.

27. Setup time is the time before a clock edge that a synchronous input must be held steady. Hold time is the time after an applied clock edge that an input level must be held constant.

28. Propagation delay is the time for an input change, such as on CLK or CLR, to have an effect on an output, such as Q. Propagation time is always indicated with respect to the change in output level: \( t_{phL} \) for a LOW-to-HIGH output transition and \( t_{phH} \) for a HIGH-to-LOW output change.

29. Minimum pulse width, \( t_w \), indicates how long a CLK or CLR input must be held after an active edge or level is applied before returning to the original level.

30. Recovery time is the minimum time required from the end of an active level on one input (such as CLR) to an active CLK edge.

**Glossary**

**Asynchronous** Not synchronized to the system clock.

**Asynchronous inputs** The inputs of a flip-flop that change the flip-flop’s \( Q \) outputs immediately, without waiting for a pulse at the CLK input. Examples include preset and clear inputs.

**Clear** An asynchronous reset function.

**CLOCK** An enabling input to a sequential circuit that is sensitive to the positive- or negative-going edge of a waveform.

**Edge** The HIGH-to-LOW (negative edge) or LOW-to-HIGH (positive edge) transition of a pulse waveform.

**Edge detector** A circuit in an edge-triggered flip-flop that converts the active edge of a CLOCK input to an active-level pulse at the internal latch’s SET and RESET inputs.

**Edge-sensitive** Edge-triggered.

**Edge-triggered** Enabled by the positive or negative edge of a digital waveform.

**Flip-flop** A sequential circuit based on a latch whose output changes when its CLOCK input receives either an edge or a pulse, depending on the device.

**Gated SR latch** An SR latch whose ability to change states is controlled by an extra input called the ENABLE input.

**Generic map** A VHDL construct that maps one or more parameters of a component to a value for that instance of the component.

**Hold time** \( t_h \) The time that the synchronous inputs of a flip-flop must remain stable after the active CLK transition is finished.

**Latch** A sequential circuit with two inputs called SET and RESET, which make the latch store a logic 0 (reset) or 1 (set) until actively changed.

**Level-sensitive** Enabled by a logic HIGH or LOW level.

**Library of Parameterized Modules (LPM)** A standardized set of components for which certain properties can be specified when the component is instantiated.

**Master Reset** An asynchronous reset input used to set a sequential circuit to a known initial state.

**Parameter (in an LPM component)** A property of a component that can be specified when the component is instantiated.

**Preset** An asynchronous set function.

**Port map** A VHDL construct that maps the name of a port in a component to the name of a port, variable, or signal in a design entity that uses the component.

**Propagation delay** The time required for the output of a digital circuit to change states after a change at one or more of its inputs.

**Pulse width** \( t_w \) Minimum time required for an active-level pulse applied to a CLK, CLR, or PRE input, as measured from the midpoint of the leading edge of the pulse to the midpoint of the trailing edge.

**Recovery time** \( t_{rec} \) Minimum time from the midpoint of the trailing edge of a CLR or PRE pulse to the midpoint of an active CLK edge.

**Reset** 1. The stored LOW state of a latch circuit. 2. A latch input that makes the latch store a logic 0.

**Sequential circuit** A digital circuit whose output depends not only on the present combination of inputs, but also on the history of the circuit.

**Set** 1. The stored HIGH state of a latch circuit. 2. A latch input that makes the latch store a logic 1.

**Setup time** \( t_u \) The time required for the synchronous inputs of a flip-flop to be stable before a CLOCK pulse is applied.
**Steering gates** Logic gates, controlled by the *ENABLE* input of a gated latch, that steer a *SET* or *RESET* pulse to the correct input of an SR latch circuit.

**Synchronous** Synchronized to the system clock.

**Synchronous inputs** The inputs of a flip-flop that do not affect the flip-flop’s *Q* outputs unless a clock pulse is applied. Examples include *D*, *J*, and *K* inputs.

**Toggle** Alternate between binary states with each applied clock pulse.

**T (toggle) flip-flop** A flip-flop whose output toggles between HIGH and LOW states on each applied clock pulse when a synchronous input, called *T*, is active.

**Transparent latch (gated D latch)** A latch whose output follows its data input when its *ENABLE* input is active.

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**PROBLEMS**

**Section 7.1   Latches**

7.1 Complete the timing diagram in Figure 7.66 for the active-HIGH latch shown. The latch is initially set.

7.2 Repeat Problem 7.1 for the timing diagram shown in Figure 7.67.

**FIGURE 7.66**
Problem 7.1
Timing Diagram

**FIGURE 7.67**
Problem 7.2
Timing Diagram

7.3 Complete the timing diagram in Figure 7.68 for the active-LOW latch shown.

7.4 Figure 7.69 shows an active-LOW latch used to control a motor starter. The motor runs when *Q* = 1 and stops when *Q* = 0. (Problem continues . . .)

**FIGURE 7.68**
Problem 7.3
Timing Diagram

**FIGURE 7.69**
Problem 7.4
Latch for Motor Starter
The motor is housed in a safety enclosure that has an access hatch for service. A safety interlock prevents the motor from running when the hatch is open. The **HATCH** switch opens when the hatch opens, supplying a logic HIGH to the circuit. The **START** switch is a normally open momentary-contact pushbutton (LOW when pressed). The **STOP** switch is a normally closed momentary-contact pushbutton (HIGH when pressed).

Draw the timing diagram of the circuit, showing **START**, **STOP**, **HATCH**, **S**, **R**, and **Q** for the following sequence of events:

a. **START** is pressed and released.
b. The hatch cover is opened.
c. **START** is pressed and released.
d. The hatch cover is closed.
e. **START** is pressed and released.
f. **STOP** is pressed and released.

Briefly describe the functions of the three switches and how they affect the motor operation.

### Section 7.2 NAND/NOR Latches

#### 7.5 Draw a NAND latch, correctly labeling the inputs and outputs. Describe the operation of a NAND latch for all four possible combinations of **S** and **R**.

#### 7.6 Draw a NOR latch, correctly labeling the inputs and outputs. Describe the operation of a NOR latch for all four possible combinations of **S** and **R**.

#### 7.7 The timing diagram in Figure 7.70 shows the input waveforms of a NAND latch. Complete the diagram by showing the output waveforms.

---

**FIGURE 7.70**
Problem 7.7
Timing Diagram

---

**FIGURE 7.71**
Problem 7.8
Input Waveforms to a NOR Latch

---

**FIGURE 7.72**
Problem 7.9
Input Waveforms to a Latch

---

7.8 Figure 7.71 shows the input waveforms to a NOR latch. Draw the corresponding output waveforms.

7.9 Figure 7.72 represents two input waveforms to a latch circuit.

a. Draw the outputs **Q** and **\( \bar{Q} \)** if the latch is a NAND latch.
b. Draw the output waveforms if the latch is a NOR latch.

(Note that in each case, the waveforms will produce the forbidden state at some point. Even under this condition, it is still possible to produce unambiguous output waveforms. Refer to Figures 7.18 and 7.19 for guidance.)

#### 7.10 a. Draw a timing diagram for a NAND latch showing each of the following sequences of events:

i. **S** and **R** are both LOW; **S** goes HIGH before **R**.
ii. **\( \bar{S} \)** and **\( \bar{R} \)** are both LOW; **\( \bar{R} \)** goes HIGH before **\( \bar{S} \)**.
iii. **S** and **R** are both LOW; **S** and **R** go LOW simultaneously.

b. State why **S** = **R** = 0 is a forbidden state for the NAND latch.
c. Briefly explain what the final result is for each of the above transitions.

#### 7.11 a. Draw a timing diagram for a NOR latch showing each of the following sequences of events:

i. **S** and **R** are both HIGH; **S** goes LOW before **R**.
ii. **S** and **R** are both HIGH, **R** goes LOW before **S**.
iii. **S** and **R** are both HIGH, **S** and **R** go LOW simultaneously.
b. Briefly explain what the final result is for each of the transitions listed in part a of this question.

c. State why $S = R = 1$ is a forbidden state for the NOR latch.

7.12 Figure 7.73 shows the effect of mechanical bounce on the switching waveforms of a single-pole double-throw (SPDT) switch.

a. Briefly explain how this effect arises.

**FIGURE 7.73**  
Problem 7.12  
Effect of Mechanical Bounce on a SPDT Switch

b. Draw a NAND latch circuit that can be used to eliminate this mechanical bounce, and briefly explain how it does so.

**Section 7.3 Gated Latches**

7.13 Complete the timing diagram for the gated latch shown in Figure 7.74.

7.14 Complete the timing diagram for the gated latch shown in Figure 7.75.
7.15 A pump motor can be started at two different locations with momentary-contact pushbuttons $S_1$ and $S_2$. It can be stopped by momentary-contact pushbuttons $ST_1$ and $ST_2$. As in Problem 7.4, a $RUN$ input on the motor controller must be kept HIGH to keep the motor running. After the motor is stopped, a timer prevents the motor from starting for 5 minutes.

Draw a circuit block diagram showing how an SR latch and some additional gating logic can be used in such an application. The timer can be shown as a block activated by the $STOP$ function. Assume that the timer output goes HIGH for 5 minutes when activated.

7.16 The $S$ and $R$ waveforms in Figure 7.76 are applied to two different gated latches. The $ENABLE$ waveforms for the latches are shown as $EN_1$ and $EN_2$. Draw the output waveforms $Q_1$ and $Q_2$, assuming that $S$, $R$, and $EN$ are all active HIGH. Which output is least prone to synchronization errors? Why?
7.17 Figure 7.77 represents the waveforms of the EN and D inputs of a 4-bit transparent latch. Complete the timing diagram by drawing the waveforms for \( Q_1 \) to \( Q_4 \).

7.18 An electronic direction finder aboard an aircraft uses a 4-bit number to distinguish 16 different compass points as follows:

<table>
<thead>
<tr>
<th>Direction</th>
<th>Degrees</th>
<th>Gray Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>0/360</td>
<td>0000</td>
</tr>
<tr>
<td>NNE</td>
<td>22.5</td>
<td>0001</td>
</tr>
<tr>
<td>NE</td>
<td>45</td>
<td>0011</td>
</tr>
<tr>
<td>ENE</td>
<td>67.5</td>
<td>0010</td>
</tr>
<tr>
<td>E</td>
<td>90</td>
<td>0110</td>
</tr>
<tr>
<td>ESE</td>
<td>112.5</td>
<td>0111</td>
</tr>
<tr>
<td>SE</td>
<td>135</td>
<td>0101</td>
</tr>
<tr>
<td>SSE</td>
<td>157.5</td>
<td>0100</td>
</tr>
<tr>
<td>S</td>
<td>180</td>
<td>1100</td>
</tr>
<tr>
<td>SSW</td>
<td>202.5</td>
<td>1101</td>
</tr>
<tr>
<td>SW</td>
<td>225</td>
<td>1111</td>
</tr>
<tr>
<td>WSW</td>
<td>247.5</td>
<td>1110</td>
</tr>
<tr>
<td>W</td>
<td>270</td>
<td>1010</td>
</tr>
<tr>
<td>WNW</td>
<td>295.5</td>
<td>1011</td>
</tr>
<tr>
<td>NW</td>
<td>315</td>
<td>1001</td>
</tr>
<tr>
<td>NNW</td>
<td>337.5</td>
<td>1000</td>
</tr>
</tbody>
</table>

The output of the direction finder is stored in a 4-bit latch so that the aircraft flight path can be logged by a computer. The latch is periodically updated by a continuous pulse on the latch enable line.

Figure 7.78 shows a sample reading of the direction finder’s output as presented to the latch. (Problem continues . . .)
a. Complete the timing diagram by filling in the data for the \( Q \) outputs.

b. Based on the completed timing diagram of Figure 7.78, make a rough sketch of the aircraft's flight path for the monitored time.

7.19 Write a VHDL file for an 8-bit latch, using LATCH primitives. Create a simulation file that demonstrates the operation of all eight bits.

7.20 Write a VHDL file for an 8-bit latch, using a component from the Library of Parameterized Modules. Create a simulation file that tests the latch for all eight bits.

Section 7.4 Edge-Triggered D Flip-Flops

7.21 The waveforms in Figure 7.79 are applied to the inputs of a positive edge-triggered D flip-flop and a gated D latch. Complete the timing diagram where \( Q_1 \) is the output of the flip-flop and \( Q_2 \) is the output of the gated latch. Account for any differences between the \( Q_1 \) and \( Q_2 \) waveforms.

7.22 Complete the timing diagram for a positive edge-triggered D flip-flop if the waveforms shown in Figure 7.80 are applied to the flip-flop inputs.

7.23 Repeat Problem 7.22 for the waveforms shown in Figure 7.81.

7.24 Repeat Problem 7.22 for the waveforms shown in Figure 7.82.

7.25 Draw a logic diagram of a D flip-flop configured for toggle mode. (Hint: The \( D \) input must always be the opposite of the \( Q \) output.)

7.26 Write a VHDL file that defines a 12-bit D flip-flop with a clock common to all flip-flops, using MAX+PLUS II primitives. The component declaration for the DFF component is as follows:

![Figure 7.79](image1)

![Figure 7.80](image2)

![Figure 7.81](image3)
COMPONENT DFF
PORT (d : IN STD_LOGIC;
      clk : IN STD_LOGIC;
      clrn : IN STD_LOGIC;
      prn : IN STD_LOGIC;
      q : OUT STD_LOGIC);
END COMPONENT;

Disregard the clrn (active-LOW clear) and prn (active-LOW preset) ports for this problem. (Hint: you may have to use a component declaration in your file that only declares the ports d, clk, and q.)

7.27 Write a VHDL file that creates a 12-bit D flip-flop, using the LPM component lpm_ff. (This component is instantiated as a D flip-flop by default. The required LPM component port names are: data, clock, and q.)

Section 7.5 Edge-Triggered JK Flip-Flops

7.28 The waveforms in Figure 7.83 are applied to a negative edge-triggered JK flip-flop. Complete the timing diagram by drawing the Q waveform.

7.29 Repeat Problem 7.28 for the waveforms in Figure 7.84.

7.30 Given the inputs x, y, and z to the circuit in Figure 7.85, draw the waveform for output Q.
7.31 Assume that all flip-flops in Figure 7.86 are initially set. Draw a timing diagram showing the CLK, Q₀, Q₁, and Q₂ waveforms when eight clock pulses are applied. Make a table showing each combination of Q₂, Q₁, and Q₀. What pattern do the outputs form over the period shown on the timing diagram?

7.32 Refer to the JK flip-flop circuit in Figure 7.87. Is the circuit synchronous or asynchronous? Explain your answer.

7.33 Assume all flip-flops in the circuit in Figure 7.87 are reset. Analyze the operation of the circuit when sixteen clock pulses are applied by making a table showing the sequence of states of Q₃Q₂Q₁Q₀, beginning at 0000.

7.34 Draw a timing diagram showing the sequence of states from the table derived in Problem 7.33.

7.35 The waveforms shown in Figure 7.88 are applied to a negative edge-triggered JK flip-flop. The flip-flop’s Preset and Clear inputs are active LOW. Complete the timing diagram by drawing the output waveforms.
7.36 Repeat Problem 7.35 for the waveforms in Figure 7.89.

7.37 Create a MAX+PLUS II Graphic Design File for the synchronous circuit in Figure 7.87. Modify the circuit to add an asynchronous Master Reset function. Create a simulation file to verify the circuit operation.

7.38 Modify the gdf created in Problem 7.37 to include a Master Reset function and an asynchronous preset function that will set the state of the circuit to $Q_3Q_2Q_1Q_0 = 1010$ when activated. Create a simulation file to verify the circuit operation.

7.39 The term asynchronous is sometimes used to refer to the configuration of a circuit (e.g., a 3-bit asynchronous counter) and sometimes to a type of input to a device (e.g., an asynchronous clear input). Briefly explain how these two usages are similar and how they are different.

7.40 Write a VHDL file for a 12-bit D flip-flop that uses MAX+PLUS II DFF primitives, similar to that in Problem 7.26. Include active-LOW asynchronous clear (CLRN) and preset (PRN) inputs. Create a simulation file to verify the operation of your design.

7.41 Write a VHDL file for a 12-bit D flip-flop with asynchronous preset and clear, using the LPM component lpm_ff, similar to that in Problem 7.27. Required ports: data, clock, aclr (asynchronous clear), aset (asynchronous set), and q. Ports aset and aclr are active-HIGH. Add two signals to the VHDL design to make them active-LOW. Create a simulation file to verify the operation of your design.

Section 7.6 Edge-Triggered T Flip-Flops

7.42 The $T$ and CLK waveforms for a positive-edge triggered T flip-flop is shown in Figure 7.90. Complete the timing diagram.

7.43 The $T$ and CLK waveforms for a positive-edge triggered T flip-flop is shown in Figure 7.91. Complete the timing diagram.

FIGURE 7.89
Problem 7.36
Waveforms

FIGURE 7.90
Problem 7.42
Timing Diagram

FIGURE 7.91
Problem 7.43
Timing Diagram
ANSWERS TO SECTION REVIEW PROBLEMS

Section 7.1
7.1 The latch resets (i.e., \( Q \) goes LOW) upon receiving the first reset pulse. At that point, the latch is already reset, so further pulses are ignored.

Section 7.2
7.2 The NOR latch has active-HIGH inputs. If you make both inputs HIGH, you are attempting to set and reset the latch at the same time, which is a contradictory action. A NAND latch has active-LOW inputs. Therefore, if both inputs are HIGH, neither the set nor reset function activates and there is no change on the latch output.

Section 7.3
7.3

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;
ENTITY lch16prm IS
PORT(d_in : IN STD_LOGIC_VECTOR (15 downto 0);
     enable : IN STD_LOGIC;
     q_out : OUT STD_LOGIC_VECTOR (15 downto 0 ));
END lch16prm;

ARCHITECTURE a OF lch16prm IS
BEGIN
  -- Instantiate a latch from a MAX+PLUS II primitive
  latch4:
  FOR i IN 15 downto 0 GENERATE
    latchPrimitive: latch
    PORT MAP (d => d in (i),
              ena => enable, q => q out (i ) );
  END GENERATE;
END a;

Section 7.4
7.4 The edge detector circuit in the clock circuit accounts for the operational difference between a D flip-flop and a D latch. It works by using the difference in internal delay times between the gates that comprise the flip-flop’s clock input circuit.

Section 7.5
7.5 The flip-flops in asynchronous circuits are not all clocked at the same time; they are asynchronous with respect to the system clock. The flip-flops in a synchronous circuit have a common clock connection, which makes them synchronous to the system clock. The disadvantage to asynchronous circuits is that the internal delays of flip-flops can lead to unwanted intermediate states, since the flip-flops do not all change at the same time.
Section 7.6
7.6 The circuit is shown in Figure 7.94.

Section 7.7
7.7 The parameter is called propagation delay. For the specified output transition, the symbol is $t_{pu}$. 

FIGURE 7.94
Solution to Section Review Problem 7.6
CHAPTER 8

Introduction to Programmable Logic Architectures

OUTLINE

8.1 Programmable Sum-of-Products Arrays
8.2 PAL Fuse Matrix and Combinational Outputs
8.3 PAL Outputs with Programmable Polarity
8.4 PAL Devices with Programmable Polarity
8.5 Universal PAL and Generic Array Logic
8.6 MAX7000S CPLD
8.7 FLEX10K CPLD

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

• Draw a diagram showing the basic hardware conventions for a sum-of-products-type programmable logic device.
• Describe the structure of a programmable array logic (PAL) AND matrix.
• Draw fuses on the logic diagram of a PAL to implement simple logic functions.
• Describe the structures of combinational, programmable polarity, and registered PAL outputs.
• Determine the number and type of outputs from a PAL/GAL part number.
• Explain the structure of an output logic macrocell (OLMC).
• State differences between Universal PAL and generic array logic (GAL) and standard PAL.
• Interpret the logic diagrams of Universal PAL and GAL devices to determine the number of outputs and product terms and the type of control signals available in a device.
• Interpret block diagrams to determine the basic structure of an Altera MAX7000S CPLD, including macrocell configuration, Logic Array Blocks (LABs), control signals, and product term expanders.
• State the differences between PLDs based on sum-of-products (SOP) architecture versus look-up table (LUT) architecture.
• Interpret block diagrams to determine the basic structure of a logic element in an Altera FLEX10K CPLD, including look-up tables, cascade chains, carry chains, and control signals.
• Interpret block diagrams to determine how a logic element in a FLEX10K device relates to the overall structure of the device.
• Interpret block diagrams to determine how logic array blocks and embedded array blocks relate to the overall structure of a FLEX10K CPLD.

In the past several chapters, we have been using Altera’s MAX+PLUS II software to make circuit designs for downloading into a complex programmable logic device (CPLD). We have treated this device as a black box—something whose function we design, but whose structure we do not really understand. In this chapter, we will look inside the box.
Before we examine the structure of an Altera MAX7000S CPLD, we will look at the internal structure of several simpler devices that are based on similar technologies, such as the PAL16L8 and PAL16R8 low-density PLDs (largely for an historical overview), the PALCE16V8, and the GAL22V10.

These devices are based on programmable matrices of sum-of-products (SOP) circuits, as is the Altera MAX series of devices. The main programming element is the EEPROM (electrically erasable programmable read-only memory) cell. EEPROM-based devices will retain their programmed data when power is removed from the device.

The Altera FLEX series of CPLDs is based on another technology altogether. It stores logic functions in look-up tables (LUTs) that act as truth tables with four input bits. The main logic element of the FLEX series is the SRAM (static random access memory) cell. SRAM-based CPLDs must have their programming data loaded every time they are powered up. They have the advantage of being faster than EEPROM devices, with a higher bit capacity.

8.1 Programmable Sum-of-Products Arrays

**KEY TERMS**

- **Product line** A single line on a logic diagram used to represent all inputs to an AND gate (i.e., one product term) in a PLD sum-of-products array.
- **Input line** A line that applies the true or complement form of an input variable to the AND matrix of a PLD.
- **PAL** Programmable array logic. Programmable logic with a fixed OR matrix and a programmable AND matrix.

The original programmable logic devices (PLDs) consisted of a number of AND and OR gates organized in sum-of-products (SOP) arrays in which connections were made or broken by a matrix of fuse links. An intact fuse allowed a connection to be made; a blown fuse would break a connection.

Figure 8.1a shows a simple fuse matrix connected to a 4-input AND gate. True and complement forms of two variables, \( A \) and \( B \), can be connected to the AND gate in any combination by blowing selected fuses. In Figure 8.1a, fuses for \( A \) and \( B \) are blown. The output of the AND gate represents the product term \( AB \), the logical product of the intact fuse lines.

Figure 8.1b shows a more compact notation for the AND-gate fuse matrix. Rather than showing each AND input individually, a single line, called the **product line**, goes into the AND gate, crossing the true and complement input lines. An intact connection to an input line is shown by an “X” on the junction between the input line and the product line. A symbol convention similar to Figure 8.1b has been developed for programmable logic. Figure 8.2 shows an example.

The circuit shown in Figure 8.2 is a sum-of-products network whose Boolean expression is given by:

\[
F = \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}
\]

The product terms are accumulated by the AND gates as in Figure 8.1b. A buffer having true and complement outputs applies each input variable to the AND matrix, thus producing two input lines. Each product line can be joined to any input line by leaving the corresponding fuse intact at the junction between the input and product lines.

If a product line, such as for the third AND gate, has all its fuses intact, we do not show the fuses on that product line. Instead, this condition is indicated by an “X” through the gate. The output of the third AND gate is a logic 0, since \((\overline{A} \overline{B} \overline{C} \overline{C})\) = 0. This is necessary to enable the OR gate output:

\[
\overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C} + 0 = \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} \overline{C}
\]
Unconnected inputs are HIGH (e.g., \( \overline{A} \cdot 1 \cdot \overline{B} \cdot 1 \cdot C = \overline{A} \overline{B} C \) for the first product line).

If the unused AND output was HIGH, the function \( F \) would be:

\[
A \overline{B} C + A \overline{B} \overline{C} + 1 = 1
\]

The configuration in Figure 8.2, with a programmable AND matrix and a hardwired OR connection, is called PAL (programmable array logic) architecture.\(^1\)

Since any combinational logic function can be written in SOP form, any Boolean function can be programmed into these PLDs by blowing selected fuses. The programming

---

\(^1\)PAL is a registered trademark of Vantis Semiconductor.
is done by special equipment and its associated software. The hardware and software selects each fuse individually and applies a momentary high-current pulse if the fuse is to be blown.

The main problem with fuse-programmable PLDs is that they can be programmed one time only; if there is a mistake in the design and/or programming or if the design is updated, we must program a new PLD. More recent technology has produced several types of erasable PLDs, based not on fuses but on floating-gate metal-oxide-semiconductor transistors. These transistors also form the basis of memory technologies such as electrically erasable programmable read-only memory (EEPROM or E²PROM).

### 8.2 PAL Fuse Matrix and Combinational Outputs

#### **KEY TERMS**

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>JEDEC</td>
<td>Joint Electron Device Engineering Council</td>
</tr>
<tr>
<td>JEDEC file</td>
<td>An industry-standard form of text file indicating which fuses are blown and which are intact in a programmable logic device.</td>
</tr>
<tr>
<td>Text file</td>
<td>An ASCII-coded document stored on disk.</td>
</tr>
<tr>
<td>Checksum</td>
<td>An error-checking code derived from the accumulated sum of the data being checked.</td>
</tr>
<tr>
<td>Cell</td>
<td>A programmable location in a PLD, specified by the intersection of an input line and a product line.</td>
</tr>
<tr>
<td>Product line first cell number</td>
<td>The lowest cell number on a particular product line in a PAL AND matrix where all cells are consecutively numbered.</td>
</tr>
<tr>
<td>Input line number</td>
<td>A number assigned to a true or complement input line in a PAL AND matrix.</td>
</tr>
<tr>
<td>Multiplexer</td>
<td>A circuit that selects one of several signals to be directed to a single output.</td>
</tr>
</tbody>
</table>

Figure 8.3 shows the logic diagram of a PAL16L8 PAL circuit. This device can produce up to eight different sum-of-products expressions, one for each group of AND and OR gates. The device has active-LOW tristate outputs, as indicated by the “L” in the part number. Each is controlled by a product line from the related AND matrix.

The pins that can be used only as inputs or outputs are marked “I” or “O,” respectively. Six of the pins can be used as inputs or outputs and are marked “I/O.” The I/O pins can also feed back a derived Boolean expression into the matrix, where it can be employed as part of another function. A detail of an I/O section is shown in Figure 8.4.

The part number of a PAL device gives the designer information about the number of inputs and outputs and their configurations, as follows:

```
  +----------------+    +----------------+    +----------------+    +----------------+    +----------------+    +----------------+
  | Number of inputs|    | Output type:   |    | Number of (registered) outputs |
  +----------------+    | H = Active HIGH |    +----------------+    +----------------+    | C = Complementary (both HIGH and LOW) |
                   |    | L = Active LOW  |    +----------------+    +----------------+    |                                |
                   |    | P = Programmable polarity |    +----------------+    +----------------+    |                                |
                   |    | R = Registered (D flip-flop) |    +----------------+    +----------------+    |                                |
                   |    | X = XOR registered |    +----------------+    +----------------+    |                                |
                   +----------------+    +----------------+    +----------------+    +----------------+    +----------------+  
        PAL 16 R 8  
```
The numbering system has some potential ambiguities. For example, it is not possible to use 16 inputs and 8 outputs in a PAL16L8 device at the same time; 6 of the inputs are actually input/output pins. Some possible configurations are as follows:

- 16 inputs (10 dedicated + 6 I/O) and 2 dedicated outputs
- 10 dedicated inputs and 8 outputs (2 dedicated + 6 I/O)
- 12 inputs (10 dedicated + 2 I/O) and 6 outputs (2 dedicated + 4 I/O)

Each of the outputs of the PAL16L8 is buffered by a tristate inverter, whose \textit{ENABLE} input is controlled by its own product line. When the \textit{ENABLE} line of the tristate inverter is
HIGH, the inverter output is the same as it would normally be—a logic HIGH or LOW, determined by the state of the corresponding OR gate output.

When the ENABLE line is LOW, the inverter output is in the high-impedance state. The output acts as an open circuit, neither HIGH nor LOW; it is as though the output was completely disconnected from the circuit. The inverter is permanently enabled if all fuses on the ENABLE product line are blown, and permanently disabled if these fuses are all intact.

Published logic diagrams of PAL devices generally do not have fuses drawn on them. This allows us to draw fuses for any application. In practice, PLDs have become too complex to manually draw fuse maps for most applications.

Historically, PLD programming would begin with fuses drawn on a logic diagram, and each fuse would be selected and blown individually by someone operating a hardware device constructed for such a purpose.

Fuse assignment is now done with special software such as ABEL, CUPL, or PALASM. These programs will take inputs such as Boolean equations, truth tables, or other forms and produce the simplest SOP solution to the particular problem. (MAX+PLUS II is not configured to generate programming data for low-density PALs, although it can generate data for similar devices in the Altera Classic PLD series.)

The end result of such software is a JEDEC file, an industry-standard way of listing which fuses in the PLD should remain intact and which should be blown. The JEDEC file is stored on disk as an ASCII text file. Most PLD programmers will accept the JEDEC file and use it as a template for blowing fuses in the target device.

Fuse locations, called cells, are specified by two numbers: the product line first cell number, shown along the left side of the diagram, and the input line number, shown along the top. The address of any particular fuse is the sum of its product line first cell number and its input line number. The fuses on the PAL16L8 device are numbered from 0000 to 2047 ( = 2016 + 31).

Figure 8.5 shows an example of a JEDEC file for a PAL16L8 application. The file starts with an ASCII “Start Text” character (^B). Next is some information required by the PAL programmer about the type of device (PAL16L8), number of fuses (2048), and so forth. The fuse information starts with the line L0000, which is the first product line. The 1s and 0s which follow show the programmed state of each cell in each product line; a 1 is a blown fuse and a 0 is an intact fuse. In other words, each 0 in the JEDEC file represents an X in the same position on the PAL logic diagram.

The product terms for first sum-of-products output are set by the states of fuses 0000 to 0255 (eight product lines). In the file shown, all fuses are blown in the first product
line, the second product line shows three intact fuses, and so forth. Since all fuses are intact in the last three lines, they need not be shown in the JEDEC file.

Whenever some unprogrammed product lines are omitted from the fuse map, the last fuse line shown ends with an asterisk (*). The next line with programmed fuses is indicated by a new fuse number. For example, the second group of fuses (0256 to 0511) in Figure 8.5 begins after the line marked L0256 in the JEDEC file. The remaining fuse lines are similarly indicated.

The JEDEC file in Figure 8.5 ends with a hexadecimal checksum (C8DCF), an error-checking code derived from the programming data, and an ASCII “End Text” code (^C).
In order to examine the general principle of fuse programming, let us develop the programmed logic diagram for a common combinational circuit: a 4-to-1 multiplexer. (After developing the fuse maps for several examples, we will not refer to this technique again.)

This circuit, shown in Figure 8.6a, directs one of four input logic signals, \( D_0 \) to \( D_3 \), to output \( Y \), depending on the state of two select inputs \( S_0 \) and \( S_1 \). The circuit works on the enable/inhibit principle; each AND gate is enabled by a different combination of \( S_1 \) \( S_0 \). The binary state of the select inputs is the same as the decimal subscript of the selected data input. For instance, \( S_1 \ S_0 = 10 \) selects data input \( D_2 \); the AND gate corresponding to \( D_2 \) is enabled and the other three ANDs are inhibited.

The logic equation for output \( Y \) is given by:

\[
Y = D_0 \overline{S_1} \overline{S_0} + D_1 \overline{S_1} S_0 + D_2 S_1 \overline{S_0} + D_3 S_1 S_0
\]

Since the outputs of the PAL16L8 are active LOW, as illustrated in Figure 8.6b, we should rewrite the equation as follows:

\[
\overline{Y} = D_0 S_1 S_0 + D_1 S_1 S_0 + D_2 \overline{S_1} S_0 + D_3 S_1 S_0
\]

The \( D \) inputs must be complemented to reverse the effect of the active-LOW output. The output is enabled when the \( EN \) input is HIGH. Figure 8.7 shows the PAL16L8A logic diagram with fuses for the multiplexer application.

### 8.3 PAL Outputs With Programmable Polarity

The multiplexer application developed in the previous section uses a PAL device whose output is always fixed at the active-LOW polarity. This fixed polarity is suitable for most applications, but Boolean functions that would normally have active-HIGH outputs must be implemented in DeMorgan equivalent form, which is not always very straightforward.

Some applications require both active-HIGH and active-LOW outputs. In such cases, it is useful to have a device whose output polarity is fuse programmable.

Figure 8.8 shows the logic diagram of a PAL20P8 PAL device. This device is the same as a PAL16L8, except that there are four more dedicated inputs, and the polarity of each output is programmable. The Exclusive OR gate on each output is programmed to act as either an inverter or a buffer. When its associated fuse is intact, the XOR input is grounded and passes the output of its related SOP network in true form. When combined with the output inverter, this produces an active-LOW output. When the polarity fuse is blown, the fused XOR input floats to the HIGH state, inverting the SOP output; the output pin becomes active HIGH.
8.3 • PAL Outputs With Programmable Polarity

The polarity fuses are given numbers higher than those of the main fuse array. In this case, the product line fuses are numbered 0000 to 2559 and the output polarity fuses are numbered 2560 to 2567.

Figure 8.9 illustrates the selection of output polarity. Two Boolean functions, $F1$ and $F2$, are programmed into the fuse array, with outputs at pins (17) and (15), respectively. The equations are:

$$F1 = A \overline{B} + \overline{A} \overline{B}$$
$$F2 = A \overline{B} + A \overline{B}$$
We could, if we chose, rewrite $F_2$ to show the output as active LOW:

$$\overline{F_2} = A \overline{B} + \overline{A} B$$

The portion of the PAL20P8 logic diagram shown in Figure 8.9 represents the fuses required to program $F_1$ and $F_2$. Pins (14) and (16) supply inputs $A$ and $B$ to the matrix. The $ENABLE$ lines of the tristate output buffers float HIGH, since all fuses are blown on the corresponding product lines, thus permanently enabling the output buffers.

The fuses numbered 2565 and 2567 select the polarity at pins (15) and (17). Fuse 2565 is blown. The fused input to the corresponding XOR gate floats HIGH, thus making the gate into an inverter. Combined with the tristate buffer, this makes pin (17) active HIGH.

Fuse 2567 is intact. This grounds the input to the corresponding XOR gate, making the gate into a noninverting buffer. Combined with the tristate output buffer, this makes pin (15) active LOW.
EXAMPLE 8.1

Show how a PAL20P8 device can be used to implement the following logic functions by drawing fuses on the device’s logic diagram.

<table>
<thead>
<tr>
<th>Function</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>NOT:</td>
<td>$F_1 = \overline{A}$</td>
</tr>
<tr>
<td>AND:</td>
<td>$F_2 = BC$</td>
</tr>
<tr>
<td>OR:</td>
<td>$F_3 = D + E$</td>
</tr>
<tr>
<td>NAND:</td>
<td>$F_4 = \overline{FG}$</td>
</tr>
<tr>
<td>NOR:</td>
<td>$F_5 = \overline{H + J}$</td>
</tr>
<tr>
<td>XOR:</td>
<td>$F_6 = K \oplus L = \overline{K}L + KL$</td>
</tr>
<tr>
<td>XNOR:</td>
<td>$F_7 = M \oplus N = M\overline{N} + MN$</td>
</tr>
</tbody>
</table>

How would the implementation of these logic functions differ if only active-LOW outputs were available, as in a PAL16L8?

SOLUTION

The PAL20P8 has 14 dedicated inputs, 2 dedicated outputs, and 6 lines that can be used as inputs or outputs. Our functions need 13 input variables and 7 output variables. We will use six I/O pins (pins (16) through (21)) and one dedicated output (pin (15)) for the output variables.
All functions must be in SOP form. Outputs for NOT, AND, OR, Exclusive OR, and Exclusive NOR are active HIGH. Therefore, polarity fuses on the outputs for F1, F2, F3, F6, and F7 are blown. NAND and NOR outputs are active LOW; the polarity fuses for F4 and F5 remain intact.

Figure 8.10 shows the logic diagram of the programmed PAL. If only active-LOW outputs were available, we would need to rewrite some of the equations to make the outputs correspond to their DeMorgan equivalent forms, as follows:

**AND:** \( F_2 = \overline{B + C} \)

**OR:** \( F_3 = \overline{D \cdot E} \)

**XOR:** \( F_6 = K \oplus L = \overline{K \cdot L} + \overline{K} \cdot L \)

**XNOR:** \( F_7 = M \oplus \overline{N} = MN + MN \)

**FIGURE 8.10**
Programmed Logic Diagram for Seven Logic Functions
Flip-flops are generally found in programmable logic devices as registered outputs. A register is one or more flip-flops used to store data. Registered outputs in programmable array logic (PAL) devices can be used for the same functions as individual flip-flops.

Figure 8.11 shows the logic diagram of a PAL device with eight registered outputs: a PAL16R8. The fuse matrix is identical to that of a PAL16L8 device; the differences
between the two devices are the registered outputs, a dedicated clock input (pin 1), and a pin for enabling all registered outputs (pin 11).

With Registered PAL, the number of outputs shown in the part number indicates the number of registered outputs. For example, a PAL16R4 device has four registered outputs and four combinational I/O pins, a PAL16R6 device has six registered outputs and two combinational I/O pins, and a PAL16R8 has eight registered outputs.

EXAMPLE 8.2

A common data operation is that of “rotation.” Figure 8.12 illustrates how a 4-bit number can be rotated to the right by 0, 1, 2, or 3 places by a circuit called a “barrel shifter.” To rotate the data, move all bits the required number of places to the right. As data reach the rightmost position, move them to the beginning so that they are transferred in a closed loop.

This operation is usually performed by serially shifting the data the required number of places and feeding back the last output to the first input of a serial shift register.

Rotation can also be accomplished by a parallel transfer operation. We can load the bits of the input into four D flip-flops in the order determined by two select inputs, \( S_1 \) and \( S_0 \). Assume that the binary number \( S_1 S_0 \) is the same as the rotation number in Figure 8.12. Table 8.1 summarizes the contents of the circuit after one clock pulse is applied.

<table>
<thead>
<tr>
<th>( S_1 )</th>
<th>( S_0 )</th>
<th>( Q_A )</th>
<th>( Q_B )</th>
<th>( Q_C )</th>
<th>( Q_D )</th>
<th>Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( A )</td>
<td>( B )</td>
<td>( C )</td>
<td>( D )</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>( D )</td>
<td>( A )</td>
<td>( B )</td>
<td>( C )</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>( C )</td>
<td>( D )</td>
<td>( A )</td>
<td>( B )</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( B )</td>
<td>( C )</td>
<td>( D )</td>
<td>( A )</td>
<td>3</td>
</tr>
</tbody>
</table>

Sketch a circuit, using gates and flip-flops, that can accomplish this rotation as a parallel transfer function. Briefly explain its operation.

Write the Boolean expression(s) for the circuit.

Show how the circuit can be implemented by a PAL16R4 device by drawing fuses on its logic diagram.

SOLUTION Figure 8.13 shows a parallel transfer circuit (barrel shifter) that will perform the specified rotation. The circuit works by enabling one AND gate in each group of four for each combination of \( S_1 \) and \( S_0 \). For example, when \( S_1 S_0 = 00 \), the rotation is 0 and the leftmost AND gate of each group is enabled, transferring the parallel data into the flip-flops so that \( D_A = A \), \( D_B = B \), \( D_C = C \), and \( D_D = D \). After one clock pulse, \( Q_A \), \( Q_B \), \( Q_C \), \( Q_D \) = \( ABCD \).
Similarly, if $S_1 S_0 = 10$, we select a rotation of 2. The third AND gate from the left is selected in each group of four. This makes the data $D_A = C$, $D_B = D$, $D_C = A$, and $D_D = B$ appear at the flip-flop inputs. After one clock pulse, $Q_A Q_B Q_C Q_D = CDAB$.

The same principle governs the circuit operation for the other two select codes. The Boolean equations for the circuit are:

$$Q_A = S_1 S_0 A + S_1 S_0 D + S_1 S_0 C + S_1 S_0 B$$
$$Q_B = S_1 S_0 B + S_1 S_0 A + S_1 S_0 D + S_1 S_0 C$$
$$Q_C = S_1 S_0 C + S_1 S_0 B + S_1 S_0 A + S_1 S_0 D$$
$$Q_D = S_1 S_0 D + S_1 S_0 C + S_1 S_0 B + S_1 S_0 A$$
These equations imply that each registered output requires us to use four product lines, one for each product term. The programmed logic diagram is shown in Figure 8.14.
There are several limitations of standard low-density PALs. First, these devices are one-time programmable (OTP). Since the AND matrix of a PAL is programmable by blowing metal fuse links, programming is permanent; there is no opportunity to correct or update a design. In development of a new design, where many modifications must be made to the original design, this can be particularly wasteful. Second, standard PAL outputs are permanently configured either as combinational or registered. A given PAL has a certain number of each type of output, which may not be optimum for the design. Third, a standard PAL cannot be programmed while it is installed in a circuit.

A number of low-density PLDs have been developed to address these concerns. Devices such as the PALCE16V8 Universal PAL (Vantis Corporation), and the GAL16V8 and GAL22V10 Generic Array Logic (Lattice Semiconductor)* are based on sum-of-products fuse matrices, just as the earlier-version PALs. However, these devices are based on electrically erasable read only memory (EEPROM or E²PROM) cells, rather than fuses, which allow them to be erased and reprogrammed about 10,000 times. A programmed device will hold its data for about 20 years.

Universal PALs and GALs also have programmable input/output configurations. An I/O pin can be configured as a registered output, a combinational output, or a dedicated input, as required. Additionally, an output can be specified as active-HIGH or active-LOW.

---

*Vantis has recently been acquired by Lattice, so these devices are really produced by the same company
Devices such as the ispGAL22V10 or the Altera MAX7000S series can be programmed while installed in a circuit via a standard four-wire interface called a JTAG port. This property is known as **in-system programmability (ISP)**.

### PALCE16V8

Figure 8.15 shows one I/O pin and its associated circuitry for a PALCE16V8 Universal PAL. (The “V” stands for “variable” or “versatile” architecture.) It consists of a programmable SOP array with 8 product terms and an **output logic macrocell (OLMC)**, or just “macrocell”, which determines the I/O configuration for that pin. The various configuration options are selected by a network of four multiplexers that are programmed by a set of **architecture cells** that set the MUX select inputs HIGH or LOW.

![Output Logic Macrocell for a PALCE16V8 PLD](image)

A **global architecture cell**, SG1, selects configuration options for all macrocells in the device. Two **local configuration cells**, SL0_n and SL1_n, select configurations for I/O_n only. (In this case, the cells shown are SL0_3 and SL1_3 for configuration of I/O_3.)

Figure 8.16 shows the different macrocell configurations for a PALCE16V8 Universal PAL. Most of these configurations are designed to emulate an I/O of a standard PAL, so that an old-style PAL can be replaced by a Universal PAL, and can be programmed by data for the older PAL. The macrocells can also be configured in a pattern that does not conform to an older device.

Figure 8.17 shows the logic diagram of a PALCE16V8 Universal PAL. The device has eight dedicated inputs, eight macrocells, a Clock pin and an Output Enable pin. The latter two signals are shown in the macrocell diagram of Figure 8.15 as the lines labeled **CLK** and **OE**.

If there are registered outputs, the clock input (pin 1) provides a **global clock** function. That is, all registered outputs are clocked simultaneously by this signal. (Some other PLDs provide an option to clock a registered output from a product term in the AND matrix, allowing several clock functions in one chip.) If there are no registered outputs used in the PLD, pin 1 can be used as an input.
Pin 11 provides an active-LOW Output Enable function. This is selected by local architecture cells to provide control of the output tristate buffer, either from the OE pin or from a product term in the AND matrix. If the OE function is unused, the pin can be used as an input.

**GAL22V10**

Figure 8.18 shows the logic diagram of a GAL22V10 generic array logic device. This industry-standard device has a number of features that make it superior to the PALCE16V8.
FIGURE 8.17 (a)
PALCE16V8 Logic Diagram (Courtesy of Lattice Semiconductor Corporation)
FIGURE 8.17 (b)
(PALCE16V8 Logic Diagram)
1. There are more outputs (10 as opposed to 8 for the 16V8).
2. There are more inputs (11 dedicated inputs, plus any I/O lines used as inputs).
3. The output logic macrocells are of different sizes, allowing expressions with larger numbers of product terms in some OLMCs than others. There are two OLMCs with each of the following numbers of product lines: 8, 10, 12, 14, and 16. This allows more flexibility in design, while minimizing the number of product lines.
4. OLMC configuration is much simpler than that of a PALCE16V8. Two architecture cells per macrocell, \( S_0 \) and \( S_1 \), select the output type, as shown in Figure 8.19.

5. There are product lines for Synchronous Preset (SP) and Asynchronous Reset (AR). The SP line sets all flip-flops HIGH on the first clock pulse after it becomes active. The AR line sets all flip-flops LOW as soon as it activates, without waiting for the clock pulse. (Note that these lines set or reset the \( Q \) output of each flip-flop. An active-LOW registered output inverts this state at the output pin.)

8.6 MAX7000S CPLD

KEY TERMS

CPLD Complex programmable logic device. A programmable logic device consisting of several interconnected programmable blocks.

Logic Array Block (LAB) A group of macrocells that share common resources in a CPLD.

Programmable Interconnect Array (PIA) An internal bus with programmable connections that link together the Logic Array Blocks of a CPLD.
Buried logic Logic circuitry in a PLD that has no connection to the input or output pins of the PLD, but is used solely as internal logic.

I/O Control Block A circuit in an Altera CPLD that controls the type of tristate switching used in a macrocell output.

Parallel logic expanders Product terms that are borrowed from neighboring macrocells in the same LAB.

Shared logic expanders Product terms that are inverted and fed back into the programmable AND matrix of an LAB for use by any other macrocell in the LAB.

Figure 8.20 shows the block diagram of an Altera MAX7000S Complex PLD (CPLD). A device of this type—the EPM7128SLC84—is one of the two devices installed on the Altera UP-1 University Program board, so we will use it as a specific example of the MAX7000S family of devices.

The part number breaks up as follows:

- EPM7 MAX7000 family
- 128 number of macrocells
- S in-system programmable
- LC84 84-pin PLCC package
The main structure of the MAX7000S is a series of Logic Array Blocks (LABs),
linked by a Programmable Interconnect Array (PIA). Each LAB is a group of
16 macrocells that can share common product terms and lend or borrow unused product
terms among each other. A single LAB has similar I/O and programming capability to a
low-density PLD, so a CPLD like the MAX7000S can be thought of as an array of inter-
connected PALs or GALs on a single chip.

An EPM7128S has 8 LABs, for a total of $8 \times 16 = 128$ macrocells. However, these
are not all available to the user as I/Os; the number of available I/O pins depends on the de-
vice package. Figure 8.20 indicates that each LAB in a MAX7000S device has from 6 to
16 I/O pins. For an EPM7128S in a 160-pin PQFP package, there are 12 I/Os per LAB, for
a total of 96 available pins. For the same device in an 84-pin PLCC package, there are only
8 I/Os per LAB, for a total of 64 pins.

In practice, if an EPM7128SLC84 is to be programmed in-circuit (i.e., while installed
on a circuit board), there are only 60 I/Os available, as four pins are required for the pro-
gramming interface. The macrocells that are not connected to user I/O pins can only be
used for buried logic, or logic that is internal to the chip only.

As implied in Figure 8.20, all I/O pins connect to and from their associated LAB via
an I/O Control Block (a circuit that controls the tristate switching of signals at an I/O pin).
The I/O pin signals also connect directly to the PIA, where they are available for use in
other LABs. Sixteen lines connect the macrocell outputs of each LAB to the PIA, again for
use throughout the device. The PIA communicates to each LAB via 36 product lines to
provide connections from other LABs.

The MAX7000S family has four pins that can be configured as control signals or
inputs. $GCLK1$ is a global clock that is common to all macrocells in the device and can
be used to synchronously clock all registers. $OE1$ is an output enable that can globally
activate or disable the tristate outputs of the device macrocells. $GCLRn$ is an active-
LOW global clear function. The fourth control pin can be configured as an input, as can
the other three pins, or as a second global clock ($GCLK2$) or output enable ($OE2$). If
the control functions are not used, these pins add four inputs to the available total.
These assignments can be made by the MAX+PLUS II software during the design
process.

Figure 8.21 shows a macrocell from a MAX7000S device. The macrocell is similar to
that of a GAL or Universal PAL in that it provides a sum-of-products function with active-
HIGH or -LOW options and the choice of registered or combinational output. Registered outputs can be clocked with one of two global clocks or by a product term from the AND matrix.
The register can be cleared globally or by a product term and preset with a product term.

The macrocell has five dedicated product terms, which is fewer than found in the PAL
and GAL matrices we examined earlier. This is generally sufficient to implement most
logic functions. If more terms are required, they can be supplied by a set of shared logic
expanders or parallel logic expanders.

Shared logic expanders do not add more product terms to a given macrocell. They
do make the programming of the entire LAB more efficient by allowing a product term
to be programmed once and used in several macrocells of the same LAB. One product
term per macrocell is inverted and fed back into the shared expander pool of product
terms. Since there are 16 macrocells per LAB, the shared logic expander pool has up to
16 product terms.

Parallel logic expanders allow a macrocell to borrow up to 15 product terms from its
three lower-numbered neighbors (5 product terms per neighboring macrocell). For exam-
ple, macrocell 4 can borrow up to 5 terms each from macrocells 3, 2, and 1. By using its 5
dedicated product terms and the maximum number of parallel expanders, a macrocell can
have up to 20 product terms at its disposal. These borrowed terms are not usable by the
macrocell from which they were borrowed. The parallel expanders are set up so that a
lower-number cell lends product terms to a higher-number cell, so the number of available
terms depends on how close to the end of a chain a macrocell is. Expander assignments are
done automatically by MAX+PLUS II at compile time.
8.7 FLEX10K CPLD

**KEY TERMS**

- **Look-up table (LUT)**: A circuit that implements a combinational logic function by storing a list of output values that correspond to all possible input combinations.
- **Logic element (LE)**: A circuit internal to a CPLD used to implement a logic function as a look-up table.
- **Cascade chain**: A circuit in a CPLD that allows the input width of a Boolean function to expand beyond the width of one logic element.
- **Carry chain**: A circuit in a CPLD that is optimized for efficient operation of carry functions between logic elements.
- **Embedded array block (EAB)**: A relatively large block of storage elements in a CPLD (2048 bits in a FLEX10K device), used for implementing complex logic functions in look-up table format.

All programmable logic devices we have seen until now have been based on sum-of-products arrays. Another major type of PLD is based on look-up table (LUT) architecture. In this architecture, a number of storage elements are used to synthesize logic functions by storing each function as a truth table. To illustrate the look-up table concept, let us use the truth table of a 2-bit equality comparator, shown in Table 8.2.

The comparator examines inputs $A_1A_0$ and $B_1B_0$ and makes output $AEQB$ equal to logic 1 if $A_1A_0 = B_1B_0$. If we were to implement the circuit as an SOP array, we would first find the Boolean expression by combining the four product terms from the truth table and then program the appropriate cells in a CPLD AND matrix. The look-up table implementation of this function is based on a totally different concept.
Table 8.2  Truth Table for a 2-bit Equality Comparator

<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$A_0$</th>
<th>$B_1$</th>
<th>$B_0$</th>
<th>Decimal</th>
<th>AEQB</th>
</tr>
</thead>
<tbody>
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<td>0</td>
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<td>1</td>
<td>1</td>
<td>15</td>
<td>1</td>
</tr>
</tbody>
</table>

**FIGURE 8.22**  Look-up Table

Figure 8.22 shows the structural concept of a 4-bit look-up table circuit. An array of 16 flip-flops ($Q_0$ through $Q_{15}$) contain data for all possible combinations of $A_1A_0B_1B_0$, one flip-flop per combination. The LUT inputs $A_1A_0B_1B_0$ are decoded by an internal **address decoder**. Each decoder output activates a tristate buffer that passes or blocks the output of one flip-flop. The active buffer passes the contents of the flip-flop to $AEQB$; all other buffers are in the high-impedance state, blocking the data from the other flip-flops.

The contents of the flip-flops are loaded when the look-up table is configured (programmed) with the required function. After that the flip-flops retain their information until they are reconfigured. For our comparator example, flip-flops 0, 5, 10, and 15 are all set ($Q = 1$). All other flip-flops are reset ($Q = 0$). Examine Table 8.2 to confirm that this is true.

The 16-bit storage element in Figure 8.22, combined with switching to choose a combinational or registered output and to interconnect with other parts of the chip, is called a **logic element (LE)**. A logic element performs a function similar to that of a macrocell in SOP-type PLDs.

Figure 8.23 shows the structure of a logic element in an Altera FLEX10K CPLD. In addition to the LUT, the LE has circuitry to select various control functions, such as clock and reset, a flip-flop for registered output, some expansion circuitry (cascade and carry), and interconnections to local and global busses.

The **cascade chain** circuit, shown in Figure 8.24 allows the user to program Boolean functions with more than four inputs, thus requiring more than one LUT. The
FIGURE 8.23
FLEX10K Logic Element (Courtesy of Altera)

FIGURE 8.24
Cascade Chain Operation (Courtesy of Altera)
cascade chain can be AND- or OR-type, depending on what DeMorgan equivalent form is most appropriate.

The carry chain, shown in Figure 8.25 allows for efficient fast-carry implementation of adders, comparators, and other circuits that depend on the combination of low-order bits to define high-order functions (i.e., circuits whose inputs become wider with higher-order bits). Figure 8.25 shows the carry chain as implemented by an n-bit adder.

A Logic Array Block (LAB), shown in Figure 8.26, consists of eight logic elements and a local interconnect. The LAB is connected to the rest of the device by a series of row and column interconnects, which Altera calls a FastTrack Interconnect. Figure 8.27 shows the overall structure of a FLEX10K device, with several LABs and a number of
Embedded Array Blocks (EABs). An EAB is an array of 2048 storage elements that can be used to efficiently implement complex logic functions.

The FLEX10K device found on the Altera UP-1 board—the EPF10K20RC240-4—has an array of 6 rows by 24 columns of LABs, which gives a total of 144 LABs (6 × 144 = 1152 logic elements). The device also has 6 EABs (6 × 2048 = 12288 bits of EAB storage). Note that one EAB has significantly more storage capacity than all LABs combined.

The FLEX10K series of CPLDs (and LUT-based devices generally) are based on static random access memory (SRAM) technology. The advantage of this configuration is that it can be manufactured with a very high density of storage cells and it programs quickly compared to an EEPROM-based SOP device. The disadvantage is that SRAM cells are volatile; that is, they do not retain their data when power is removed from the circuit. An SRAM-based device must be reconfigured every time it is powered up.

1. Programmable logic devices (PLDs) are configured in two basic architectures: sum-of-products (SOP), which usually consist of a se-
Summary

2. Programmable array logic (PAL) is an SOP-type architecture in which there are a series of programmable AND gates that have a fixed connection to an OR-gate output.

3. Connections from PLD inputs to PAL AND arrays were historically made by leaving intact selected fuses in a crosspoint fuse array. In modern PLDs, these connections are made by programming EEPROM (electrically erasable programmable read only memory) cells.

4. An AND-gate input in a PAL array is called a product line.

5. A PAL16L8 PLD is an SOP device with up to 16 inputs and up to 8 outputs. There are 10 dedicated inputs, 2 dedicated outputs, and 6 pins that can be configured as input or output. All outputs in the PAL16L8 are active-LOW.

6. A PAL is programmed by a computer and programming hardware that uses a JEDEC file as a template for determining which fuses to blow and which to leave intact.

7. Some PAL devices have programmable-polarity outputs. This is achieved with an XOR gate that has a programmable cell or fuse on one input to switch the output between inverting and noninverting levels.

8. A registered PLD output consists of a flip-flop (usually D-type) on the output of an SOP matrix.

9. A PAL part number indicates the number of registered outputs (e.g., a PAL16R8 has eight registered outputs).

10. Early-version standard PALs are limited in that they are one-time programmable (OTP), their outputs are permanently configured as combinational or registered, and they cannot be programmed in-system. Later-version PALs (e.g., PAL16CE16V8 Universal PAL) and GALs (generic array logic such as GAL22V10) overcome these limitations.

11. PALs and GALs with configurable architecture have outputs that can be combinational or registered, with various input or
feedback options.
12. Configurable output circuits in a PLD are called output logic
macrocells (OLMCs) or just macrocells.
13. Macrocells are configured by programming architecture
cells. Global architecture cells affect all macrocells in a de-
vice. A local architecture cell affects only the macrocell in
which it is found.
14. GALs and Universal PALs have global control signals, such
as clock, clear, and output enable, that can be applied to all
macrocells in the device.
15. A GAL22V10 has ten macrocells, a global clock that can be
used as a combinational input for nonclocked designs, and
eleven dedicated inputs.
16. The GAL22V10 macrocells are not all the same size. There
are two macrocells with each of the following numbers of
product terms: 8, 10, 12, 14, 16.
17. PLDs that can be programmed while installed in a circuit are
called in-system programmable (ISP). They are programmed
by a 4-wire interface that complies to a standard published
by the Joint Test Action Group (JTAG) and the IEEE (Std.
1149.1).
18. An Altera MAX7000S CPLD consists of groups of 16
macrocells, called Logic Array Blocks (LABs), that are
interconnected by an internal bus called a Programmable
Interconnect Array (PIA).
19. The number of macrocell outputs in an LAB that are con-
ected to I/O pins depends on the CPLD package type.
Macrocells that do not have external connections can still be
used for buried logic function.
20. MAX7000S devices have four programmable control pins:
global clock (GCLK1), Global Output Enable (OE1), Global
Clear (GCLRn), and a pin that can be configured as a second
global clock (GCLK2) or as a second global output enable
(OE2). If these functions are not used, the associated pins
can be used as standard I/Os.
21. If the ISP capability of a CPLD is to be used, there are four
fewer pins available on the CPLD for user I/O.

G L O S S A R Y

Architecture cell  A programmable cell that, in combination
with other architecture cells, sets the configuration of a macrocell.
Buried logic  Logic circuitry in a PLD that has no connection

Carry chain  A circuit in a CPLD that is optimized for efficient
operation of carry functions between logic elements.
Cascade chain  A circuit in a CPLD that allows the input

Cell  A fuse location in a programmable logic device, specified
by the intersection of an input line and a product line.
Checksum  An error-checking code derived from the accumulat-
ing sum of the data being checked.
CPLD  Complex programmable logic device. A programmable
logic device consisting of several interconnected programmable
blocks.

Embedded array block (EAB)  A relatively large block of stor-
age elements in a CPLD (2048 bits in a FLEX10K device), used
for implementing complex logic functions in look-up table format.
Generic array logic (GAL)  A type of programmable logic de-
vice whose outputs can be configured as combinational or regis-
tered and whose programming matrix is based on electrically
erasable logic cells.
Global architecture cell  An architecture cell that affects the
configuration of all macrocells in a device.
Global clock  A clock signal in a PLD that clocks all registered
outputs in the device.
I/O Control Block  A circuit in an Altera CPLD that controls
the type of tristate switching used in a macrocell output.
Input line  A line which applies the true or complement form
of an input variable to the AND matrix of a PLD.
Input line number  A number assigned to a true or comple-
ment input line in a PAL AND matrix.
In-system programmability (ISP)  The ability of a PLD to be programmed through a standard four-wire interface while installed in a circuit.

JEDEC  Joint Electron Device Engineering Council

JEDEC file  An industry standard form of text file indicating which fuses are blown and which are intact in a programmable logic device.

JTAG Port  A four-wire interface specified by the Joint Test Action Group (JTAG) used for loading test data or programming data into a PLD installed in a circuit.

Local architecture cell  An architecture cell that affects the configuration of one macrocell only.

Logic Array Block (LAB)  A group of macrocells that share common resources in a CPLD.

Logic element (LE)  A circuit internal to a CPLD used to implement a logic function as a look-up table.

Look-up table (LUT)  A circuit that implements a combinational logic function by storing a list of output values that correspond to all possible input combinations.

Multiplexer  A circuit which selects one of several signals to be directed to a single output.

One-time programmable (OTP)  A property of some PLDs that allows them to be programmed, but not erased.

Output logic macrocell (OLMC)  An input/output circuit that can be programmed for a variety of input or output configurations, such as active HIGH or active LOW, combinational, or registered. Often just called a macrocell.

PAL  Programmable array logic. Programmable logic with a fixed OR matrix and a programmable AND matrix.

Parallel logic expanders  Product terms that are borrowed from neighboring macrocells in the same LAB.

Product line  A single line on a logic diagram used to represent all inputs to an AND gate (i.e., one product term) in a PLD sum-of-products array.

Product line first cell number  The lowest cell number on a particular product line in a PAL AND matrix where all cells are consecutively numbered.

Programmable Interconnect Array (PIA)  An internal bus with programmable connections that link together the Logic Array Blocks of a CPLD.

Programmable logic device (PLD)  A logic device whose function can be programmed by the user, usually in sum-of-products form.

Register  A digital circuit such as a flip-flop that stores one or more bits of digital information.

Registered output  An output of a programmable array logic (PAL) device having a flip-flop (usually D-type) which stores the output state.

Shared logic expanders  Product terms that are inverted and fed back into the programmable AND matrix of an LAB for use by any other macrocell in the LAB.

Text file  An ASCII-coded document stored on a magnetic disk.

Universal PAL  A PLD based on erasable cells and configurable outputs, much like GAL, but primarily designed to emulate PAL devices, such as PAL16L8.

PROBLEMS

Problem numbers set in color indicate more difficult problems; those with underlines indicate most difficult problems.

Section 8.1  Introduction to Programmable Logic

Section 8.2  PAL Fuse Matrix and Combinational Outputs

Section 8.3  PAL Outputs With Programmable Polarity

8.1 Draw a diagram showing the basic configuration and symbology for a PLD sum-of-products array.

8.2 Draw a basic PAL circuit having four inputs, eight product terms, and one active-LOW combinational output. Draw fuses on your diagram showing how to make the following Boolean expression:

\[ f = \overline{A}B\overline{C} + \overline{B}CD + \overline{A}CD + AD \]

8.3 Modify the PAL circuit drawn in Problem 8.2 to make two outputs having eight product terms and programmable polarity. Draw fuses on the diagram for each of the following functions:

\[ f_1 = AB\overline{C} + CD + \overline{A}CD + AD \]

\[ f_2 = AB\overline{C} + \overline{B}CD + \overline{A}CD + AD \]

8.4 Make a photocopy of Figure 8.8 (PAL20P8 logic diagram). Draw fuses on the PAL20P8 logic diagram showing how to make a BCD-to-2421 code converter, as developed in Example 3.22.

Table 8.3 shows how the two codes relate to each other. The equations are listed on page 362.

<table>
<thead>
<tr>
<th>Decimal Equivalent</th>
<th>BCD Code</th>
<th>2421 Code</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D_4</td>
<td>D_3</td>
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<td>0</td>
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<td>9</td>
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</tr>
</tbody>
</table>
The Boolean equations for the BCD-to-2421 decoder are:

\[ Y_4 = D_4 + D_3D_2 + D_3D_1 \]
\[ Y_3 = D_4 + D_2D_2 + D_3\bar{D}_1 \]
\[ Y_2 = D_4 + \bar{D}_3D_2 + D_3\bar{D}_2D_1 \]
\[ Y_1 = D_1 \]

Repeat Problem 8.4 for a 2421-to-BCD code converter.

What is a registered output?

State the number of registered outputs for each of the following PAL devices:
- a. PAL16R4
- b. PAL16R6
- c. PAL16R8

Name two features of a PALCE16V8 that make it superior to a PAL16L8.

State the difference between a global architecture cell and a local architecture cell in a PALCE16V8.

How many macrocells are there in a GAL22V10? How many product lines do these macrocells have?

State the four configurations possible with a macrocell in a GAL22V10.

Is there a global output enable function available for a PALCE16V8? For a GAL22V10?

Can the registered outputs of a PALCE16V8 be clocked by a product term function from the PAL AND matrix?

Can the registered outputs of a GAL22V10 be clocked by a product term function from the GAL AND matrix?

Are the Asynchronous Reset (AR) and Synchronous Preset (SP) functions in a GAL22V10 global or local? Explain your answer in one sentence.

How many macrocells are available in the following CPLDs:
- a. EPM7032
- b. EPM7064
- c. EPM7128S
- d. EPM7160S

Which of the CPLDs listed in Problem 8.17 are in-system programmable? What does it mean when a device is in-system programmable?

How many logic array blocks (LABs) are there in an Altera MAX7000S CPLD?

How many user I/O pins are there in an EPM7128SLC84 CPLD? How many pins per LAB does this represent?

What can be done with the macrocells in an LAB that do not connect to I/O pins?

State the possible clock configurations of a MAX7000S macrocell.

State the possible reset configurations of a MAX7000S macrocell.

State the possible preset configurations of a MAX7000S macrocell.

How many dedicated product terms are available in a MAX7000S macrocell? How can this number of product terms be supplemented? What is the maximum number of product terms available to a macrocell?

How many shared logic expanders are available in an LAB?

Briefly state the difference between CPLDs having sum-of-products architecture and look-up table architecture.

How many inputs can a look-up table accept in an Altera FLEX10K logic element? How can this be expanded?

What is the purpose of the carry chain in a FLEX10K CPLD?

How many logic elements are there in a FLEX10K LAB?

How many bits of storage are there in an Embedded Array Block in a FLEX10K CPLD?
CHAPTER 9

Counters and Shift Registers

OUTLINE

9.1 Basic Concepts of Digital Counters
9.2 Synchronous Counters
9.3 Design of Synchronous Counters
9.4 Programming Binary Counters in VHDL
9.5 Control Options for Synchronous Counters
9.6 Programming Presettable and Bidirectional Counters in VHDL
9.7 Shift Registers
9.8 Programming Shift Registers in VHDL
9.9 Shift Register Counters

CHAPTER OBJECTIVES

Upon successful completion of this chapter you will be able to:

• Determine the modulus of a counter.
• Determine the number of outputs required by a counter for a given modulus.
• Determine the maximum modulus of a counter, given the number of circuit outputs.
• Draw the count sequence table, state diagram, and timing diagram of a counter.
• Determine the recycle point of a counter’s sequence.
• Calculate the frequencies of each counter output, given the input clock frequency.
• Draw a circuit for any full sequence synchronous counter.
• Determine the count sequence, state diagram, timing diagram, and modulus of any synchronous counter.
• Complete the state diagram of a synchronous counter to account for unused states.
• Design the circuit of a truncated sequence synchronous counter, using flip-flops and logic gates.
• Use MAX+PLUS II to create a graphic design file for any synchronous counter circuit.
• Use behavioral descriptions in VHDL to design synchronous counters of any modulus.
• Use a parameterized counter from the Library of Parameterized Modules in a VHDL file.
• Use the MAX+PLUS II simulation tool to verify the operation of synchronous counters.
• Implement various counter control functions, such as parallel load, clear, count enable, and count direction, both in Graphic Design Files and in VHDL.
• Design a circuit to decode the output of the counter, both in a MAX+PLUS II Graphic Design File or in VHDL.
• Draw a logic circuit of a serial shift register and determine its contents over time given any input data.
Counters and shift registers are two important classes of sequential circuits. In the simplest terms, a counter is a circuit that counts pulses. As such, it is used in many circuit applications, such as event counting and sequencing, timing, frequency division, and control. A basic counter can be enhanced to incorporate functions such as synchronous or asynchronous parallel loading, synchronous or asynchronous clear, count enable, directional control, and output decoding. In this chapter, we will design counters using schematic entry, VHDL, and counters from the Library of Parameterized Modules and verify their operation using the MAX+PLUS II simulator.

Shift registers are circuits that store and move data. They can be used in serial data transfer, serial/parallel conversion, arithmetic functions, and delay elements. As with counters, many shift registers have additional functions such as parallel load, clear, and directional control. We can implement these circuits using schematic entry, VHDL, and LPM components.

9.1 Basic Concepts of Digital Counters

**KEY TERMS**

**Counter** A sequential digital circuit whose output progresses in a predictable repeating pattern, advancing by one state for each clock pulse.

**Recycle** To make a transition from the last state of the count sequence to the first state.

**Count sequence** The specific series of output states through which a counter progresses.

**State diagram** A diagram showing the progression of states of a sequential circuit.

**Modulus** The number of states through which a counter sequences before repeating.

**Modulo-n (or mod-n) counter** A counter with a modulus of $n$.

**UP counter** A counter with an ascending sequence.

**DOWN counter** A counter with a descending sequence.

- Draw a timing diagram showing the operation of a serial shift register.
- Draw the logic circuit of a general parallel-load shift register.
- Draw a timing diagram showing the operation of a parallel-load shift register.
- Draw the general logic circuit of a bidirectional shift register and explain the concepts of right-shift and left-shift.
- Use timing diagrams to explain the operation of a bidirectional shift register.
- Describe the operation of a universal shift register.
- Design shift registers, ring counters, and Johnson counters with the MAX+PLUS II Graphic Editor or VHDL.
- Verify the operation of shift registers, ring counters, and Johnson counters using the MAX+PLUS II simulation tool.
- Design a decoder for a Johnson counter.
- Use a ring counter or a Johnson counter as an event sequencer.
- Compare binary, ring, and Johnson counters in terms of the modulus and the required decoding for each circuit.
The simplest definition of a **counter** is “a circuit that counts pulses.” Knowing only this, let us look at an example of how we might use a counter circuit.

**Example 9.1**

Figure 9.1 shows a 10-bit binary counter that can be used to count the number of people passing by an optical sensor. Every time the sensor detects a person passing by, it produces a pulse. Briefly describe the counter’s operation. What is the maximum number of people it can count? What happens if this number is exceeded?

**Solution**

The counter has a 10-bit output, allowing a binary number from 00 0000 0000 to 11 1111 1111 (0 to 1023) to appear at its output. The sensor causes the counter to advance by one binary number for every pulse applied to the counter’s clock (CLK) input. If the counter is allowed to register no people (i.e., 00 0000 0000), then the circuit can count 1023 people, since there are 1024 unique binary combinations of a 10-bit number, including 0. (This is because $2^{10} = 1024$.) When the 1024th pulse is applied to the clock input, the counter rolls over to 0 (or **recycles**) and starts counting again. (After this point, the counter would not accurately reflect the number of people counted.)

The counter is labeled CTR DIV 1024 to indicate that one full cycle of the counter requires 1024 clock pulses (i.e., the frequency of the MSB output signal ($Q_9$) is the clock frequency divided by 1024).

A counter is a digital circuit that has a number of binary outputs whose states progress through a fixed sequence. This **count sequence** can be ascending, descending, or nonlinear.

The output sequence of a counter is usually defined by its **modulus**, that is, the number of states through which the counter progresses. An **UP counter** with a modulus of 12 counts through 12 states from 0000 up to 1011 (0 to 11 in decimal), recycles to 0000, and continues. A **DOWN counter** with a modulus of 12 counts from 1011 down to 0000, recycles to 1011, and continues downward. Both types of counter are called **modulo-12**, or just **mod-12** counters, since they both have sequences of 12 states.

**State Diagram**

The states of a counter can be represented by a **state diagram**. Figure 9.2 compares the state diagram of a mod-12 UP counter to an analog clock face. Each counter state is illustrated in the state diagram by a circle containing its binary value. The progression is shown by a series of directional arrows.

Both the clock face and the state diagram represent a closed system of counting. In each case, when we reach the end of the count sequence, we start over from the beginning of the cycle.

For instance, if it is 10:00 a.m. and we want to meet a friend in four hours, we know we should turn up for the appointment at 2:00 p.m. We arrive at this figure by starting at 10 on the clock face and counting 4 digits forward in a “clockwise” circle. This takes us two digits past 12, the “recycle point” of the clock face.

Similarly, if we want to know the 8th state after 0111 in a mod-12 UP counter, we start at state 0111 and count 8 positions in the direction of the arrows. This brings us to state 0000 (the recycle point) in 5 counts and then on to state 0011 in another 3 counts.
Number of Bits and Maximum Modulus

The state diagram of Figure 9.2 represents the states of a mod-12 counter as a series of 4-bit numbers. Counter states are always written with a fixed number of bits, since each bit represents the logic level of a physical location in the counter circuit. A mod-12 counter requires four bits because its highest count value is a 4-bit number: 1011.

The maximum modulus of a 4-bit counter is 16 \( (m = 2^4) \). The count sequence of a mod-16 UP counter is from 0000 to 1111 (0 to 15 in decimal), as illustrated in the state diagram of Figure 9.3.

In general, an \( n \)-bit counter has a maximum modulus of \( 2^n \) and a count sequence from 0 to \( 2^n - 1 \) (i.e., all 0s to all 1s). Since a mod-16 counter has a modulus of \( 2^4 \) (\( = m_{max} \)), we say that it is a full-sequence counter. We can also call this a binary counter if it generates the sequence in binary order. A counter, such as a mod-12 counter, whose modulus is less than \( 2^n \), is called a truncated sequence counter.

Count-Sequence Table and Timing Diagram

Two ways to represent a count sequence other than a state diagram are by a count sequence table and by a timing diagram. The count sequence table is simply a list of counter states in the same order as the count sequence. Tables 9.1 and 9.2 show the count sequence tables of a mod-16 UP counter and a mod-12 UP counter, respectively.
We can derive timing diagrams from each of these tables. We know that each counter advances by one state with each applied clock pulse. The mod-16 count sequence shows us that the $Q_0$ waveform changes state with each clock pulse. $Q_1$ changes with every two clock pulses, $Q_2$ with every four, and $Q_3$ with every eight. Figure 9.4 shows this pattern for the mod-16 UP counter, assuming the counter is a positive edge-triggered device.

![FIGURE 9.4 Mod-16 Timing Diagram](image)
A divide-by-two ratio relates the frequencies of adjacent outputs of a binary counter. For example, if the clock frequency is $f_c = 16$ MHz, the frequencies of the output waveforms are: 8 MHz ($f_0 = f_c/2$); 4 MHz ($f_1 = f_c/4$); 2 MHz ($f_2 = f_c/8$); 1 MHz ($f_3 = f_c/16$).

We can construct a similar timing diagram, illustrated in Figure 9.5, for a mod-12 UP counter. The changes of state can be monitored by noting where $Q_0$ (the least significant bit) changes. This occurs on each positive edge of the $CLK$ waveform. The sequence progresses by 1 with each $CLK$ pulse until the outputs all go to 0 on the first $CLK$ pulse after state $Q_3Q_2Q_1Q_0 = 1011$.

The output waveform frequencies of a truncated sequence counter do not necessarily have a simple relationship to one another as do binary counters. For the mod-12 counter the relationships between clock frequency, $f_c$, and output frequencies are: $f_0 = f_c/2$; $f_1 = f_c/4$; $f_2 = f_c/12$; $f_3 = f_c/12$. Note that both $Q_2$ and $Q_3$ have the same frequencies ($f_2$ and $f_3$), but are out of phase with one another.

**EXAMPLE 9.2**

Draw the state diagram, count sequence table, and timing diagram for a mod-12 DOWN counter.

**Solution**  Figure 9.6 shows the state diagram for the mod-12 DOWN counter. The states are identical to those of a mod-12 UP counter, but progress in the opposite direction. Table 9.3 shows the count sequence table of this circuit.

**FIGURE 9.5**
Mod-12 Timing Diagram

**FIGURE 9.6**
Example 9.2
State Diagram of a Mod-12 DOWN Counter
The timing diagram of this counter is illustrated in Figure 9.7. The output starts in state $Q_3Q_2Q_1Q_0 = 1011$ and counts DOWN until it reaches 0000. On the next pulse, it recycles to 1011 and starts over.

**SECTION 9.1 REVIEW PROBLEM**

9.1 How many outputs does a mod-24 counter require? Is this a full-sequence or a truncated sequence counter? Explain your answer.

**9.2 Synchronous Counters**

**KEY TERMS**

**Synchronous counter** A counter whose flip-flops are all clocked by the same source and thus change in synchronization with each other.

**Present state** The current state of flip-flop outputs in a synchronous sequential circuit.

**Next state** The desired future state of flip-flop outputs in a synchronous sequential circuit after the next clock pulse is applied.

**Memory section** A set of flip-flops in a synchronous circuit that hold its present state.
In Chapter 7, we briefly examined the circuits of a 3-bit and a 4-bit synchronous counter (Figures 7.53 and 7.87, respectively). A synchronous counter is a circuit consisting of flip-flops and control logic, whose outputs progress through a regular predictable sequence, driven by a clock signal. The counter is synchronous because all flip-flops are clocked at the same time.

Figure 9.8 shows the block diagram of a synchronous counter, which consists of a memory section to keep track of the present state of the counter and a control section to direct the counter to its next state. The memory section is a sequential circuit (flip-flops) and the control section is combinational (gates). They communicate through a set of status lines that go from the Q outputs of the flip-flops to the control gate inputs and command lines that connect the control gate outputs to the synchronous inputs (J, K, D, or T) of the flip-flops. Outputs can be tied directly to the status lines or can be decoded to give a sequence other than that of the flip-flop output states. The circuit might have inputs to implement one or more control functions, such as changing the count direction, clearing the counter, or presetting the counter to a specific value.

Analysis of Synchronous Counters

A 3-bit synchronous binary counter based on JK flip-flops is shown in Figure 9.9. Let us analyze its count sequence in detail so that we can see how the J and K inputs are affected by the Q outputs and how transitions between states are made. Later we will look at the function of truncated sequence counter circuits and counters that are made from flip-flops other than JK.

The synchronous input equations are given by:

\[ J_2 = K_2 = Q_1 \cdot Q_0 \]
\[ J_1 = K_1 = Q_0 \]
\[ J_0 = K_0 = 1 \]
For reference, the JK flip-flop function table is shown in Table 9.4:

<table>
<thead>
<tr>
<th>$J$</th>
<th>$K$</th>
<th>$Q_{t+1}$</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$Q_t$</td>
<td>No change</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>Reset</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>Set</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$\bar{Q}_t$</td>
<td>Toggle</td>
</tr>
</tbody>
</table>

$Q_t$ indicates the state of $Q$ before a clock pulse is applied. $Q_{t+1}$ indicates the state of $Q$ after the clock pulse.

Assume the counter output is initially $Q_2Q_1Q_0 = 000$. Before any clock pulses are applied, the $J$ and $K$ inputs are at the following states:

$J_2 = K_2 = Q_1Q_0 = 0\cdot0 = 0$  (No change)
$J_1 = K_1 = Q_0 = 0$  (No change)
$J_0 = K_0 = 1$  (Constant)  (Toggle)

The transitions of the outputs after the clock pulse are:

$Q_2$: 0 → 0  (No change)
$Q_1$: 0 → 0  (No change)
$Q_0$: 0 → 1  (Toggle)

The output goes from $Q_2Q_1Q_0 = 000$ to $Q_2Q_1Q_0 = 001$ (see Figure 9.10). The transition is defined by the values of $J$ and $K$ before the clock pulse, since the propagation delays of the flip-flops prevent the new output conditions from changing the $J$ and $K$ values until after the transition.

The new conditions of the $J$ and $K$ inputs are:

$J_2 = K_2 = Q_1Q_0 = 0\cdot1 = 0$  (No change)
$J_1 = K_1 = Q_0 = 1$  (Toggle)
$J_0 = K_0 = 1$  (Constant)  (Toggle)
The transitions of the outputs generated by the second clock pulse are:

\[ Q_2: 0 \rightarrow 0 \quad \text{(No change)} \]
\[ Q_1: 0 \rightarrow 1 \quad \text{(Toggle)} \]
\[ Q_0: 1 \rightarrow 0 \quad \text{(Toggle)} \]

The new output is \( Q_2Q_1Q_0 = 010 \), since both \( Q_0 \) and \( Q_1 \) change and \( Q_2 \) stays the same. The \( J \) and \( K \) conditions are now:

\[ J_2 = K_2 = Q_1Q_0 = 1 \cdot 0 = 0 \quad \text{(No change)} \]
\[ J_1 = K_1 = Q_0 = 0 \quad \text{(No change)} \]
\[ J_0 = K_0 = 1 \quad \text{(Constant)} \]

(No change) (No change) (Toggle)

The output transitions are:

\[ Q_2: 0 \rightarrow 0 \quad \text{(No change)} \]
\[ Q_1: 1 \rightarrow 1 \quad \text{(No change)} \]
\[ Q_0: 0 \rightarrow 1 \quad \text{(Toggle)} \]

The output is now \( Q_2Q_1Q_0 = 011 \), which results in the \( JK \) conditions:

\[ J_2 = K_2 = Q_1Q_0 = 1 \cdot 1 = 1 \quad \text{(Toggle)} \]
\[ J_1 = K_1 = Q_0 = 1 \quad \text{(Toggle)} \]
\[ J_0 = K_0 = 1 \quad \text{(Constant)} \]

(No change) (Toggle) (Toggle)

The above conditions result in output transitions:

\[ Q_2: 0 \rightarrow 1 \quad \text{(Toggle)} \]
\[ Q_1: 1 \rightarrow 0 \quad \text{(Toggle)} \]
\[ Q_0: 0 \rightarrow 1 \quad \text{(Toggle)} \]

All the outputs toggle and the new output state is \( Q_2Q_1Q_0 = 100 \). The \( J \) and \( K \) values repeat the above pattern in the second half of the counter cycle (states 100 to 111). Go through the exercise of calculating the \( J, K, \) and \( Q \) values for the rest of the cycle. Compare the result with the timing diagram in Figure 9.10.

![FIGURE 9.10 Timing Diagram for a Synchronous 3-bit Binary Counter](image)
\( K_0 = 0 \) and \( Q_1 \) and \( Q_2 \) reset (\( J_1 = 0, K_1 = 1 \) and \( J_2 = 0, K_2 = 1 \)). To do so we would need a different set of combinational logic in the circuit.

The simplest synchronous counter design uses only the no change (\( JK = 00 \)) or toggle (\( JK = 11 \)) modes, since the \( J \) and \( K \) inputs of each flip-flop can be connected together. The no change and toggle modes allow us to make any transition (i.e., not just in a linear sequence), even though for truncated sequence and nonbinary counters this is not usually the most efficient design.

There is a simple progression of algebraic expressions for the \( J \) and \( K \) inputs of a synchronous binary (full sequence) counter, which uses only the no change and toggle states:

\[
\begin{align*}
J_0 &= K_0 = 1 \\
J_1 &= K_1 = Q_0 \\
J_2 &= K_2 = Q_1 \cdot Q_0 \\
J_3 &= K_3 = Q_2 \cdot Q_1 \cdot Q_0 \\
J_4 &= K_4 = Q_3 \cdot Q_2 \cdot Q_1 \cdot Q_0 \\
&\text{etc.}
\end{align*}
\]

The \( J \) and \( K \) inputs of each stage are the ANDed outputs of all previous stages. This implies that a flip-flop toggles only when the outputs of all previous stages are HIGH. For example, \( Q_2 \) doesn’t change unless both \( Q_1 \) AND \( Q_0 \) are HIGH (and therefore \( J_2 = K_2 = 1 \)) before the clock pulse. In a 3-bit counter, this occurs only at states 011 and 111, after which \( Q_2 \) will toggle, along with \( Q_1 \) and \( Q_0 \), giving transitions to states 100 and 000 respectively. Look at the timing diagram of Figure 9.10 to confirm this.

**Determining the Modulus of a Synchronous Counter**

We can use a more formal technique to analyze any synchronous counter, as follows.

1. Determine the equations for the synchronous inputs (\( JK, D, \) or \( T \)) in terms of the \( Q \) outputs for all flip-flops. (For counters other than straight binary full sequence types, the equations will *not* be the same as the algebraic progressions previously listed.)

2. Lay out a table with headings for the Present State of the counter (\( Q \) outputs before \( CLK \) pulse), each Synchronous Input before \( CLK \) pulse, and Next State of the counter (\( Q \) outputs after the clock pulse).

3. Choose a starting point for the count sequence, usually 0, and enter the starting point in the Present State column.

4. Substitute the \( Q \) values of the initial present state into the synchronous input equations and enter the results under the appropriate columns.

5. Determine the action of each flip-flop on the next \( CLK \) pulse (e.g., for a JK flip-flop, the output either will not change (\( JK = 00 \)), or will reset (\( JK = 01 \)), or set (\( JK = 10 \)), or toggle (\( JK = 11 \))).

6. Look at the \( Q \) values for every flip-flop. Change them according to the function determined in Step 5 and enter them in the column for the counter’s next state.

7. Enter the result from Step 6 on the next line of the column for the counter’s present state (i.e., this line’s next state is the next line’s present state).

8. Repeat the above process until the result in the next state column is the same as the initial state.

**EXAMPLE 9.3**

Find the count sequence of the synchronous counter shown in Figure 9.11 and, from the count sequence table, draw the timing diagram and state diagram. What is the modulus of the counter?
Solution  The $J$ and $K$ equations are:

\[
\begin{align*}
J_2 &= Q_1 \overline{Q}_0 & J_1 &= Q_0 & J_0 &= \overline{Q}_2 \\
K_2 &= 1 & K_1 &= Q_0 & K_0 &= 1
\end{align*}
\]

The output transitions can be determined from the values of the $J$ and $K$ functions before each clock pulse, as shown in Table 9.5.

<table>
<thead>
<tr>
<th>Present State</th>
<th>Synchronous Inputs</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_2Q_1Q_0$</td>
<td>$J_2K_2$</td>
<td>$J_1K_1$</td>
</tr>
<tr>
<td>000</td>
<td>01 (R)</td>
<td>00 (NC)</td>
</tr>
<tr>
<td>001</td>
<td>01 (R)</td>
<td>11 (T)</td>
</tr>
<tr>
<td>010</td>
<td>01 (R)</td>
<td>00 (NC)</td>
</tr>
<tr>
<td>011</td>
<td>11 (T)</td>
<td>11 (T)</td>
</tr>
<tr>
<td>100</td>
<td>01 (R)</td>
<td>00 (NC)</td>
</tr>
</tbody>
</table>

Since there are five unique output states, the counter’s modulus is 5.

The timing diagram and state diagram are shown in Figure 9.12. Since this circuit produces one pulse on $Q_2$ for every 5 clock pulses, we can use it as a divide-by-5 circuit.
The analysis in Example 9.3 did not account for the fact that the counter uses only 5 of a possible 8 output states. In any truncated sequence counter, it is good practice to determine the next state for each unused state to ensure that if the counter powers up in one of these unused states, it will eventually enter the main sequence.

### EXAMPLE 9.4

Extend the analysis of the counter in Example 9.3 to include its unused states. Redraw the counter’s state diagram to show how these unused states enter the main sequence (if they do).

**Solution**

The synchronous input equations are:

\[
\begin{align*}
J_2 &= Q_1Q_0 \\
J_1 &= Q_0 \\
J_0 &= \overline{Q_2} \\
K_2 &= 1 \\
K_1 &= Q_0 \\
K_0 &= 1
\end{align*}
\]

The unused states are \(Q_2Q_1Q_0 = 101, 110, \) and \(111\). Table 9.6 shows the transitions made by the unused states. Figure 9.13 shows the completed state diagram.

**Table 9.6**

State Table for Mod-5 Counter Including Unused States

<table>
<thead>
<tr>
<th>Present State</th>
<th>Synchronous Inputs</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q_2Q_1Q_0)</td>
<td>(J_2K_2)</td>
<td>(J_1K_1)</td>
</tr>
<tr>
<td>000</td>
<td>01 (R)</td>
<td>00 (NC)</td>
</tr>
<tr>
<td>001</td>
<td>01 (R)</td>
<td>11 (T)</td>
</tr>
<tr>
<td>010</td>
<td>01 (R)</td>
<td>00 (NC)</td>
</tr>
<tr>
<td>011</td>
<td>11 (T)</td>
<td>11 (T)</td>
</tr>
<tr>
<td>100</td>
<td>01 (R)</td>
<td>00 (NC)</td>
</tr>
</tbody>
</table>

| 101 | 01 (R) | 11 (T) | 01 (R) | 010 |
| 110 | 01 (R) | 00 (NC) | 01 (R) | 010 |
| 111 | 11 (T) | 11 (T) | 01 (R) | 000 |

**FIGURE 9.13**
Example 9.4
Complete State Diagram

### SECTION 9.2 REVIEW PROBLEM

9.2 A 4-bit synchronous counter based on JK flip-flops is described by the following set of equations:

\[
\begin{align*}
J_3 &= Q_2Q_1Q_0 \\
J_2 &= Q_1Q_0 \\
J_1 &= \overline{Q_2}Q_0 \\
J_0 &= 1 \\
K_3 &= Q_0 \\
K_2 &= Q_1Q_0 \\
K_1 &= Q_0 \\
K_0 &= 1
\end{align*}
\]
Assume the counter output is at 1000 in the count sequence. What will the output be after one clock pulse? After two clock pulses?

9.3 Design of Synchronous Counters

**KEY TERMS**

<table>
<thead>
<tr>
<th><strong>Excitation table</strong></th>
<th>A table showing the required input conditions for every possible transition of a flip-flop output.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>State machine</strong></td>
<td>A synchronous sequential circuit.</td>
</tr>
</tbody>
</table>

A synchronous counter can be designed using established techniques that involve the derivation of Boolean equations for the counter’s next state logic. Alternatively, several VHDL structures can be used to define counters; we can use a behavioral description of the counter, or we can use a state machine definition in VHDL that specifies each present and next state explicitly.

In addition to the classical counter design techniques, we will examine the design of a counter through a behavioral description in VHDL. We will leave the state machine design for the following chapter.

**Classical Design Technique**

There are several steps involved in the classical design of a synchronous counter.

1. Define the problem. Before you can begin design of a circuit, you have to know what its purpose is and what it should do under all possible conditions.
2. Draw a state diagram showing the progression of states under various input conditions and what outputs the circuit should produce, if any.
3. Make a state table which lists all possible Present States and the Next State for each one. List the present states in binary order.
4. Use flip-flop excitation tables to determine at what states the flip-flop synchronous inputs must be to make the circuit go from each Present State to its Next State.
5. The logic levels of the synchronous inputs are Boolean functions of the flip-flop outputs and the control inputs. Simplify the expression for each input and write the simplified Boolean expression.
6. Use the Boolean expressions found in step 5 to draw the required logic circuit.

**Flip-flop Excitation Tables**

In the synchronous counter circuits we examined earlier in this chapter, we used JK flip-flops that were configured to operate only in toggle or no change mode. We can use any type of flip-flop for a synchronous sequential circuit. If we choose to use JK flip-flops, we can use any of the modes (no change, reset, set, or toggle) to make transitions from one state to another.

A flip-flop excitation table shows all possible transitions of a flip-flop output and the synchronous input levels needed to effect these transitions. Table 9.7 is the excitation table of a JK flip-flop.

If we want a flip-flop to make a transition from 0 to 1, we can use either the toggle function ($JK = 11$) or the set function ($JK = 10$). It doesn’t matter what $K$ is, as long as $J = 1$. This is reflected by the variable pair ($JK = 1X$) beside the $0 \rightarrow 1$ entry in Table 9.7. The $X$ is a don’t care state, a 0 or 1 depending on which is more convenient for the simplification of the Boolean function of the $J$ or $K$ input affected.

Table 9.8 shows a condensed version of the JK flip-flop excitation table.
Design of Synchronous Counters

### Design of a Synchronous Mod-12 Counter

We will follow the procedure outlined above to design a synchronous mod-12 counter circuit, using JK flip-flops. The aim is to derive the Boolean equations of all J and K inputs and to draw the counter circuit.

1. **Define the problem.** The circuit must count in binary sequence from 0000 to 1011 and repeat. The output progresses by 1 for each applied clock pulse. Since the outputs are 4-bit numbers, we require 4 flip-flops.

2. **Draw a state diagram.** The state diagram for this problem is shown in Figure 9.14.

3. **Make a state table showing each present state and the corresponding next state.**

4. **Use flip-flop excitation tables to fill in the J and K entries in the state table.** Table 9.9 shows the combined result of steps 3 and 4. Note that all present states are in binary order.

   We assume for now that states 1100 to 1111 never occur. If we assign their corresponding next states to be don’t care states, they can be used to simplify the J and K expressions we derive from the state table.

#### Table 9.7 JK Flip-Flop Excitation Table

<table>
<thead>
<tr>
<th>Transition</th>
<th>Function</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 → 0</td>
<td>No change or</td>
<td>00</td>
<td>0X</td>
</tr>
<tr>
<td></td>
<td>reset</td>
<td>01</td>
<td></td>
</tr>
<tr>
<td>0 → 1</td>
<td>Toggle or</td>
<td>11</td>
<td>1X</td>
</tr>
<tr>
<td></td>
<td>set</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>1 → 0</td>
<td>Toggle or</td>
<td>11</td>
<td>X1</td>
</tr>
<tr>
<td></td>
<td>reset</td>
<td>01</td>
<td></td>
</tr>
<tr>
<td>1 → 1</td>
<td>No change or</td>
<td>00</td>
<td>X0</td>
</tr>
<tr>
<td></td>
<td>set</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 9.8 Condensed Excitation Table for a JK Flip-Flop

<table>
<thead>
<tr>
<th>Transition</th>
<th>JK</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 → 0</td>
<td>0X</td>
</tr>
<tr>
<td>0 → 1</td>
<td>1X</td>
</tr>
<tr>
<td>1 → 0</td>
<td>X1</td>
</tr>
<tr>
<td>1 → 1</td>
<td>X0</td>
</tr>
</tbody>
</table>

**FIGURE 9.14**

State Diagram for a Mod-12 Counter
Let us examine one transition to show how the table is completed. The transition from $Q_3Q_2Q_1Q_0 = 0100$ to $Q_3Q_2Q_1Q_0 = 0110$ consists of the following individual flip-flop transitions.

- $Q_3: 0 \rightarrow 0$ (No change or reset; $J_3K_3 = 0X$)
- $Q_2: 1 \rightarrow 1$ (No change or set; $J_2K_2 = X0$)
- $Q_1: 0 \rightarrow 1$ (Toggle or set; $J_1K_1 = 1X$)
- $Q_0: 1 \rightarrow 0$ (Toggle or reset; $J_0K_0 = X1$)

The other lines of the table are similarly completed.

5. **Simplify the Boolean expression for each input.** Table 9.9 can be treated as eight truth tables, one for each $J$ or $K$ input. We can simplify each function by Boolean algebra or by using a Karnaugh map.

Figure 9.15 shows K-map simplification for all 8 synchronous inputs. These maps yield the following simplified Boolean expressions.

- $J_0 = 1$
- $K_0 = 1$
- $J_1 = Q_0$
- $K_1 = Q_0$
- $J_2 = \overline{Q}_0Q_1Q_0$
- $K_2 = Q_1Q_0$
- $J_3 = Q_2Q_1Q_0$
- $K_3 = Q_1Q_0$

6. **Draw the required logic circuit.** Figure 9.16 shows the circuit corresponding to the above Boolean expressions.

We have assumed that states 1100 to 1111 will never occur in the operation of the mod-12 counter. This is normally the case, but when the circuit is powered up, there is no guarantee that the flip-flops will be in any particular state.

If a counter powers up in an unused state, the circuit should enter the main sequence after one or more clock pulses. To test whether or not this happens, let us make a state
FIGURE 9.15
K-Map Simplification of Table 9.9
FIGURE 9.16
Synchronous Mod-12 Counter
Table 9.10 Unused States in a Mod-12 Counter

<table>
<thead>
<tr>
<th>Present State</th>
<th>Synchronous Inputs</th>
<th>Next State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_3Q_2Q_1Q_0$</td>
<td>$J_3K_3$</td>
<td>$J_2K_2$</td>
</tr>
<tr>
<td>0000</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>1101</td>
<td>00</td>
<td>00</td>
</tr>
<tr>
<td>1111</td>
<td>11</td>
<td>01</td>
</tr>
</tbody>
</table>

Figure 9.17 shows the complete state diagram for the designed mod-12 counter. If the counter powers up in an unused state, it will enter the main sequence in no more than four clock pulses.

If we want an unused state to make a transition directly to 0000 in one clock pulse, we have a couple of options:

1. We could reset the counter asynchronously and otherwise leave the design as is.
2. We could rewrite the state table to specify these transitions, rather than make the unused states don’t cares.

Option 1 is the simplest and is considered perfectly acceptable as a design practice. Option 2 would yield a more complicated set of Boolean equations and hence a more complex circuit, but might be worthwhile if a direct synchronous transition to 0000 were required.

FIGURE 9.17
Complete State Diagram of Mod-12 Counter in Figure 9.16
EXAMPLE 9.5

Derive the synchronous input equations of a 4-bit synchronous binary counter based on D flip-flops. Draw the corresponding counter circuit.

Solution

The first step in the counter design is to derive the excitation table of a D flip-flop. Recall that \( Q \) follows \( D \) when the flip-flop is clocked. Therefore the next state of \( Q \) is the same as the input \( D \) for any transition. This is illustrated in Table 9.11.

<table>
<thead>
<tr>
<th>Transition</th>
<th>( D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ( \rightarrow ) 0</td>
<td>0</td>
</tr>
<tr>
<td>0 ( \rightarrow ) 1</td>
<td>1</td>
</tr>
<tr>
<td>1 ( \rightarrow ) 0</td>
<td>0</td>
</tr>
<tr>
<td>1 ( \rightarrow ) 1</td>
<td>1</td>
</tr>
</tbody>
</table>

Next, we must construct a state table, shown in Table 9.12, with present and next states for all possible transitions. Note that the binary value of \( D_3D_2D_1D_0 \) is the same as the next state of the counter.

<table>
<thead>
<tr>
<th>Present State</th>
<th>Next State</th>
<th>Synchronous Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_3Q_2Q_1Q_0 )</td>
<td>( Q_3Q_2Q_1Q_0 )</td>
<td>( D_3D_2D_1D_0 )</td>
</tr>
<tr>
<td>0000</td>
<td>0001</td>
<td>0001</td>
</tr>
<tr>
<td>0001</td>
<td>0010</td>
<td>0010</td>
</tr>
<tr>
<td>0010</td>
<td>0011</td>
<td>0011</td>
</tr>
<tr>
<td>0011</td>
<td>0100</td>
<td>0100</td>
</tr>
<tr>
<td>0100</td>
<td>0101</td>
<td>0101</td>
</tr>
<tr>
<td>0101</td>
<td>0110</td>
<td>0110</td>
</tr>
<tr>
<td>0110</td>
<td>0111</td>
<td>0111</td>
</tr>
<tr>
<td>0111</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>1000</td>
<td>1001</td>
<td>1001</td>
</tr>
<tr>
<td>1001</td>
<td>1010</td>
<td>1010</td>
</tr>
<tr>
<td>1010</td>
<td>1011</td>
<td>1011</td>
</tr>
<tr>
<td>1011</td>
<td>1100</td>
<td>1100</td>
</tr>
<tr>
<td>1100</td>
<td>1101</td>
<td>1101</td>
</tr>
<tr>
<td>1101</td>
<td>1110</td>
<td>1110</td>
</tr>
<tr>
<td>1110</td>
<td>1111</td>
<td>1111</td>
</tr>
<tr>
<td>1111</td>
<td>0000</td>
<td>0000</td>
</tr>
</tbody>
</table>

This state table yields four Boolean equations, for \( D_3 \) through \( D_0 \), in terms of the present state outputs. Figure 9.18 shows four Karnaugh maps used to simplify these functions. The simplified equations are:

\[
D_3 = \overline{Q}_3Q_2Q_1Q_0 + Q_3\overline{Q}_2 + Q_3\overline{Q}_1 + Q_3\overline{Q}_0 \\
D_2 = \overline{Q}_2Q_1Q_0 + Q_2\overline{Q}_1 + Q_1\overline{Q}_0 \\
D_1 = \overline{Q}_1Q_0 + Q_1\overline{Q}_0 \\
D_0 = \overline{Q}_0
\]
These equations represent the maximum SOP simplifications of the input functions. However, we can rewrite them to make them more compact. For example, the equation for $D_3$ can be rewritten, using DeMorgan’s theorem ($x/H_3$33526 $y/H_1$1001 $z/H_3$33526 $x/H_3$33526 $y/H_1$1005 $z$), and our knowledge of Exclusive OR (XOR) functions ($x y$).

$$D_3 = \overline{Q_3}Q_2Q_1Q_0 + Q_3\overline{Q_2} + Q_3Q_1 + Q_3\overline{Q_0}$$

We can write similar equations for the other $D$ inputs as follows:

$$D_2 = Q_2 \oplus Q_1Q_0$$
$$D_1 = Q_1 \oplus Q_0$$
$$D_0 = Q_0 \oplus 1$$

These equations follow a predictable pattern of expansion. Each equation for an input $D_n$ is simply $Q_n$ XORed with the logical product (AND) of all previous $Q$s.

Figure 9.19 shows the circuit for the 4-bit counter, including an asynchronous reset.
In Section 7.6 (Edge-Triggered T Flip-Flops) of Chapter 7, we saw how a D flip-flop could be configured for a switchable toggle function (refer to Figure 7.59). The flip-flops in Figure 9.19 are similarly configured. Each flip-flop output, except $Q_0$, is fed back to its input through an Exclusive OR gate. The other input to the XOR controls whether this feedback is inverted (for toggle mode) or not (for no change mode). Recall that $x \oplus 0 = x$ and $x \oplus 1 = \overline{x}$.

For example, $Q_3$ is fed back to $D_3$ through an XOR gate. The feedback is inverted only if the 3-input AND gate has a HIGH output. Thus, the $Q_3$ output toggles only if all previous bits are HIGH ($Q_3Q_2Q_1Q_0 = 0111$ or $1111$). The flip-flop toggle mode is therefore controlled by the states of the XOR and AND gates in the circuit.

### SECTION 9.3 REVIEW PROBLEM

9.3 A 4-bit synchronous counter must make a transition from state $Q_3Q_2Q_1Q_0 = 1011$ to $Q_3Q_2Q_1Q_0 = 1100$. Write the required states of the synchronous inputs for a set of four JK flip-flops used to implement the counter. Write the required states of the synchronous inputs if the counter is made from D flip-flops.
9.4 Programming Binary Counters in VHDL

**KEY TERMS**

- **If statement**: A VHDL construct in which statements within the IF statement are executed only when a specified Boolean condition is satisfied.
- **Attribute**: A property associated with a named identifier in VHDL. (For example, the attribute `EVENT`, when associated with the identifier `clk` (written `clk’EVENT`), indicates, when true, that a transition has occurred on the input called `clk`.)

When using VHDL to create a counter, we can take several approaches. We can encode the Boolean equations of the counter directly with concurrent signal assignment statements; we can use VHDL code to describe the behavior of the counter; we can use a CASE statement to implement the state diagram of the counter; or we can use a predefined counter, such as those found in the MAX+PLUS II Library of Parameterized Modules (LPM) and map its ports to the ports of a VHDL design entity.

If we chose to use concurrent signal assignments to encode the Boolean equations of a counter, we could derive the following equations for a 4-bit counter with D flip-flops.

\[
\begin{align*}
    d(3) &= q(3) \text{ xor } (q(2) \text{ and } q(1) \text{ and } q(0)), \\
    d(2) &= q(2) \text{ xor } (q(1) \text{ and } q(0)), \\
    d(1) &= q(1) \text{ xor } q(1), \\
    d(0) &= \text{ not } q(0);
\end{align*}
\]

In Chapter 5, we saw that using concurrent signal assignment statements is an inefficient way to code many digital functions. (For one thing, if we use this procedure, we must know what the equations are. Getting to that point requires a lot of work that can be done by the VHDL compiler.) While acknowledging this as a possible option, we will not examine this method any further for the count logic of binary counters.

In this section, we will design a counter using a behavioral description and using an LPM counter. The design of a counter as a state machine will be examined in the next chapter.

**Behavioral Description of Counters**

The following VHDL code shows the behavioral description of a simple 8-bit counter (`ct_simp.vhd`) with asynchronous clear.

```vhdl
ENTITY ct_simp IS
  PORT(
    clk : IN BIT;
    clear : IN BIT;
    q : OUT INTEGER RANGE 0 TO 255);
END ct_simp;

ARCHITECTURE a OF ct_simp IS
BEGIN
  PROCESS (clk, clear)
  VARIABLE count : INTEGER RANGE 0 TO 255;
  BEGIN
    IF (clear = '0') THEN
      count := 0;
    ELSE
      IF (clk’EVENT AND clk = '1') THEN
        count := count + 1;
      END IF;
    END IF;
  END PROCESS;
END a;
```

```vhdl
```
END IF;
q <= count;
END PROCESS;
END a;

Recall that the PROCESS statement has the following syntax:

PROCESS (sensitivity list)
[VARIBALE variable name :type [range]; ]
BEGIN
Process statements
END PROCESS;

Square brackets [] indicate an optional part of the code.

When there is a change in an item in the sensitivity list, the process statements are executed. For a synchronous counter, the list would often only include clock, since any action in a synchronous circuit depends on a clock transition. Since the clear function in this counter is asynchronous, the clear input must also be monitored for any changes.

To hold the accumulating output value of the counter, we define a variable called count, presumed to have an initial value of 0, but defined for the range of 0 to 255. (This 8-bit value rolls over to 0 when the count exceeds 255.) The variable (any variable) is local to the process in which it is defined. We update the value of count by an IF statement, with the form:

IF (condition) THEN
  Statement[s];
[ELSIF (condition) THEN
  statement[s];]
[ELSE
  statement[s];]
END IF;

The clause (IF (clear='0') THEN) monitors the asynchronous clear function independently of the clock and executes the variable assignment that sets the output to 0 if the Boolean condition (clear='0') is true. Otherwise, the clock is monitored for a positive edge by the condition (clk'EVENT AND clk = '1'). The clause clk'EVENT (pronounced “clock tick event”) is a predefined attribute of the clock signal and is true if there has just been a change on clock. The combination of this and the condition clk = '1' indicates that a positive edge has just occurred. If this is true, the count is incremented.

As a final step, the accumulated count must be assigned to an output port. This is done in the concurrent signal assignment q <= count at the end of the process.

Note the difference in types of assignments. A variable is assigned by the := operator (e.g., count := count + 1); A signal is assigned by the <= operator (e.g., q <= count).

LPM Counters in VHDL

We can use a component (lpm_counter) from the Library of Parameterized Modules (LPM) to instantiate a counter in VHDL. When using an LPM counter, we don’t need to describe the behavior of the counter, as this has been done for us in the module itself. All we need to do is map the ports and parameters of the LPM component to the ports of the VHDL design entity. We do this by using a generic map to specify the parameters we need and a port map to map the ports of the LPM device either to an external port or an internal signal. The VHDL code below shows the VHDL implementation (lpm_simp.vhd) of the same 8-bit counter as in the previous behavioral example.

-- lpm_simp.vhd
-- Eight-bit binary counter based on a component
9.4 • Programming Binary Counters in VHDL

-- from the Library of Parameterized Modules (LPM)
-- Counter has an active-LOW asynchronous clear.

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY lpm_simp IS
  PORT(
    clk, clear : IN  STD_LOGIC;
    q          : OUT STD_LOGIC_VECTOR (7 downto 0));
END lpm_simp;

ARCHITECTURE count OF lpm_simp IS
  SIGNAL clrn : STD_LOGIC;
BEGIN
  count8: lpm_counter
    GENERIC MAP (LPM_WIDTH => 8)
    PORT MAP ( clock  => clk,
               aclr   => clrn,
               q      => q(7 downto 0));
  clrn <= not clear;
END count;

LPM components require us to use two packages: the std_logic_1164 package in the ieee library to define STD_LOGIC types used in the LPM components and the lpm_components package in the lpm library to define the components themselves. Since LPM components are defined using STD_LOGIC and STD_LOGIC_VECTOR types, we should use these types for our other identifiers as well.

The entity declaration defines the inputs and outputs of our counter and need not correspond to the port names for the LPM counter. That correspondence is defined in the architecture body, where we instantiate the counter module. The counter is defined in a component instantiation statement, which takes the following form:

__instance_name: __component_name
  GENERIC MAP (__parameter_name => __parameter_value,
               __parameter_name => __parameter_value)
  PORT MAP (__component_port => __connect_port,
            __component_port => __connect_port);

The component name is the name of the LPM component. Parameter names are those defined in the LPM component, such as LPM_WIDTH. Parameter values are those values assigned in the instance of the component. Component ports are the LPM port names. Connect ports are the names of identifiers declared in the entity or as signals or variables.

If we want to invert the active level of an LPM input port, we must use a signal assignment statement. (e.g., clrn <= not clear;) We need to do this because a VHDL input port cannot be “updated” (modified); only an output can be assigned a new value as a result of a Boolean expression. Thus, we create a signal called clrn that maps to the aclr (asynchronous clear) port of the LPM counter. This is connected to the clear input of the counter circuit via an inverter. Figure 9.20 shows the graphic equivalent of this mapping.

SECTION 9.4 REVIEW PROBLEM

9.4 Write a VHDL code segment that increments a variable called count upon detection of a negative edge of an input called clock.
9.5 Control Options for Synchronous Counters

**KEY TERMS**

- **Parallel load** A function that allows simultaneous loading of binary values into all flip-flops of a synchronous circuit. Parallel loading can be synchronous or asynchronous.

- **Presettable counter** A counter with a parallel load function.

- **Clear** Reset (synchronous or asynchronous).

- **Count enable** A control function that allows a counter to progress through its count sequence when active and disables the counter when inactive.

- **Bidirectional counter** A counter that can count up or down, depending on the state of a control input.

- **Terminal count** The last state in a count sequence before the sequence repeats (e.g., 1111 is the terminal count of a 4-bit binary UP counter; 0000 is the terminal count of a 4-bit binary DOWN counter).

- **Ripple carry out or ripple clock out (RCO)** An output that produces one pulse with the same period as the clock upon terminal count.

Synchronous counters can be designed with a number of features other than just straight counting. Some of the most common features include:

- Synchronous or asynchronous **parallel load**, which allows the count to be set to any value whenever a LOAD input is asserted

- Synchronous or asynchronous **clear** (reset), which sets all of the counter outputs to zero

- **Count enable**, which allows the count sequence to progress when asserted and inhibits the count when deasserted

- **Bidirectional** control, which determines whether the counter counts up or down

- **Output decoding**, which activates one or more outputs when detecting particular states on the counter outputs

- **Ripple carry out or ripple clock out (RCO)**, a special case of output decoding that produces a pulse upon detecting the **terminal count**, or last state, of a count sequence.
We will examine the implementation of these functions, first as Graphic Design Files in MAX+PLUS II, and then, in the next section, in VHDL, both as behavioral descriptions and as functions of LPM counters.

Parallel Loading

Figure 9.21 shows the symbol of a 4-bit **presettable counter** (i.e., a counter with a parallel load function). The parallel inputs, $P_3$ to $P_0$, have direct access to the flip-flops of the counter. When the \( LOAD \) input is asserted, the values at the \( P \) inputs are loaded directly into the counter and appear at the \( Q \) outputs.

**NOTE**

Parallel loading requires at least two sets of inputs: the load data \((P_3 \text{ to } P_0)\) and the load command \((LOAD)\). If the load function is synchronous, as described below, it also requires a clock input.

![Figure 9.21](4b_al_sl.scf)

4-bit Counter with Parallel Load

Parallel loading can be synchronous or asynchronous. The MAX+PLUS II simulation in Figure 9.22 shows the difference. Two waveforms, \( QS[3..0] \) and \( QA[3..0] \), represent the outputs of two 4-bit counters with synchronous and asynchronous load, respectively. Both counters have the same clock, load, and \( P \) inputs. The count is already in progress at the beginning of the simulation window and shows both counters advancing with each clock pulse: 4, 5, 6.

When \( LOAD \) goes HIGH at 500 ns, the value of \( P[3..0] (= AH) \) is loaded into the asynchronously loading counter \((QA[3..0])\) immediately after a short propagation delay (12.5 ns). The counter with synchronous load \((QS[3..0])\) is not loaded until the next positive clock edge, shown at 560 ns.

**FIGURE 9.22**

Synchronous vs. Asynchronous Load

**Synchronous Load**

The logic diagram of Figure 9.23 shows the concept of synchronous parallel load. Depending on the status of the \( LOAD \) input, the flip-flop will either count according to its
Count logic (the next-state combinational circuit) or load an external value. The flip-flop shown is the most significant bit of a 4-bit binary counter, such as shown in Figure 9.19, but with the count logic represented only by an input pin. (For the fourth bit of a counter, the Boolean equation of the count logic is given by \( D_3 = Q_3 \oplus Q_2Q_1Q_0 \). It is left out in order to more clearly show the operation of the count/load function select circuit.)

The LOAD input selects whether the flip-flop synchronous input will be fed by the count logic or by the parallel input \( P_3 \). When LOAD = 0, the upper AND gate steers the count logic to the flip-flop, and the count progresses with each clock pulse. When LOAD = 1, the lower AND gate loads the logic level at \( P_3 \) directly into the flip-flop on the next clock pulse.

Figure 9.24 shows the same circuit, but includes the count logic. If we leave out the 3-input AND gate, as in Figure 9.25, we have a circuit that can be used as a general element (called \texttt{sl\_count}) in a synchronous presettable counter. Figure 9.26 shows the logic diagram of a 4-bit synchronously presettable counter consisting of four instances of the counter element of Figure 9.25 and appropriate AND gates for a synchronous counter. This diagram implements a synchronous counter like that of Figure 9.19, but also incorporates a synchronous load function.

Figure 9.27 shows a simulation of the counter in Figure 9.26. The first 19 clock pulses drive the counter through its normal 4-bit cycle from 0H to FH, then up to 2H. At this point, we set the LOAD input HIGH and the value at the \( P \) inputs (9H) is loaded into the counter on the rising edge of the next clock pulse. An asynchronous \texttt{RESET} pulse at 880 ns drives the counter outputs to 0H, after which the count resumes.
9.5 • Control Options for Synchronous Counters

FIGURE 9.25
Counter Element with Synchronous Load and Asynchronous Reset (sl_count)

FIGURE 9.26
4-bit Counter with Synchronous Load and Asynchronous Reset
Asynchronous Load

The asynchronous load function of a counter makes use of the asynchronous preset and clear inputs of the counter’s flip-flops. Figure 9.28 shows the circuit implementation of the asynchronous load function, without any count logic.

When $ALOAD$ (Asynchronous LOAD) is HIGH, both NAND gates in Figure 9.28 are enabled. If the $P$ input is HIGH, the output of the upper NAND gate goes LOW, activating the flip-flop’s asynchronous $PRESET$ input, thus setting $Q = 1$. The lower NAND gate has a HIGH output, thus deactivating the flip-flop’s $CLEAR$ input.

If $P$ is LOW the situation is reversed. The upper NAND output is HIGH and the lower NAND has a LOW output, activating the flip-flop’s $CLEAR$ input, resetting $Q$. Thus, $Q$ will be the same value as $P$ when the $ALOAD$ input is asserted. When $ALOAD$ is not asserted ($= 0$), both NAND outputs are HIGH and thus do not activate either the preset or clear function of the flip-flop.

Figure 9.29 shows the asynchronous load circuit with an asynchronous clear (reset) function added. The flip-flop can be cleared by a logic LOW either from the $P$ input (via the lower NAND gate) or the $CLEAR$ input pin. The clear function disables the upper NAND gate when it is LOW, preventing the flip-flop from being cleared and preset simultaneously. This extra connection also ensures that the clear function has priority over the load function.
EXAMPLE 9.6

Use MAX+PLUS II to redraw the circuit in Figure 9.29 to create a general element called \texttt{al\_count} that can be used in a synchronous counter with asynchronous load and clear. (Refer to Figure 9.25 for a similar element with \textit{synchronous} load.)

Solution

Figure 9.30 shows the modified circuit, which includes an XOR gate for part of the count logic. The remainder of the count logic must be supplied externally to this element for each bit of the counter.

EXAMPLE 9.7

Draw a circuit with four instances of \texttt{al\_count} (from Example 9.6) to make a 4-bit synchronous counter with asynchronous load and reset. Create a simulation that tests the function of the counter.

Solution

Figure 9.31 shows the circuit. (Compare this circuit to the counter with synchronous load in Figure 9.26. This difference between the two is in the load function, not the count logic.)

The Boolean function applied to the \textit{COUNT} input of each instance of \texttt{al\_count} consists of the logical product of all previous output bits. (\(COUNT_3 = Q_2 Q_1 Q_0\), \(COUNT_2 = Q_1 Q_0\), \(COUNT_1 = Q_0\), \(COUNT_0 = 1\).) When combined with the XOR at the \textit{COUNT} input
of each element, this yields the Boolean equations for a binary counter based on D flip-flops, as derived in Example 9.5. The circuitry inside each instance of \texttt{al\_count} also generates the asynchronous load and clear functions.

Figure 9.32 shows a MAX+PLUS II simulation of the counter. The counter cycles through its full range and continues. A pulse at 700 ns loads the counter with the value \(9H\) (\(=\ 1001_2\)), after which the count continues from that point.

**FIGURE 9.31**
Example 9.7
4-bit Counter with Asynchronous Load and Reset

**FIGURE 9.32**
Example 9.7
Simulation of a 4-bit Counter with Asynchronous Load and Reset
The reset pulse at 900 ns clears the counter. The LOAD pulse starting at 1.02 μs shows how the load function has precedence over the count function. When LOAD is asserted, 9H is loaded and the count does not increase until LOAD is deasserted. The RESET pulse at 1.08 μs overrides both load and count functions. When RESET is deasserted, 9H is asynchronously reloaded.

Count Enable

The counter elements in Figures 9.25 (sl_count) and 9.30 (al_count) are just D flip-flops configured for switchable toggle operation with additional circuitry for load and clear functions. Normally, when these elements are used in synchronous counters, the count progresses when the input to the element’s XOR gate goes HIGH. In other words, the count progresses when the counter element is switched from a no change to a toggle mode.

In order to arrest the count sequence, we must disable the count logic of the counter circuit. Figure 9.33 shows a simple modification to the 4-bit counter circuit of Figure 9.26 that can achieve this function. Each AND gate has an extra input which is used to enable or inhibit the count logic function to each flip-flop.

Figure 9.34 shows a simulation of the counter. Note that the count progresses normally when COUNT_ENA is HIGH and stops when COUNT_ENA is LOW, even though the clock pulses remain constant throughout the simulation.

Also note that the count enable has no effect on the synchronous load and asynchronous reset functions. In the latter part of the simulation, the count stops at AH (Q3Q2Q1Q0 = 10102), when COUNT_ENA goes LOW. At 760 ns, the synchronous load function loads the value of 9H into the counter. The counter stays at this value, even after LOAD is no longer active, since the count is still disabled. At 880 ns, an asynchronous reset pulse clears the counter. The count resumes on the first clock pulse after COUNT_ENA goes HIGH again.

Bidirectional Counters

Figure 9.35 shows the logic diagram of a 4-bit synchronous DOWN counter. Its count sequence starts at 1111 and counts backwards to 0000, then repeats. The Boolean equations for this circuit will not be derived at this time, but will be left for an exercise in an end-of-chapter problem.

We can intuitively analyze the operation of the counter if we understand that the upper three flip-flops will each toggle when their associated XOR gates have a HIGH input from the rest of the count logic.

Q0 is set to toggle on each clock pulse. Q1 toggles whenever Q0 is LOW (every second clock pulse, at states 1110, 1100, 1010, 1000, 0110, 0100, 0010, and 0000). Q2 toggles when Q1 AND Q0 are LOW (1100, 1000, 0100, and 0000). Q3 toggles when Q2 AND Q1 AND Q0 are LOW (1000 and 0000). The result of this analysis can be represented by a timing diagram, such as the simulation shown in Figure 9.36. As we expect, the counter will count down from 1111 (FH) to 0000 (0H) and repeat.

We can create a bidirectional counter by including a circuit to select count logic for an UP or DOWN sequence. Figure 9.37 shows a basic synchronous counter element that can be used to create a synchronous counter. The element is simply a D flip-flop configured for switchable toggle mode.

Four of these elements can be combined with selectable count logic to make a 4-bit bidirectional counter, as shown in Figure 9.38. Each counter element has a pair of AND-shaped gates and an OR gate to steer the count logic to the XOR in the element. When DIR = 1, the upper gate in each pair is enabled and the lower gates disabled,
FIGURE 9.33
4-bit Counter with Synchronous Load, Asynchronous Reset, and Count Enable

FIGURE 9.34
Simulation of 4-bit Counter with Synchronous Load, Asynchronous Reset, and Count Enable
FIGURE 9.35
4-bit Synchronous DOWN Counter

FIGURE 9.36
4-bit DOWN Counter Simulation
steering the UP count logic to the counter element. When $DIR = 0$, the lower gate in each
pair is enabled, steering the DOWN count logic to the counter element. The directional
function can also be combined with the load and count enable functions, as was shown for
unidirectional UP counters.

Figure 9.39 shows a simulation of the bidirectional counter of Figure 9.38. The
waveforms show the UP count when $DIR$ is HIGH and the DOWN count when $DIR$
is LOW.
Decoding the Output of a Counter

Figure 9.40 shows a graphic design file of a 4-bit bidirectional counter with an output decoder. The counter is the one shown in Figure 9.38, represented as a logic circuit symbol. The decoder component decode16 is a module written in VHDL, as listed below.

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY decode16 IS
  PORT(
    sel : IN   INTEGER RANGE 0 to 15;
    y   : OUT  BIT_VECTOR (0 to 15));
END decode16;

ARCHITECTURE a OF decode16 IS
BEGIN
  WITH sel SELECT
  y <= x"7FFF" WHEN 0,
       x"BFFF" WHEN 1,
       x"DFFF" WHEN 2,
       x"EFFF" WHEN 3,
       x"F7FF" WHEN 4,
       x"FBFF" WHEN 5,
       \text{otherwise} y;
END a;
```

FIGURE 9.39
Simulation of 4-bit Bidirectional Counter

FIGURE 9.40
4-bit Bidirectional Counter with Output Decoder
The decoder has 16 outputs, one for each state of the counter. For each state, one and only one output will be low. (Refer to the section on binary decoders in Chapter 5 for a more detailed description of $n$-line-to-$m$-line binary decoders.)

Figure 9.41 shows a portion of the simulation waveforms (i.e., only the count value and the decoder outputs) for the circuit in Figure 9.40. As the count progresses up or down, as shown by the waveform for $Q[3..0]$, the decoder outputs respond by going LOW in sequence.

Output decoders for binary counters can also be configured to have active HIGH outputs. In this case, one and only one output would be HIGH for each output state of the counter.

**Terminal Count and RCO**

A special case of output decoding is a circuit that will detect the terminal count, or last state, of a count sequence and activate an output to indicate this state. The terminal count depends on the count sequence. A 4-bit binary UP counter has a terminal count of 1111; a 4-bit binary DOWN counter has a terminal count of 0000. A circuit to detect these conditions must detect the *maximum value* of an UP count and the *minimum value* of a DOWN count.

![FIGURE 9.41](Simulation of 4-bit Decoder)
The decoder shown in Figure 9.42 fulfills both of these conditions. The directional input \( \text{DIR} \) enables the upper gate when HIGH and the lower gate when LOW. Thus, the upper gate generates a HIGH output when \( \text{DIR} = 1 \) AND \( Q_3Q_2Q_1Q_0 = 1111 \). The lower gate generates a HIGH when \( \text{DIR} = 0 \) AND \( Q_3Q_2Q_1Q_0 = 0000 \).

Figure 9.43 shows the terminal count decoder combined with a 4-bit bidirectional counter. The decoder is also used to enable a NAND gate output that generates an \( \text{RCO} \) signal. \( \text{RCO} \) stands for ripple carry out or ripple clock out. The purpose of \( \text{RCO} \) is to produce exactly one clock pulse upon terminal count and have the positive edge of \( \text{RCO} \) at the end of the counter cycle, for a counter that has a positive edge-triggered clock.

This function is generally found in counters with a fixed number of bits (i.e., fixed-function counter chips, not PLDs) and is used to asynchronously clock a further counter stage, as in Figure 9.44. This allows us to extend the width of the counter beyond the number of bits available in the fixed-function device. This is not necessary when designing synchronous counters in programmable logic, but is included for the sake of completeness.
The NAND gate in Figure 9.43 is enabled upon terminal count and passes the clock signal through to $R_{CO}$. The NAND output sits HIGH when inhibited. The clock is inverted in the RCO circuit so that when the NAND gate inverts it again, the circuit generates a clock pulse in true form.

Figure 9.45 shows the simulation of the circuit of Figure 9.43. In the first half of the simulation, the counter is counting DOWN. The terminal count decoder output, $MAX\_MIN$, goes HIGH when $Q_3Q_2Q_1Q_0 = 0000$. $R_{CO}$ generates a pulse at that time. For the second half, the counter is counting UP. $MAX\_MIN$ is HIGH when $Q_3Q_2Q_1Q_0 = 1111$ and $R_{CO}$ generates a pulse at that time.

Note that the $R_{CO}$ pulse appears to be half the width of the $MAX\_MIN$ pulse. Although the NAND gate that generates $R_{CO}$ is enabled for the whole $MAX\_MIN$ pulse, the clock input is HIGH for the first half-period, which is the same as the $R_{CO}$ inhibit level.

The positive edge of $R_{CO}$ is at the end of the pulse. The idea is to synchronize the positive edge of the clock with the positive edge of $R_{CO}$. However, since the RCO decoder is combinational, a propagation delay of about 7 ns is introduced.

SECTION 9.5 REVIEW PROBLEM

9.5 Figure 9.46 shows two presettable counters, one with asynchronous load and clear, the other with synchronous load and clear. The counter with asynchronous functions has a 4-bit output labeled $QA$. The synchronously loaded counter has a 4-bit output labeled $QS$. The load and reset inputs to both counters are active LOW.
9.6 Programming Presettable and Bidirectional Counters in VHDL

Figure 9.47 shows a partial timing diagram for the counters. Complete the diagram.

9.6 Programming Presettable and Bidirectional Counters in VHDL

The presettable counters and bidirectional counters described in the previous section can be easily implemented in VHDL, either as behavioral descriptions or as LPM components. We will initially examine the behavioral descriptions of two counters, one with asynchronous load and clear and one with synchronous load and clear. We will then examine some options available in the module `lpm_counter`.
**Behavioral Description**

The following lists the VHDL code for an 8-bit bidirectional counter with count enable, terminal count decoding, and asynchronous load and clear:

```vhdl
ENTITY pre_ct8a IS
  PORT (
    clk, count_ena : IN BIT;
    clear, load, direction : IN BIT;
    p : IN INTEGER RANGE 0 TO 255;
    max_min : OUT BIT;
    qd : OUT INTEGER RANGE 0 TO 255);
END pre_ct8a;
ARCHITECTURE a OF pre_ct8a IS
BEGIN
  PROCESS (clk, clear, load)
  VARIABLE cnt : INTEGER RANGE 0 TO 255;
  BEGIN
    IF (clear = '0') THEN              —— Asynchronous clear
      cnt := 0;
    ELSIF (load = '1' and clear = '1') THEN —— Asynchronous load
      cnt := p;
    ELSE
      IF (clk'EVENT and clk = '1') THEN
        IF (count_ena = '1' and direction = '0') THEN
          cnt :=cnt - 1;
        ELSIF (count_ena = '1' and direction = '1') THEN
          cnt :=cnt + 1;
        END IF;
      END IF;
    END IF;
    qd <= cnt;
    —— Terminal count decoder
    IF (cnt = 0 and direction = '0') THEN
      max_min <= '1';
    ELSIF (cnt = 255 and direction = '1') THEN
      max_min <= '1';
    ELSE
      max_min <= '0';
    END IF;
  END PROCESS;
END a;
```

The load and clear functions of this counter are asynchronous, so these identifiers are part of the sensitivity list of the PROCESS statement; the statements in the process will execute if there is a change on the clear, load, or clock inputs. Load and clear are checked by IF statements, independently of the clock. Since load and clear are checked first, they have precedence over the clock. Clear has precedence over load since load can only activate if clear is not active.

If clear and load are not asserted, the clock status is checked by a clause in an IF statement: \( \text{IF } (\text{clk'EVENT and CLK } = '1') \text{ THEN}\). If this condition is true, then a count variable is incremented or decremented, depending on the states of a count enable input and a directional control input. If the count enable input is not asserted, the count is neither incremented nor decremented.
The count value is assigned to the counter outputs by the signal assignment statement
(qd <= cnt;) after the clear, load, clock, count enable, and direction inputs have been
evaluated. Possible results from the signal assignment are:

- \( qd = 0 \) (clear = 0),
- \( qd = p \) (load = 1 AND clear = 1),
- **increment** \( qd \) (count_ena = 1 AND direction = 1),
- **decrement** \( qd \) (count_ena = 1 AND direction = 0), or
- **no change on** \( qd \) (count_ena = 0).

The terminal count is decoded by determining the count direction and value of the
count variable. If the count is UP and the count value is maximum (255\(_{10} = \text{FFH}\)) or the
count is DOWN and the count value is minimum (0\(_{10} = 00H\)), a terminal count decoder output called \( \text{max\_min} \) goes HIGH.

The code for the same 8-bit counter, but with synchronous clear and load, is shown next.

```
ENTITY pre_ct8s IS
    PORT (
        clk, count_ena : IN BIT;
        clear, load, direction : IN BIT;
        p : IN INTEGER RANGE 0 TO 255;
        max_min : OUT BIT;
        qd : OUT INTEGER RANGE 0 TO 255);
END pre_ct8s;
ARCHITECTURE a OF pre_ct8s IS
BEGIN
    PROCESS (clk)
    VARIABLE cnt : INTEGER RANGE 0 TO 255;
    BEGIN
        IF (clk'EVENT AND clk = '1') THEN
            IF (clear = '0') THEN —— Synchronous clear
                cnt := 0;
            ELSIF (load = '1') THEN —— Synchronous load
                cnt := p;
            ELSIF (count_ena = '1' and direction = '0') THEN
                cnt := cnt - 1;
            ELSIF (count_ena = '1' and direction = '1') THEN
                cnt := cnt + 1;
            END IF;
            END IF;
            qd <= cnt;
            —— Terminal count decoder
            IF (cnt = 0 and direction = '0') THEN
                max_min <= '1';
            ELSIF (cnt = 255 and direction = '1') THEN
                max_min <= '1';
            ELSE
                max_min <= '0';
            END IF;
        END PROCESS;
    END a;
```

The PROCESS statement in the synchronous counter has only one identifier in its sens-
itivity list—that of the clock input. Load and clear status are not evaluated until after the
process checks for a positive clock edge. Otherwise the code is the same as for the asynchronously loading counter.

Figure 9.48 shows a detail of a simulation of the asynchronously loading counter. It shows the point where the count rolls over from FFH to 00H and activates the max_min output. The directional output changes shortly after this point and shows the terminal count decoding for a DOWN count, the point where the counter rolls over from 00H to FFH. In the UP count, max_min is HIGH when the counter output is FFH, but not 00H. In the DOWN count, max_min goes HIGH when the counter output is 00H, but not FFH.

Figure 9.49 shows the operation of the asynchronous load and clear functions. Figure 9.50 show the synchronous load and clear. The inputs are identical for each simulation; each has two pairs of load pulses and a pair of clear pulses. The first pulse of each pair is arranged so that it immediately follows a positive clock edge; the second pulse of each pair immediately precedes a positive clock edge.

In the counter with asynchronous load and clear, these functions are activated by the first pulse of each pair and again on the second pulse. For the counter with synchronous load and clear, only the second pulse of each pair has an effect, since the load and clear functions must be active during or just prior to an active clock edge, in order to satisfy
setup-time requirements of the counter flip-flops. The end of the load and clear pulse can correspond to the positive clock edge, as the flip-flop hold time is zero.

Also note that the counter with synchronous load and clear has no intermediate glitch states on its outputs. (The simulation for the asynchronously loading counter shows glitch states on output \( qd \) at 21.04 μs, 21.10 μs, 21.22 μs, and 21.44 μs. Refer to the section on Synchronous versus Asynchronous Circuits in Chapter 7 for further discussion of intermediate states in asynchronous circuits.)

Figures 9.51 and 9.52 show further simulation details for our two VHDL counters. Both show the priority of the load, clear, and count enable functions. Both diagrams show that load and clear are independent of count enable and that clear has precedence over load. Again note that the counter with synchronous load and clear is free of intermediate glitch states.
LPM Counters

Earlier in this chapter, we saw how a parameterized counter from the Library of Parameterized Modules (LPM) could be used as a simple 8-bit counter. The component lpm_counter has a number of other functions that can be implemented using specific ports and parameters. These functions are indicated in Table 9.13.

<table>
<thead>
<tr>
<th>Function</th>
<th>Ports</th>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic count operation</td>
<td>clock, q[]</td>
<td>LPM_WIDTH</td>
<td>Output q[] increases by one with each positive clock edge. LPM_WIDTH is the number of output bits.</td>
</tr>
<tr>
<td>Synchronous load</td>
<td>slod, data[]</td>
<td>none</td>
<td>When slod = 1, output q[] goes to the value at input data[] on the next positive clock edge. data[] has the same width as q[].</td>
</tr>
<tr>
<td>Synchronous clear</td>
<td>sclr</td>
<td>none</td>
<td>When sclr = 1, output q[] goes to zero on positive clock edge.</td>
</tr>
<tr>
<td>Synchronous set</td>
<td>sset</td>
<td>LPM_SVALUE</td>
<td>When sset = 1, output goes to value of LPM_SVALUE on positive clock edge. If LPM_SVALUE is not specified, q[] goes to all 1s.</td>
</tr>
<tr>
<td>Asynchronous load</td>
<td>aload, data[]</td>
<td>none</td>
<td>Output goes to value at data[] when aload = 1.</td>
</tr>
<tr>
<td>Asynchronous clear</td>
<td>aclr</td>
<td>none</td>
<td>Output goes to zero when aclr = 1.</td>
</tr>
<tr>
<td>Asynchronous set</td>
<td>aset</td>
<td>LPM_AVALUE</td>
<td>Output goes to value of LPM_AVALUE when aset = 1. If LPM_AVALUE is not specified, outputs all go HIGH when aset = 1.</td>
</tr>
<tr>
<td>Directional control</td>
<td>updown</td>
<td>LPM_DIRECTION</td>
<td>Optional direction control. Default direction is UP. Only one of updown and LPM_DIRECTION can be used. updown = 1 for UP count, updown = 0 for DOWN count. LPM_DIRECTION = “UP”, “DOWN”, or “DEFAULT”</td>
</tr>
<tr>
<td>Count enable</td>
<td>cnt_en</td>
<td>none</td>
<td>When cnt_en = 1, count proceeds upon positive clock edges. No effect on other synchronous functions (sload, sclr, sset). Defaults to “enabled” when not specified.</td>
</tr>
<tr>
<td>Clock enable</td>
<td>clk_en</td>
<td>none</td>
<td>All synchronous functions are enabled when clk_en = 1. Defaults to “enabled” when not specified.</td>
</tr>
<tr>
<td>Modulus control</td>
<td>none</td>
<td>LPM_MODULUS</td>
<td>Modulus of counter is set to value of LPM_MODULUS.</td>
</tr>
<tr>
<td>Output decoding</td>
<td>eq[15..0]</td>
<td>none</td>
<td>Sixteen active-HIGH decoded outputs, one for each internal counter value from 0 to 15.</td>
</tr>
</tbody>
</table>

The only ports that are required by an LPM counter are clock, and one of q[] (counter outputs) or eq[] (decoder outputs). The only required parameter is LPM_WIDTH, which specifies the number of counter output bits. All other ports and parameters are optional, although certain ones must be used together. (For instance, ports sload and data[] are optional, but both must be used for the synchronous load function.) If unused, a port or parameter will be held at a default logic level.

To use any of the functions of an LPM component in a VHDL file, we use a component instantiation statement and specify the required parameters in a generic map and the ports in a port map.
The VHDL component declaration, shown below, indicates that all parameters except LPM_WIDTH are defined as having type STRING, which requires the parameter value to be written in double quotes, even if numeric. (e.g., LPM_MODULUS => "12"). Since LPM_WIDTH is defined as type POSITIVE (i.e., any integer > 0) it must be written without quotes (e.g., LPM_WIDTH => 8). Default values of all ports and parameters are also included in the component declaration (e.g., clk_en: IN STD_LOGIC := '1'; the default value of the clock enable input is ‘1’). The LPM component declaration can also be found in the MAX+PLUS II Help menu (Help; Megafunctions/LPM; lpm_counter).

VHDL Component Declaration for lpm_counter:

COMPONENT lpm_counter
  GENERIC (LPM_WIDTH: POSITIVE;
    LPM_MODULUS: STRING := "UNUSED";
    LPM_AVALUE: STRING := "UNUSED";
    LPM_SVALUE: STRING := "UNUSED";
    LPM_DIRECTION: STRING := "UNUSED";
    LPM_TYPE: STRING := "L_COUNTER";
    LPM_PVALUE: STRING := "UNUSED";
    LPM_HINT : STRING := "UNUSED";
  )
  PORT (data: IN STD_LOGIC_VECTOR (LPM_WIDTH-1 DOWNTO 0) := (OTHERS => '0');
    clock: IN STD_LOGIC;
    cin: IN STD_LOGIC := '0';
    clk_en: IN STD_LOGIC := '1';
    cnt_en: IN STD_LOGIC := '1';
    updown: IN STD_LOGIC := '1';
    sload: IN STD_LOGIC := '0';
    sset: IN STD_LOGIC := '0';
    sclr: IN STD_LOGIC := '0';
    aclr: IN STD_LOGIC := '0';
    cout: OUT STD_LOGIC;
    q: OUT STD_LOGIC_VECTOR (LPM_WIDTH-1 DOWNTO 0) );
END COMPONENT;

EXAMPLE 9.8
Write a VHDL file for an 8-bit LPM counter with ports for the following functions: asynchronous load, asynchronous clear, directional control, and count enable.

Solution  The required VHDL file is shown below. Note that no behavioral descriptions are required for the functions, only a mapping from the defined port names to the entity inputs and outputs.

-- pre_lpm8
-- 8-bit presettable counter with asynchronous clear and load,
-- count enable, and a directional control port

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;
ENTITY pre_lpm8 IS
  PORT (  
    clk, count_ena : IN     STD_LOGIC;  
    clear, load, direction : IN    STD_LOGIC;  
    p                : IN     STD_LOGIC_VECTOR(7 downto 0);  
    qd               : OUT    STD_LOGIC_VECTOR(7 downto 0));
END pre_lpm8;

ARCHITECTURE a OF pre_lpm8 IS
BEGIN
  counter1: lpm_counter  
    GENERIC MAP (LPM_WIDTH => 8)  
    PORT MAP ( clock  => clk,  
               updown => direction,  
               cnt_en => count_ena,  
               data   => p,  
               aload  => load,  
               aclr   => clear,  
               q      => qd);
END a;

EXAMPLE 9.9

Write a VHDL file that uses an LPM counter to generate a DOWN counter with a modulus of 500. Create a MAX+PLUS II simulation file to verify the counter’s operation.

Solution  A mod-500 counter requires nine bits since $2^8 < 500 < 2^9$. Since the counter always counts DOWN, we can use the parameter LPM_DIRECTION to specify the DOWN counter rather than using an unnecessary port. The required VHDL code is given below.

Note that the value of LPM_WIDTH is written without quotes, since it is defined as type POSITIVE in the component declaration. LPM_MODULUS and LPM_DIRECTION are written in double quotes, since the component declaration defines them as type STRING.

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY mod5c_lpm IS
  PORT (  
    clk : IN      STD_LOGIC;  
    q   : OUT     STD_LOGIC_VECTOR (8 downto 0) );
END mod5c_lpm;

ARCHITECTURE a OF mod5c_lpm IS
BEGIN
  counter1: lpm_counter  
    GENERIC MAP (LPM_WIDTH => 9,  
                  LPM_DIRECTION => "DOWN",  
                  LPM_MODULUS => "500")  
    PORT MAP ( clock => clk,  
                q    => q);
END a;

Figure 9.53 shows a partial simulation of the counter, indicating the point at which the output rolls over from 0 to 499 (decimal).
If we are designing a counter for the Altera UP-1 circuit board, we can simulate the on-board oscillator by choosing a clock period of 40 ns, which corresponds to a clock frequency of 25 MHz. The default simulation period is from 0 to 1 \( \mu s \), which only gives 1 \( \mu s \) \( \div 40 \text{ ns/clock period} = 25 \text{ clock periods} \). This is not enough time to show the entire count cycle. The minimum value for the end of the simulation time is:

\[
40 \text{ ns/clock period} \times 500 \text{ clock periods} = 20000 \text{ ns} = 20 \mu s.
\]

If we wish to see a few clock cycles past the recycle point, we can set the simulation end time to 20.1 \( \mu s \). (In the MAX+PLUS II Simulator window, select File menu; End Time. Enter the value 20.1us (no spaces) into the Time window and click OK.)

To view the count waveform, \( q \), in decimal rather than hexadecimal, select the waveform by clicking on it. Either right-click to get a pop-up menu or select Enter Group from the simulator Node menu, as in Figure 9.54. This will bring up the Enter Group dialog box shown in Figure 9.55. Select DEC (for decimal) and click OK.
EXAMPLE 9.10

Write a VHDL file that instantiates a 12-bit LPM counter with asynchronous clear and synchronous set functions. Design the counter to set to 2047 (decimal). Create a simulation to verify the counter operation.

Solution The required VHDL file is:

```vhdl
-- sset_lpm.vhd
-- 12-bit LPM counter with sset and aclr
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;
ENTITY sset_lpm IS
    PORT(
        clk        : IN  STD_LOGIC;
        clear, set : IN  STD_LOGIC;
        q          : OUT STD_LOGIC_VECTOR (11 downto 0));
END sset_lpm;
ARCHITECTURE a OF sset_lpm IS
BEGIN
    counter1: lpm_counter
        GENERIC MAP (LPM_WIDTH => 12,
                      LPM_SVALUE => "2047")
        PORT MAP ( clock => clk,
                    sset => set,
                    aclr => clear,
                    q    => q);
END a;
```

Figure 9.56 shows the simulation file of the counter. The full count sequence would take over 160 μs, so we will assume the count portion of the design works properly. Only the set and clear functions are fully simulated. The count waveform is shown in decimal.

SECTION 9.6 REVIEW PROBLEM

9.6 The first part of a VHDL process statement includes a sensitivity list: PROCESS (sensitivity list). How should this be written for a counter with asynchronous clear and for a counter with synchronous clear?
9.7 Shift Registers

**KEY TERMS**

**Shift register** A synchronous sequential circuit that will store and move $n$-bit data, either serially or in parallel, in $n$ flip-flops.

**SRGn** Abbreviation for an $n$-bit shift register (e.g., SRG4 indicates a 4-bit shift register).

**Serial shifting** Movement of data from one end of a shift register to the other at a rate of one bit per clock pulse.

**Parallel transfer** Movement of data into all flip-flops of a shift register at the same time.

**Rotation** Serial shifting of data with the output(s) of the last flip-flop connected to the synchronous input(s) of the first flip-flop. The result is continuous circulation of the same data.

**Right shift** A movement of data from the left to the right in a shift register. (Right is defined in MAX+PLUS II as toward the LSB.)

**Left shift** A movement of data from the right to the left in a shift register. (Left is defined in MAX+PLUS II as toward the MSB.)

**Bidirectional shift register** A shift register that can serially shift bits left or right according to the state of a direction control input.

**Parallel-load shift register** A shift register that can be preset to any value by directly loading a binary number into its internal flip-flops.

**Universal shift register** A shift register that can operate with any combination of serial and parallel inputs and outputs (i.e., serial in/serial out, serial in/parallel out, parallel in/serial out, parallel in/parallel out). A universal shift register is often bidirectional, as well.

A shift register is a synchronous sequential circuit used to store or move data. It consists of several flip-flops, connected so that data are transferred into and out of the flip-flops in a standard pattern.

Figure 9.57 represents three types of data movement in three 4-bit shift registers. The circuits each contain four flip-flops, configured to move data in one of the ways shown.

Figure 9.57a shows the operation of serial shifting. The stored data are taken in one at a time from the input and moved one position toward the output with each applied clock pulse.

Parallel transfer is illustrated in Figure 9.57b. As with the synchronous parallel load function of a presettable counter, data move simultaneously into all flip-flops when a clock pulse is applied. The data are available in parallel at the register outputs.

Rotation, depicted in Figure 9.57c, is similar to serial shifting in that data are shifted one place to the right with each clock pulse. In this operation, however, data are continuously circulated in the shift register by moving the rightmost bit back to the leftmost flip-flop with each clock pulse.

**Serial Shift Registers**

Figure 9.58 shows the most basic shift register circuit: the serial shift register, so called because data are shifted through the circuit in a linear or serial fashion. The circuit shown consists of four D flip-flops connected in cascade and clocked synchronously.

For a D flip-flop, $Q$ follows $D$. The value of a bit stored in any flip-flop after a clock pulse is the same as the bit in the flip-flop to its left before the pulse. The result is that when a clock pulse is applied to the circuit, the contents of the flip-flops move one position to the
right and the bit at the circuit input is shifted into $Q_3$. The bit stored in $Q_0$ is overwritten by the former value of $Q_1$ and is lost. Since the data move from left to right, we say that the shift register implements a right shift function. (Data movement in the other direction, requiring a different circuit connection, is called left shift.)

Let us track the progress of data through the circuit in two cases. All flip-flops are initially cleared in each case.

**Case 1**: A 1 is clocked into the shift register, followed by a string of 0s, as shown in Figure 9.59. The flip-flop containing the 1 is shaded.

Before the first clock pulse, all flip-flops are filled with 0s. Data In goes to a 1 and on the first clock pulse, the 1 is clocked into the first flip-flop. After that, the input goes to 0. The 1 moves one position right with each clock pulse, the register filling up with 0s behind it, fed by the 0 at Data In. After four clock pulses, the 1 reaches the Data Out flip-flop. On the fifth pulse, the 0 coming behind overwrites the 1 at $Q_0$, leaving the register filled with 0s.
**Case 2:** Figure 9.60 shows a shift register, initially cleared, being filled with 1s.

As before, the initial 1 is clocked into the shift register and reaches the Data Out line on the fourth clock pulse. This time, the register fills up with 1s, not 0s, because the Data input remains HIGH.

Figure 9.61 shows a MAX+PLUS II simulation of the 4-bit serial shift register in Figure 9.58 through 9.60. The first half of the simulation shows the circuit operation for Case
FIGURE 9.60
Filling a Shift Register with “1”s (Shift Right)

FIGURE 9.61
Simulation of a 4-bit Shift Register (Shift Right)
1, above. The 1 enters the register at \( Q_3 \) on the first clock pulse after **serial_in** (Data In) goes HIGH. The 1 moves one position for each clock pulse, which is seen in the simulation as a pulse moving through the \( Q \) outputs.

Case 2 is shown in the second half of the simulation. Again, a 1 enters the register at \( Q_3 \). The 1 continues to be applied to **serial_in**, so all \( Q \) outputs stay HIGH after receiving the 1 from the previous flip-flop.

**NOTE**

Conventions differ about whether the rightmost or leftmost bit in a shift register should be considered the most significant bit. The Altera Library of Parameterized Modules uses the convention of the leftmost bit being the MSB, so this is the convention we will follow. The convention has no physical meaning; the concept of right or left shift only makes sense on a logic diagram. The actual flip-flops may be laid out in any configuration at all in the physical circuit and still implement the right or left shift functions as defined on the logic diagram. (That is to say, wires, circuit board traces, and internal programmable logic connections can run wherever you want; left and right are defined on the logic diagram.)

**EXAMPLE 9.11**

Use the MAX+PLUS II Graphic Editor to create the logic diagram of a 4-bit serial shift register that shifts left, rather than right.

**Solution** Figure 9.62 shows the required logic diagram. The flip-flops are laid out the same way as in Figure 9.58, with the MSB (\( Q_3 \)) on the left. The \( D \) input of each flip-flop is connected to the \( Q \) output of the flip-flop to its right, resulting in a looped-back connection. A bit at \( D_0 \) is clocked into the rightmost flip-flop. Data in the other flip-flops are moved one place to the left. The bit in \( Q_2 \) overwrites \( Q_3 \). The previous value of \( Q_3 \) is lost.

![Figure 9.62](image)

**FIGURE 9.62**

4-bit Serial Shift Register Configured to Shift Left

**EXAMPLE 9.12**

Draw a diagram showing the movement of a single 1 through the register in Figure 9.62. Also draw a diagram showing how the register can be filled up with 1s.

**Solution** Figures 9.63 and 9.64 show the required data movements.
FIGURE 9.63
Shifting a “1” Through a Shift Register (Shift Left)
FIGURE 9.64  Filling a Shift Register with "1"s (Shift Left)
EXAMPLE 9.13

Use the MAX+PLUS II simulator to verify the operation of the shift-left serial shift register in Figure 9.62.

Solution Figure 9.65 shows the simulation of the shift operations shown in Example 9.12. Compare this simulation to the one in Figure 9.61 to see how the opposite shift direction appears on a timing diagram.

FIGURE 9.65
Simulation of a 4-bit Shift Register (Shift Left)

Bidirectional Shift Registers

Figure 9.66 shows the logic diagram of a bidirectional shift register. This circuit combines the properties of the right shift and left shift circuits, seen earlier in Figures 9.58 and 9.62. This circuit can serially move data right or left, depending on the state of a control input, called DIRECTION.

The shift direction is controlled by enabling or inhibiting four pairs of AND-OR circuit paths that direct the bits at the flip-flop outputs to other flip-flop inputs. When DIRECTION = 0, the right-hand AND gate in each pair is enabled and the flip-flop outputs are directed to the D inputs of the flip-flops one position left. Thus the enabled pathway is from Left_Shift_In to Q₀, then to Q₁, Q₂, and Q₃.

When DIRECTION = 1, the left-hand AND gate of each pair is enabled, directing the data from Right_Shift_In to Q₃, then to Q₂, Q₁, and Q₀. Thus, DIRECTION = 0 selects left shift and DIRECTION = 1 selects right-shift.

Figure 9.67 shows a MAX+PLUS II simulation of the bidirectional shift register in Figure 9.66. The simulation shows the left shift function from 0 to 500 ns and right shift after 500 ns. Both Right_Shift_In and Left_Shift_In are applied in both parts of the simulation, but the circuit responds only to one for each function.

For the left shift function, a 1 is applied to Q₀ at 140 ns and shifted left. The Right_Shift_In pulse is ignored. Similarly, for the right shift function, a 1 is applied to Q₃ at 540 ns and shifted right. Left_Shift_In is ignored.
FIGURE 9.66
Bidirectional Shift Register

Left_Shift_In

Right_Shift_In

DIRECTION

DFF

CLRN

PRN

QD

OUTPUT Q1

OUTPUT Q0

OUTPUT Q2

OUTPUT Q3

CLOCK

INPUT

NOT
Shift Register with Parallel Load

Earlier in this chapter, we saw how a counter could be set to any value by synchronously loading a set of external inputs directly into the counter flip-flops. We can implement the same function in a shift register, as shown in Figure 9.68.

The circuit is similar to that of the bidirectional shift register in Figure 9.66. The synchronous input of each flip-flop is fed by an AND-OR circuit that directs one of two signals to the flip-flop: the output of the previous flip-flop (shift function) or a parallel input (load function). The circuit is configured such that the shift function is enabled when LO\textsubscript{D} = 0 and the load function is enabled when LO\textsubscript{D} = 1.

Figure 9.69 shows a simulation of the parallel-load shift register circuit of Figure 9.68. In the first part of the simulation, the shift function is selected. This is tested by sending a 1 through the circuit in a right-shift pattern. Next, at 400 ns, LO\textsubscript{D} goes HIGH, and the parallel input value AH (\(= 1010\)) is synchronously loaded into the circuit. The LO\textsubscript{D} input goes LOW, thus causing the circuit to revert to the shift function. The data in the register are right-shifted out, followed by 0s. At 640 ns, the value FH (\(= 1111\)) is loaded into the circuit, then right-shifted out.

Figure 9.70 shows the logic circuit of a universal shift register. This circuit can implement any combination of serial and parallel inputs and outputs. It can also serially shift data left or right or hold data, depending on the states of \(S_1\) and \(S_0\), which form a 2-bit function select input.

Each AND-OR circuit acts as a multiplexer to direct one of several possible data sources to the synchronous inputs of each flip-flop. For instance, if we trace the paths through the corresponding AND-OR circuit, we find that the possible sources of data at \(D_2\), the synchronous input of the second flip-flop, are \(Q_3\) (\(S_1S_0 = 01\)), \(P_2\) (\(S_1S_0 = 11\)), \(Q_1\) (\(S_1S_0 = 10\)), and \(Q_2\) (\(S_1S_0 = 00\)). These are the inputs required for the right-shift, parallel load, left-shift, and hold functions, respectively. All functions are synchronous, including the parallel load and hold functions.

The hold function is a synchronous no change function, implemented by feeding back the \(Q\) output of a flip-flop to its synchronous (\(D\)) input. It is necessary to have this function, so that the flip-flops will not synchronously clear when none of the other functions is selected.
FIGURE 9.68
Serial Shift Register with Parallel Load
Table 9.14 summarizes the various possible inputs to each flip-flop as a function of $S_1$ and $S_0$.

Table 9.14  Flip-Flop Inputs as a Function of $S_1S_0$ in a Universal Shift Register

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_0$</th>
<th>Function</th>
<th>$D_3$</th>
<th>$D_2$</th>
<th>$D_1$</th>
<th>$D_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Hold</td>
<td>$Q_3$</td>
<td>$Q_2$</td>
<td>$Q_1$</td>
<td>$Q_0$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>Shift Right</td>
<td>$RSI^*$</td>
<td>$Q_3$</td>
<td>$Q_2$</td>
<td>$Q_1$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>Shift Left</td>
<td>$Q_2$</td>
<td>$Q_1$</td>
<td>$Q_0$</td>
<td>$LSI^{**}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>Load</td>
<td>$P_3$</td>
<td>$P_2$</td>
<td>$P_1$</td>
<td>$P_0$</td>
</tr>
</tbody>
</table>

$^*$ $RSI$ = Right-shift input  
$^{**}$ $LSI$ = Left-shift input

---

**EXAMPLE 9.14**

Create a simulation file to verify the operation of the universal shift register of Figure 9.70.

**Solution**  Figure 9.71 shows a possible solution. The following functions are tested: hold, right shift ($LSI$ ignored), hold, left shift ($RSI$ ignored), load FH, asynchronous clear, load FH, shift right for two clocks, shift left for three clocks.
FIGURE 9.70
4-bit Universal Shift Register
SECTION 9.7 REVIEW PROBLEM

9.7 Can the D flip-flops in Figure 9.58 be replaced by JK flip-flops? If so, what modifications to the existing circuit are required?

9.8 Programming Shift Registers in VHDL

As with other circuit applications, we can take several approaches to programming shift registers in VHDL. Three basic design techniques are structural, dataflow, and behavioral descriptions. We will use each of these techniques to design a 4-bit shift register, such as the one shown in Figure 9.58.

**Structural Design**

Structural design is like taking components out of a bin and connecting them together to make a circuit. We can use the DFF component from the MAX+PLUS II primitives library and instantiate enough components to make a shift register, with connections made...
by internal signals. The code to make a 4-bit shift register using the structural design tech-
nique is shown here in the file `srg4strc.vhd`.

---- srg4strc.vhd
---- Structural description of a 4-bit serial shift register
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;

ENTITY srg4strc IS
  PORT(
      serial_in, clk : IN      STD_LOGIC;
      qo             : BUFFER  STD_LOGIC_VECTOR(3 downto 0) );
END srg4strc;

ARCHITECTURE right_shift of srg4strc IS
  COMPONENT DFF
    PORT (d : IN STD_LOGIC;
          clk : IN STD_LOGIC;
          q   : OUT STD_LOGIC);
  END COMPONENT;
  BEGIN
    flip_flop_3: dff
      PORT MAP (serial_in, clk, qo(3) );
    dffs:
      FOR i IN 2 downto 0 GENERATE
        flip_flops_2_to_0: dff
          PORT MAP (qo(i + 1), clk, qo(i) );
      END GENERATE;
  END right_shift;

The design entity `srg4strc.vhd` instantiates four D flip-flops from the `altera.maxplus2` package and connects them by assigning common inputs and outputs to related components. A different way of writing the component instantiations would be as follows.

```
flip_flop_3: dff
  PORT MAP (serial_in, clk, qo(3) );
flip_flop_2: dff
  PORT MAP(qo(3), clk, qo(2) );
flip_flop_1: dff
  PORT MAP(qo(2), clk, qo(1) );
flip_flop_0: dff
  PORT MAP(qo(1), clk, qo(0) );
```

Since the component ports are in the sequence (D, clk, Q), the component instantia-
tions shown above imply that the D input of a flip-flop is fed by the Q of the previous flip-flop.

The port identifier `qo` is defined as mode BUFFER, not as OUT, because it is some-
times used as an input and sometimes as an output. A port of mode OUT can only be used
as an output. A port of mode BUFFER has a feedback connection so that the output can be
reused in the programmed AND matrix of the CPLD macrocell. Figure 9.72 illustrates the
difference between these modes.

Rather than defining connections in the component instantiations, we would also be
able to use an internal signal to connect the flip-flops. This method allows us to use a
port of mode OUT, rather than BUFFER. The file `srg4str2.vhd` shows this alternative
way.
--srg4str2.vhd
-- Structural description of a 4-bit serial shift register
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;
ENTITY srg4str2 IS
  PORT (
    serial_in, clk : IN STD_LOGIC;
    qo        : OUT STD_LOGIC_VECTOR(3 downto 0) );
END srg4str2;
ARCHITECTURE right_shift of srg4str2 IS
  COMPONENT DFF
    PORT (d   : IN STD_LOGIC;
          clk : IN STD_LOGIC;
          q   : OUT STD_LOGIC);
  END COMPONENT;
  SIGNAL connect : STD_LOGIC_VECTOR(3 downto 0);
BEGIN
  flip_flop_3: dff
    PORT MAP (serial_in, clk, connect(3) );
  dffs:
    FOR i IN 2 downto 0 GENERATE
      flip_flops_2_to_0: dff
        PORT MAP (connect(i + 1), clk, connect(i) );
    END GENERATE;
  qo <= connect;
END right_shift;

In this case, the internal signal connect is used to tie the flip-flops together. The circuit output derives from a signal assignment statement at the end of the file. Since the internal signal connect is used to fulfil the flip-flop input/output functions, qo can be defined solely as an output.

Dataflow Design

Dataflow design describes a design entity in terms of the Boolean relationships between different parts of the circuit. The Boolean relationships in a 4-bit shift register are defined by the expressions for the flip-flop synchronous inputs:

\[
\text{QD} \land \text{Matrix} \lor \text{CLK} \land \text{PIN} \land n \lor \text{Connection to AND matrix}
\]
The design entity *srg4dflw.vhd* illustrates the use of the dataflow design method for a 4-bit serial shift register.

Before the flip-flops can be connected, they must be defined in a PROCESS statement. The statements inside the process are sequential, as they must be to define a flip-flop, but the process itself is a concurrent statement. Signals are applied concurrently (simultaneously) to the construct implied by the process (the flip-flops) and all other concurrent constructs in the design entity (the connections between \( q \) and \( d \) and the serial input).

A signal assignment statement implements the Boolean equations for the shift register. It is written as a single statement for efficiency, but could also be written as four separate assignment statements, as follows:

\[
\begin{align*}
  d(3) &= \text{serial\_in} \\
  d(2) &= q(3) \\
  d(1) &= q(2) \\
  d(0) &= q(1)
\end{align*}
\]

We must define \( q \) as mode BUFFER, since we are using it as both input and output.

### Behavioral Design

We can create a VHDL design entity from the description of its desired behavior. In the case of a shift register, we know that after a clock pulse all data move over one position and the first flip-flop in the chain accepts a bit from a serial input, as indicated in Table 9.15. We can use this behavioral description to implement a serial shift register, as shown in the VHDL file *srg4behv.vhd*.
In the behavioral design, we are not concerned with the flip-flop inputs or other internal connections; the behavioral description is sufficient for the VHDL compiler to synthesize the required hardware. Compare this to the dataflow description, where we created a set of flip-flops, then assigned Boolean functions to the \( D \) inputs. In this case, the behavioral design method combines these two steps into one.

---

**EXAMPLE 9.15**

Write the code for a VHDL design entity that implements a 4-bit bidirectional shift register with asynchronous clear. Create a simulation that verifies the design function.

**Solution**

The VHDL code for the bidirectional shift register, `srg4bidi.vhd`, follows. A CASE statement monitors the directional control of the shift register. We require the `others` clause of the CASE statement since the identifier `direction` is of type `STD_LOGIC`; the cases ‘0’ and ‘1’ do not cover all possible values of `STD_LOGIC`. Since we want no action to be taken in the default case, we use the keyword `NULL`.

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY srg4bidi IS
  PORT (
    clk, clear : IN STD_LOGIC;
    rsi, lsi   : IN STD_LOGIC;
    direction  : IN STD_LOGIC;
    q          : BUFFER STD_LOGIC_VECTOR(3 downto 0) );
END srg4bidi;

ARCHITECTURE bidirectional_shift of srg4bidi IS
BEGIN
  PROCESS (clk, clear)
  BEGIN
    IF clear = '0' THEN
      q <= "0000"; —— asynchronous clear
    ELSIF (clk'EVENT and clk = '1') THEN
      CASE direction IS
        WHEN '0' =>
          q <= q(2 downto 0) & lsi; —— left shift
```

---

---

```vhdl
  WHEN '1' =>
    q <= q(3 downto 0) & rs; —— right shift
  END CASE;
  END IF;
  END PROCESS;
END bidirectional_shift;
```

```
WHEN '1' =>
    q <= rsi & q(3 downto 1); —— right shift
WHEN others =>
    NULL;
END CASE;
END IF;
END PROCESS;
END bidirectional_shift;

Figure 9.73 shows the simulation of the shift register, with the left shift function in the first half of the simulation and the right shift function in the second half.

FIGURE 9.73
Example 9.15
4-bit Bidirectional Shift Register

Shift Registers of Generic Width

KEY TERM

**GENERIC** A clause in the entity declaration of a VHDL component that lists the parameters that can be specified when the component is instantiated.

All multibit VHDL components we have examined until now have been of a specified width (e.g., 2-to-4 decoder, 8-bit MUX, 8-bit adder, 4-bit counter). VHDL allows us to create components having a generic, or unspecified, width or other parameter which is specified when the component is instantiated. In the entity declaration of such a component, we indicate an unspecified parameter (such as width) in a **GENERIC** clause. The unspecified parameter must be given a default value in the **GENERIC** clause, indicated by `:= value`.

When we instantiate the component, we specify the parameter value in a generic map, as we have done with components from the Library of Parameterized Modules. The design entity `srt_bhv.vhd` below behaviorally defines an \( n \)-bit right-shift register, with a default width of four bits given by the statement (`GENERIC (width : POSITIVE := 4);`).

The entity `srt8_bhv.vhd` instantiates the \( n \)-bit register as an 8-bit circuit by specifying the bit width in a generic map. If no value is specified, the component is presumed to have a default width of four, as defined in the component’s entity declaration.

--- srt_bhv.vhd
EXAMPLE 9.16

Write the code for a VHDL design entity that defines a universal shift register with a generic width. (The default width is eight bits.) Instantiate this entity as a component in a file for a 16-bit universal shift register.

Solution

--- srg_univ.vhd
--- Universal shift register with generic width
--- Default width = 8 bits
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
USE ieee.std_logic_arith.ALL;

ENTITY srg_univ IS
  GENERIC (width : POSITIVE := 8);
  PORT (
    clk, clear     : IN STD_LOGIC;
    rsi, lsi       : IN STD_LOGIC;
    function_select   : IN STD_LOGIC_VECTOR(1 downto 0);
    p              : IN STD_LOGIC_VECTOR(width-1 downto 0);
    q              : BUFFER STD_LOGIC_VECTOR(width-1 downto 0) );
END srg_univ;

ARCHITECTURE universal_shift of srg_univ IS
BEGIN
  PROCESS (clk, clear)
  BEGIN
    IF clear = '0' THEN
      -- Conversion function to convert integer 0 to vector
      -- of any width. Requires ieee.std_logic_arith package.
      q <= CONV_STD_LOGIC_VECTOR(0, width);
    ELSIF (clk'EVENT and clk = '1') THEN
      CASE function_select IS
        WHEN "00" =>
          q <= q; -- Hold
        WHEN "01" =>
          q <= rsi & q(width-1 downto 1); -- Shift right
        WHEN "10" =>
          q <= q(width-2 downto 0) & lsi; -- Shift left
        WHEN "11" =>
          q <= p; -- Load
        WHEN OTHERS =>
          NULL;
      END CASE;
    END IF;
  END PROCESS;
END universal_shift;

-- srg16uni.vhd
-- 16-bit universal shift register (instantiates srg_univ)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY srg16uni IS
  PORT ( clock, clr  : IN      STD_LOGIC;
          rsi, lsi    : IN      STD_LOGIC;
          s           : IN      STD_LOGIC_VECTOR(1 downto 0);
          parallel_in : IN      STD_LOGIC_VECTOR(15 downto 0);
          qo          : BUFFER  STD_LOGIC_VECTOR(15 downto 0) );
END srg16uni;

ARCHITECTURE universal_shift of srg16uni IS
COMPONENT srg_univ
  GENERIC (width : POSITIVE);
  PORT ( clk, clear     : IN STD_LOGIC;
rsi, lsi : IN STD_LOGIC;
function_select : IN STD_LOGIC_VECTOR(1 downto 0);
p : IN STD_LOGIC_VECTOR(width-1 downto 0);
q : BUFFER STD_LOGIC_VECTOR(width-1 downto 0));
END COMPONENT;
BEGIN
Shift_universal_16: srg_univ
GENERIC MAP (width=> 16)
PORT MAP (clk             => clock,
clear           => clr,
rsi             => rsi,
lsi             => lsi,
function_select => s,
p               => parallel_in,
q               => qo);
END universal_shift;

When we are designing the clear function in srg_univ.vhd, we must account for the fact that we must set all bits of a vector of unknown width to ‘0’. To get around this problem, we use a conversion function that changes an INTEGER value of 0 to a STD_LOGIC_VECTOR of width bits and assigns the value to the output. The required conversion function, CONV_STD_LOGIC_VECTOR(value, number_of_bits), is found in the std_logic_arith package in the ieee library. We could also use the construct

q <= (others => '0');

which states that the default case is to set all bits of q to 0 when clear is 0. Since there is no other case specified, all bits of q are cleared.

LPM Shift Registers

The Library of Parameterized Modules contains a shift register component, lpm_shiftreg, that we can instantiate in a VHDL design entity. The various functions of lpm_shiftreg are listed in Table 9.16.

The following VHDL code instantiates lpm_shiftreg as an 8-bit shift register with serial input and serial output. In this case, the LPM component is declared explicitly, with the component declaration statement listing only the ports and parameters used by the design entity. The component instantiation statement lists the port names from the design entity in the same order as the corresponding component port names. By default the register direction is LEFT (i.e., toward the MSB).

-- srg8_lpm.vhd
-- 8-bit serial shift register (shift left by default)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;
Programming Shift Registers in VHDL

ENTITY srg8_lpm IS
  PORT (
    clk        : IN  STD_LOGIC;
    serial_in  : IN  STD_LOGIC;
    serial_out : OUT STD_LOGIC);
END srg8_lpm;

ARCHITECTURE lpm_shift of srg8_lpm IS
  COMPONENT lpm_shiftreg
  GENERIC(LPM_WIDTH: POSITIVE);
  PORT ( 
    clock, shiftin : IN STD_LOGIC;
    shiftout       : OUT STD_LOGIC);
END COMPONENT;
BEGIN
  Shift_8: lpm_shiftreg
  GENERIC MAP (LPM_WIDTH=> 8)
  PORT MAP (clk, serial_in, serial_out);
END lpm_shift;

Figure 9.74 shows a simulation of the shift register, with the data shifting from right to left (LSB to MSB). Since there is no parallel output (q[]) instantiated in our design, we would not normally be able to monitor the progress of bits from flip-flop to flip-flop; we would only see shiftin, and then, eight clock cycles later, shiftout. However, we are able to monitor the flip-flop states as buried nodes (Shift_8[dfs[7..0].Q]). These buried nodes are the last eight lines in the simulation.
EXAMPLE 9.17

Modify the VHDL code just shown to make the serial shift register shift right, rather than left. Create a simulation to verify the circuit function. How do the positions of \texttt{shiftin} and \texttt{shiftout} ports relate to the internal flip-flops for the right-shift and left-shift implementations?

**Solution** The modified VHDL code is shown next as design entity \texttt{srg8lpm2}. The only difference is the addition of the parameter \texttt{LPM\_DIRECTION} to both the component declaration and component instantiation statements.

\begin{verbatim}
-- srg8_lpm2.vhd
-- 8-bit serial shift register (shift right)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY srg8lpm2 IS
  PORT (  
            clk        : IN   STD_LOGIC;
         serial_in  : IN   STD_LOGIC;
         serial_out : OUT  STD_LOGIC);
END ENTITY srg8lpm2;

ARCHITECTURE lpm_shift of srg8_lpm2 IS
  COMPONENT lpm_shiftreg
    GENERIC (LPM\_WIDTH: POSITIVE; LPM\_DIRECTION: STRING);
    PORT (  
              clock, shiftin : IN   STD_LOGIC;
           shiftout       : OUT  STD_LOGIC);
END COMPONENT;
BEGIN
  shift_8: lpm_shiftreg
    GENERIC MAP (LPM\_WIDTH=> 8, LPM\_DIRECTION => "RIGHT")
    PORT MAP (clk, serial_in, serial_out);
END lpm_shift;
\end{verbatim}
The simulation for the right-shift register is shown in Figure 9.75. The inputs are identical to those of Figure 9.74, but the internal shift direction is opposite. The LPM component configures the serial shift input and output such that they allow data to go through the entire register, regardless of shift direction. For left-shift, `serial_in (shiftin)` is applied to D0, and is shifted toward Q7. For right-shift, the same `serial_in` is applied to D7 and shifted toward Q0. Thus, there is no right shift input or left shift input in this component, and also no bidirectional shift that can be controlled by an input port. Shift direction can only be set by the value of a parameter and is therefore fixed when a component is instantiated.

**EXAMPLE 9.18**

Write the VHDL code for an 8-bit LPM shift register with both parallel and serial outputs, parallel load and asynchronous clear. Create a simulation to verify the design operation.

**Solution** The VHDL code for the parallel load shift register follows as `srg8lpm3`.

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY srg8lpm3 IS
  PORT (clk, ld, clr  : IN   STD_LOGIC;
        p             : IN   STD_LOGIC_VECTOR(7 downto 0);
        q_out         : OUT  STD_LOGIC_VECTOR(7 downto 0);
        serial_out    : OUT  STD_LOGIC);
END srg8lpm3;

ARCHITECTURE lpm_shift of srg8lpm3 IS
  COMPONENT lpm_shiftreg
    GENERIC (LPM_WIDTH: POSITIVE);
    PORT (clock, load    : IN   STD_LOGIC;
           aclr           : IN   STD_LOGIC;
           data           : IN   STD_LOGIC_VECTOR (7 downto 0);
           q              : OUT  STD_LOGIC_VECTOR (7 downto 0);
           shiftout       : OUT  STD_LOGIC);
  END COMPONENT;
BEGIN
END ARCHITECTURE lpm_shift;
```

**FIGURE 9.75**

Example 9.17
Simulation of an 8-bit LPM Shift Register (Shift Right)
Shift_8: lpm_shiftreg  
GENERIC MAP (LPM_WIDTH=> 8)  
PORT MAP (clk, ld, clr, p, q_out, serial_out);  
END lpm_shift;

The simulation for srg8lpm3 is shown in Figure 9.76. The load input is initially HIGH, causing the shift register to load 55H (= 01010101) on the first clock pulse. Since we have not instantiated the serial input shiftin, the serial input reverts to a default value of ‘1’, causing the register to be filled with 1s. If we did not want this to be the case, we would have to instantiate the shiftin port and set it to ‘0’.  

SECTION 9.8 REVIEW PROBLEM

9.8 When a shift register is encoded in VHDL, why are its outputs defined as BUFFER, not OUT?

9.9 Shift Register Counters

KEY TERMS

Ring counter A serial shift register with feedback from the output of the last flip-flop to the input of the first.

Johnson counter A serial shift register with complemented feedback from the output of the last flip-flop to the input of the first. Also called a twisted ring counter

By introducing feedback into a serial shift register, we can create a class of synchronous counters based on continuous circulation, or rotation, of data.

If we feed back the output of a serial shift register to its input without inversion, we create a circuit called a ring counter. If we introduce inversion into the feedback loop, we have a circuit called a Johnson counter. These circuits can be decoded more easily than binary counters of similar size and are particularly useful for event sequencing.

Ring Counters
Figure 9.77 shows a 4-bit ring counter made from D flip-flops. This circuit could also be constructed from SR or JK flip-flops, as can any serial shift register.

A ring counter circulates the same data in a continuous loop. This assumes that the data have somehow been placed into the circuit upon initialization, usually by synchronous or asynchronous preset and clear inputs, which are not shown.

Figure 9.78 shows the circulation of a logic 1 through a 4-bit ring counter. If we assume that the circuit is initialized to the state \( Q_3Q_2Q_1Q_0 = 1000 \), it is easy to see that the 1 is shifted one place right with each clock pulse. The feedback connection from \( Q_0 \) to \( D_3 \) ensures that the input of flip-flop 3 will be filled by the contents of \( Q_0 \), thus recirculating the initial data. The final transition in the sequence shows the 1 recirculated to \( Q_3 \).

A ring counter is not restricted to circulating a logic 1. We can program the counter to circulate any data pattern we happen to find convenient.

Figure 9.79 shows a ring counter circulating a 0 by starting with an initial state of \( Q_3Q_2Q_1Q_0 = 0111 \). The circuit is the same as before; only the initial state has changed.

Figure 9.80 shows the timing diagrams for the circuit in Figures 9.78 and 9.79.

**Ring Counter Modulus and Decoding**

The maximum modulus of a ring counter is the maximum number of unique states in its count sequence. In Figures 9.78 and 9.79, the ring counters each had a maximum modulus of 4. We say that 4 is the *maximum* modulus of the ring counters shown, since we can change the modulus of a ring counter by loading different data at initialization.

For example, if we load a 4-bit ring counter with the data \( Q_3Q_2Q_1Q_0 = 1000 \), the following unique states are possible: 1000, 0100, 0010, and 0001. If we load the same circuit with the data \( Q_3Q_2Q_1Q_0 = 1010 \), there are only two unique states: 1010 and 0101. Depending on which data are loaded, the modulus is 4 or 2.

Most input data in this circuit will yield a modulus of 4. Try a few combinations.

A ring counter requires more flip-flops than a binary counter to produce the same number of unique states. Specifically, for \( n \) flip-flops, a binary counter has \( 2^n \) unique states and a ring counter has \( n \).

This is offset by the fact that a ring counter requires no decoding. A binary counter used to sequence eight events requires three flip-flops and eight 3-input decoding gates. To perform the same task, a ring counter requires eight flip-flops and no decoding gates.

As the number of output states of an event sequencer increases, the complexity of the decoder for the binary counter also increases. A circuit requiring 16 output states can be implemented with a 4-bit binary counter and sixteen 4-input decoding gates. If you need 18 output states, you must have a 5-bit counter (\( 2^5 \approx 18 \approx 2^4 \)) and eighteen 5-input decoding gates.

The only required modification to the ring counter is one more flip-flop for each addi-
tional state. A 16-state ring counter needs 16 flip-flops and an 18-state ring counter must have 18 flip-flops. No decoding is required for either circuit.

**Johnson Counters**

Figure 9.81 shows a 4-bit Johnson counter constructed from D flip-flops. It is the same as a ring counter except for the inversion in the feedback loop where $\bar{Q}_0$ is connected to $D_3$. The circuit output is taken from flip-flop outputs $Q_3$ through $Q_0$. Since the feedback introduces a “twist” into the recirculating data, a Johnson counter is also called a “twisted ring
FIGURE 9.79
Circulating a 0 in a Ring Counter
Figure 9.82 shows the progress of data through a Johnson counter that starts cleared ($Q_3Q_2Q_1Q_0 = 0000$). The shaded flip-flops represents 1s and the unshaded flip-flops are 0s. Every 0 at $Q_0$ is fed back to $D_3$ as a 1 and every 1 is fed back as a 0. The count sequence for this circuit is given in Table 9.17. There are 8 unique states in the count sequence table.
FIGURE 9.82
Data Circulation in a 4-bit Johnson Counter
EXAMPLE 9.19

Write the VHDL code for a Johnson counter of generic width and instantiate it as an 8-bit counter. List the sequence of states in a table, assuming the counter is initially cleared, and create a simulation to verify the circuit’s operation. Include a clear input (synchronous).

Solution  The VHDL design entities for the generic-width component and the 8-bit Johnson counter follow.

--- jnsn_ct.vhd
--- Johnson counter of generic width
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY jnsn_ct IS
  GENERIC (width : POSITIVE := 4);
  PORT (
    clk, clr : IN       STD_LOGIC;
    q        : BUFFER   STD_LOGIC_VECTOR(width-1 downto 0) );
END   jnsn_ct;

ARCHITECTURE johnson_counter of jnsn_ct IS
BEGIN
  PROCESS (clk)
  BEGIN
    IF (clk'EVENT and clk = '1') THEN
      IF clr = '0' THEN
        q <= (others => '0'); — n-bit clear function (n = width)
      ELSE
        q(width-1 downto 0) <= (not q(0) ) & q(width-1 downto 1);
      END IF;
    END IF;
  END PROCESS;
END johnson_counter;

--- jnsn_ct8.vhd
--- 8-bit Johnson counter using component jnsn_ct
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY jnsn_ct8 IS
  PORT(
    clock, clear : IN      STD_LOGIC;
    qo           : BUFFER  STD_LOGIC_VECTOR(7 downto 0));
END jnsn_ct8;

ARCHITECTURE johnson_counter of jnsn_ct8 IS
COMPONENT jnsn_ct GENERIC (width : POSITIVE);
  PORT(
    clk, clr : IN     STD_LOGIC;
    q        : BUFFER STD_LOGIC_VECTOR(7 downto 0) );
END COMPONENT;
BEGIN
  johnson: jnsn_ct
    GENERIC MAP (width=> 8)
    PORT MAP (clk => clock,
      clr => clear,
      q   => qo);
END johnson_counter;
Table 9.18  Count Sequence of an 8-bit Johnson Counter

<table>
<thead>
<tr>
<th>Q7</th>
<th>Q6</th>
<th>Q5</th>
<th>Q4</th>
<th>Q3</th>
<th>Q2</th>
<th>Q1</th>
<th>Q0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000000</td>
<td>1000000</td>
<td>1100000</td>
<td>1110000</td>
<td>1111000</td>
<td>1111100</td>
<td>1111110</td>
<td>1111111</td>
</tr>
<tr>
<td>0111111</td>
<td>0011111</td>
<td>0001111</td>
<td>0000111</td>
<td>0000011</td>
<td>0000001</td>
<td>0000000</td>
<td></td>
</tr>
</tbody>
</table>

Note that in the component file (jnsn_ct.vhd), the counter is cleared synchronously by the statement \((q <= ('0'));\). Recall that the clause \((others => '0')\) can be used to set all bits of a signal aggregate to the value ‘0’. This is a simple way to clear a vector of unknown width without using a conversion function.

Table 9.18 shows the count sequence for the 8-bit Johnson counter.

The simulation of the Johnson counter, including one full cycle and a clear, is shown in Figure 9.83.

**FIGURE 9.83**
Example 9.19
Simulation of an 8-bit Johnson Counter

**Johnson Counter Modulus and Decoding**

**NOTE**
The maximum modulus of a Johnson counter is \(2n\) for a circuit with \(n\) flip-flops.

The Johnson counter represents a compromise between binary and ring counters, whose maximum moduli are, respectively, \(2^n\) and \(n\) for an \(n\)-bit counter.
If it is used for event sequencing, a Johnson counter must be decoded, unlike a ring
counter. Its output states are such that each state can be decoded uniquely by a 2-input
AND or NAND gate, depending on whether you need active-HIGH or active-LOW indica-
tion. This yields a simpler decoder than is required for a binary counter.

Table 9.19 shows the decoding of a 4-bit Johnson counter.

<table>
<thead>
<tr>
<th>( Q_3 )</th>
<th>( Q_2 )</th>
<th>( Q_1 )</th>
<th>( Q_0 )</th>
<th>Decoder Outputs</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( Q_3 \bar{Q}_0 )</td>
<td>MSB = LSB = 0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( Q_3 \bar{Q}_2 )</td>
<td>“1/0”</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( Q_2 \bar{Q}_1 )</td>
<td>Pairs</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>( Q_1 \bar{Q}_0 )</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( Q_3 \bar{Q}_0 )</td>
<td>MSB = LSB = 1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>( \bar{Q}_3 \bar{Q}_2 )</td>
<td>“0/1”</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>( \bar{Q}_2 \bar{Q}_1 )</td>
<td>Pairs</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>( \bar{Q}_1 \bar{Q}_0 )</td>
<td></td>
</tr>
</tbody>
</table>

FIGURE 9.84
4-bit Johnson Counter with Output Decoding
Decoding a sequential circuit depends on the decoder responding uniquely to every possible state of the circuit outputs. If we want to use only 2-input gates in our decoder, it must recognize two variables for every state that are both active only in that state.

A Johnson counter decoder exploits what might be called the “1/0 interface” of the count sequence table. Careful examination of Tables 9.17 and 9.18 reveals that for every state, except where the outputs are all 1s or all 0s, there is a side-by-side 10 or 01 pair which exists only in that state.

Each of these pairs can be decoded to give unique indication of a particular state. For example, the pair $Q_3Q_2$ uniquely indicates the second state since $Q_3 = 1$ AND $Q_2 = 0$ only in the second line of the count sequence table. (This is true for any size of Johnson counter; compare the second lines of Tables 9.17 and 9.18. In the second line of both tables, the MSB is 1 and the 2nd MSB is 0.)

For the states where the outputs are all 1s or all 0s, the most significant AND least significant bits can be decoded uniquely, these being the only states where MSB = LSB.

Figure 9.84 shows the decoder circuit for a 4-bit Johnson counter.

The output decoder of a Johnson counter does not increase in complexity as the modulus of the counter increases. The decoder will always consist of $2^n$ 2-input AND or NAND gates for an $n$-bit counter. (For example, for an 8-bit Johnson counter, the decoder will consist of sixteen 2-input AND or NAND gates.)

\[\text{EXAMPLE 9.20}\]

Draw the timing diagram of the Johnson counter decoder of Figure 9.84, assuming the counter is initially cleared.

**Solution**  Figure 9.85 shows the timing diagram of the Johnson counter and its decoder outputs.
13. The synchronous counter design procedure is based on the
12. A synchronous counter can be designed using a formal
11. The analysis procedure above should be applied to any un-
10. A synchronous counter can be analyzed by a formal proce-

SUMMARY

1. A counter is a circuit that progresses in a defined sequence at
the rate of one state per clock pulse.
2. The modulus of a counter is the number of states through
which the counter output progresses before repeating.
3. A counter with an ascending sequence of states is called an
UP counter. A counter with a descending sequence of states
is called a DOWN counter.
4. In general, the maximum modulus of a counter is given by 2^n
for an n-bit counter.
5. A counter whose modulus is 2^n is called a full-sequence
counter. The count progresses from 0 to 2^n – 1, which corre-
sponds to a binary output of all 0s to all 1s.
6. A counter whose output is less than 2^n is called a truncated
sequence counter.
7. The adjacent outputs of a full-sequence binary counter have
a frequency ratio of 2:1. The less significant of the two bits
has the higher frequency.
8. The outputs of a truncated sequence counter do not necessar-
ily have a simple frequency relationship.
9. A synchronous counter consists of a series of flip-flops, all
clocked from the same source, that stores the present state of
the counter and a combinational circuit that monitors the
counter’s present state and determines its next state.
10. A synchronous counter can be analyzed by a formal proce-
dure that includes the following steps:
   a. Write the Boolean equations for the synchronous inputs
      of the counter flip-flops in terms of the present state of
      the flip-flop outputs.
   b. Evaluate each Boolean equation for an initial state to find
      the states of the synchronous inputs.
   c. Use flip-flop function tables to determine each flip-flop
      next state.
   d. Set the next state to the new present state.
   e. Continue until the sequence repeats.
11. The analysis procedure above should be applied to any un-
used states of the counter to ensure that they will enter the
counter sequence properly.
12. A synchronous counter can be designed using a formal
method that relies on the excitation tables of the flip-flops used
in the counter. An excitation table indicates the required logic
levels on the flip-flop inputs to effect a particular transition.
13. The synchronous counter design procedure is based on the
following steps:
   a. Draw the state diagram of the counter and use it to list
      the relationship between the counter’s present and next
      states. The table should list the counter’s present states in
      binary order.
   b. For the initial design, unused states can be set to a known
      destination, such as 0, or treated as don’t care states.
   c. Use the flip-flop excitation table to determine the synchro-
nous input levels for each present-to-next state transition.
   d. Use Boolean algebra or Karnaugh maps to find the sim-
plest equations for the flip-flop inputs (JK, D, or T) in
   terms of Q.
   e. Unused states should be analyzed by substituting their
      values into the Boolean equations of the counter. This
      will verify whether or not an unused state will enter the
counter sequence properly.
14. If a counter must reset to 0 from an unused state, the flip-
flops can be reset asynchronously to their initial states or the
counter can be designed with the unused states always hav-
ing 0 as their next state.
15. A counter can be designed in VHDL by using a behavioral
description or a structural design that uses a component from
the Library of Parameterized Modules (LPM).
16. A behavioral counter design requires a PROCESS statement
that lists the clock signal and any asynchronous inputs in its
sensitivity list. An IF statement inside the PROCESS can
monitor the active clock edge by using the predefined
EVENT attribute (e.g., clk’EVENT) and increment a count
variable.
17. A variable is local to a PROCESS and is assigned with the :=
operator. A signal is global to the VHDL design entity and is
assigned with the <= operator. (Recall that a signal is like an
internal connecting wire and a variable is a piece of working
memory.)
18. A structural counter design can use an LPM component
(lpm_counter) and instantiate the component in a component
instantiation statement. The statement’s generic map speci-
fies the component parameters, and its port map indicates the
correspondence between the component port names and the
user port, signal, or variable names.
19. Some of the most common control features available in syn-
chronous counters include:
   a. Synchronous or asynchronous parallel load, which al-
lows the count to be set to any value whenever a LOAD
      input is asserted
   b. Synchronous or asynchronous clear (reset), which sets all
      of the counter outputs to zero
   c. Count enable, which allows the count sequence to
      progress when asserted and inhibits the count when de-
asserted
   d. Bidirectional control, which determines whether the
      counter counts up or down

SECTION 9.9 REVIEW PROBLEM

9.9 How many flip-flops are required to produce 24 unique states in each of the following
types of counters: binary counter, ring counter, Johnson counter? How many and what
type of decoding gates are required to produce an active-LOW decoder for each type
of counter?
e. Output decoding, which activates one or more outputs when detecting particular states on the counter outputs
f. Ripple carry out or ripple clock out (RCO), a special case of output decoding that produces a pulse upon detecting the terminal count, or last state, of a count sequence

20. The parallel load function of a counter requires load data (the parallel input values) and a load command input, such as LOAD, that transfer the parallel data when asserted. If the load function is synchronous, a clock pulse is also required.
21. Synchronous load transfers data to the counter outputs on an active clock edge. Asynchronous load operates as soon as the load input activates, without waiting for the clock.
22. Synchronous load is implemented by a function select circuit that selects either the count logic or the direct parallel input to be applied to the synchronous input(s) of a flip-flop.
23. Asynchronous load is implemented by enabling or inhibiting a pair of NAND gates, one of which asserts a flip-flop clear input and the other of which asserts a preset input for the same flip-flop.
24. The count enable function enables or disables the count logic of a counter without affecting other functions, such as clock or clear. This can be done by ANDing the count logic with the count enable input signal.
25. A flip-flop in an UP counter toggles when all previous bits are HIGH. A flip-flop in a DOWN counter toggles when all previous bits are LOW. A circuit that selects one of these two conditions (a pair of AND-shaped gates, combined in an OR gate; essentially a 2-to-1 multiplexer) can implement a bidirectional count.
26. An output decoder asserts one output for each counter state. A special case is a terminal count decoder that detects the last state of a count sequence.
27. RCO (ripple clock out) generates one clock pulse upon terminal count, with its positive edge at the end of the count cycle.
28. Asynchronous inputs to a behaviorally defined counter in VHDL must be included in the sensitivity list of the process defining the counter. Asynchronous inputs must be checked inside the process before the clock is checked for an active edge.
29. Synchronous inputs to a behaviorally defined counter should not be included in the sensitivity list of the process defining the counter. Synchronous inputs must be checked inside the IF statement that checks the clock edge.
30. A shift register is a circuit for storing and moving data. Three basic movements in a shift register are: serial (from one flip-flop to another), parallel (into all flip-flops at once), and rotation (serial shift with a connection from the last flip-flop output to the first flip-flop input).
31. Serial shifting can be left (toward the MSB) or right (away from the MSB). This is the convention used by MAX+PLUS II. Some data sheets indicate the opposite relationship between right/left and LSB/MSB.
32. A function select circuit can implement several shift register variations: bidirectional serial shift, parallel load with serial shift, and universal shift (parallel/serial in/out and bidirectional in one device). The circuit directs data to the D inputs of each flip-flop from one of several sources, such as from the flip-flop immediately to the left or right from an external parallel input.
33. A shift register can be created in VHDL by the structural, dataflow, or behavioral method.
34. A structural design instantiates components, such as D flip-flops, and connects them with internal signals.
35. A dataflow design uses internal Boolean relationships between inputs and outputs. It is similar to a structural model, except that it must contain a process to create the flip-flops.
36. A behavioral design method uses a description of the shift register function to generate the required hardware.
37. A VHDL component can be created with parameters (such as width) that are specified when the component is instantiated. The parameters are listed in a GENERIC clause in the component's entity declaration. Each parameter must be given a default value. The parameters are specified in a generic map in the design entity that instantiates the component.
38. A ring counter is a serial shift register with the serial output fed back to the serial input so that the internal data is continuously circulated. The initial value is generally set by asynchronous preset and clear functions.
39. The maximum modulus of a ring counter is $2^n$ for a circuit with $n$ flip-flops, as compared to $2^n$ for a binary counter. A ring counter output is self-decoding, whereas a binary counter requires $m \leq 2^n$ AND or NAND gates with $n$ inputs each.
40. A Johnson counter is a ring counter where the feedback is complemented. A Johnson counter has $2n$ states for an $n$-bit counter which can be uniquely decoded by $2n$ 2-input AND or NAND gates.

Glossary

**Attribute** A property associated with a named identifier in VHDL. (e.g., the attribute EVENT, when associated with the identifier clk (written clk'EVENT), indicates whether a transition has occurred on the input called clk.)

**Behavioral design** A VHDL design technique that uses descriptions of required behavior to describe the design.

**Bidirectional counter** A counter that can count up or down, depending on the state of a control input.

**Bidirectional shift register** A shift register that can serially shift bits left or right according to the state of a direction control input.

**Binary counter** A counter that generates a binary count sequence.

**Clear** Reset (synchronous or asynchronous)

**Command lines** Signals that connect the control section of a synchronous circuit to its memory section and direct the circuit from its present to its next state.

**Conditional signal assignment statement** A signal assignment statement that is executed only when a Boolean condition is satisfied.

**Control section** The combinational logic portion of a synchronous circuit that determines the next state of the circuit.

**Count enable** A control function that allows a counter to progress through its count sequence when active and disables the counter when inactive.
Count sequence A specific series of output states through which a counter progresses.

Counter A sequential digital circuit whose output progresses in a predictable repeating pattern, advancing by one state for each clock pulse.

Count-sequence table A list of counter states in the order of the count sequence.

Dataflow design A VHDL design technique that uses Boolean equations to define relationships between inputs and outputs.

DOWN counter A counter with a descending sequence.

Excitation table A table showing the required input conditions for every possible transition of a flip-flop output.

Full-sequence counter A counter whose modulus is the same as its maximum modulus ($m = 2^n$ for an $n$-bit counter).

GENERIC A clause in the entity declaration of a VHDL component that lists the parameters that can be specified when the component is instantiated.

Johnson counter A serial shift register with complemented feedback from the output of the last flip-flop to the input of the first. Also called a twisted ring counter

Left shift A movement of data from the right to the left in a shift register. (Left is defined in MAX+PLUS II as toward the MSB.)

Maximum modulus ($m_{\text{max}}$) The largest number of counter states that can be represented by $n$ bits ($m_{\text{max}} = 2^n$)

Memory section A set of flip-flops in a synchronous circuit that hold its present state.

Modulo-$n$ (or mod-$n$) counter A counter with a modulus of $n$.

Modulus The number of states through which a counter sequences before repeating.

Next state The desired future state of flip-flop outputs in a synchronous sequential circuit after the next clock pulse is applied.

Parallel load A function that allows simultaneous loading of binary values into all flip-flops of a synchronous circuit. Parallel loading can be synchronous or asynchronous.

Parallel-load shift register A shift register that can be preset to any value by directly loading a binary number into its internal flip-flops.

Parallel transfer Movement of data into all flip-flops of a shift register at the same time.

Present state The current state of flip-flop outputs in a synchronous sequential circuit.

UP counter A counter with an ascending sequence.

Presetable counter A counter with a parallel load function.

Recycle To make a transition from the last state of the count sequence to the first state.

Right shift A movement of data from the left to the right in a shift register. (Right is defined in MAX+PLUS II as toward the LSB.)

Ring counter A serial shift register with feedback from the output of the last flip-flop to the input of the first.

Ripple carry out or ripple clock out (RCO) An output that produces one pulse with the same period as the clock upon terminal count.

Rotation Serial shifting of data with the output(s) of the last flip-flop connected to the synchronous input(s) of the first flip-flop. The result is continuous circulation of the same data.

Serial shifting Movement of data from one end of a shift register to the other at a rate of one bit per clock pulse.

Shift register A synchronous sequential circuit that will store and move $n$-bit data, either serially or in parallel, in $n$ flip-flops.

SRGn Symbol for an $n$-bit shift register (e.g., SRG4 indicates a 4-bit shift register).

State diagram A diagram showing the progression of states of a sequential circuit.

State machine A synchronous sequential circuit.

Status lines Signals that communicate the present state of a synchronous circuit from its memory section to its control section.

Structural design A VHDL design technique that connects predesigned components using internal signals.

Synchronous counter A counter whose flip-flops are all clocked by the same source and thus change in synchronization with each other.

Terminal count The last state in a count sequence before the sequence repeats (e.g., 1111 is the terminal count of a 4-bit binary UP counter; 0000 is the terminal count of a 4-bit binary DOWN counter).

Truncated-sequence counter A counter whose modulus is less than its maximum modulus ($m < 2^n$ for an $n$-bit counter)

Universal shift register A shift register that can operate with any combination of serial and parallel inputs and outputs (i.e., serial in/parallel out, serial in/parallel out, parallel in/serial out, parallel in/parallel out). A universal shift register is often bidirectional, as well.

Problem numbers set in color indicate more difficult problems; those with underlines indicate most difficult problems.

9.1 Basic Concepts of Digital Counters

9.1 A parking lot at a football stadium is monitored before a game to determine whether or not there is available space for more cars. When a car enters the lot, the driver takes a ticket from a dispenser which also produces a pulse for each ticket taken.

The parking lot has space for 4095 cars. Draw a block diagram which shows how you can use a digital counter to light a LOT FULL sign after 4095 cars have entered. (Assume no cars leave the lot until after the game, so you don’t need to keep track of cars leaving the lot.) How many bits should the counter have?

9.2 Figure 9.86 shows a mod-16 which controls the operation of two digital sequential circuits, labeled Circuit 1 and Circuit 2. Circuit 1 is positive edge-triggered and clocked by counter output $Q_1$. Circuit 2 is negative edge-triggered and clocked by $Q_3$. (Assume no cars leave the lot until after the game, so you don’t need to keep track of cars leaving the lot.) How many bits should the counter have?
Problems

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i. The counter is at state 0111. What is the count after 7 clock pulses are applied?
ii. After 5 clock pulses, the counter output is at 0001. What was the counter state prior to the clock pulses?
iii. The counter output is at 1000 after 15 clock pulses. What was the original output state?

9.3 Draw the timing diagram for one complete cycle of the circuit operation. Draw arrows on the active edges of the waveforms that activate Circuit 1 and Circuit 2.

9.4 State how many times Circuit 1 is clocked for each time that Circuit 2 is clocked.

9.5 a. Draw the state diagram of a mod-10 UP counter.
b. Use the state diagram drawn in part a to answer the following questions:

9.11 Draw the circuit for a synchronous mod-16 UP counter made from negative edge-triggered JK flip-flops.

9.12 Write the Boolean equations required to extend the counter drawn in Problem 9.11 to a mod-64 counter.

9.13 Write the J and K equations for the MSB of a synchronous mod-256 (8-bit) UP counter.

9.14 Analyze the operation of the synchronous counter in Figure 9.87 by drawing a state table showing all transitions, including unused states. Use this state table to draw a state diagram and a timing diagram. What is the counter’s modulus?

9.15 a. Write the equations for the J and K inputs of each flip-

FIGURE 9.86
Problem 9.2
Mod-16 Counter Driving Two Sequential Circuits

FIGURE 9.87
Problem 9.14
Synchronous Counter
9.16 Analyze the operation of the counter shown in Figure 9.89. Predict the count sequence by determining the $J$ and $K$ inputs and resulting transitions for each counter output state. Draw the state diagram and the timing diagram. Assume that all flip-flop outputs are initially 0.

9.18 Design a synchronous mod-10 counter, using positive edge-triggered JK flip-flops. Check that unused states properly enter the main sequence. Draw a state diagram showing the unused states.

9.19 Design a synchronous mod-10 counter, using positive edge-triggered D flip-flops. Check that unused states properly enter the main sequence. Draw a state diagram showing the unused states.

9.20 Design a synchronous 3-bit binary counter using T flip-flops.

9.21 Table 9.20 shows the count sequence for a biquinary sequence counter. The sequence has ten states, but does not progress in binary order. The advantage of the sequence is that its most significant bit has a divide-by-10 ratio, relative to a clock input, and a 50% duty cycle. Design the
Refer to the 4-bit counter of Figure 9.26 (p. 391). The
synchronous counter circuit for this sequence, using D
flip-flops. Hint: When making the state table, list all pre-
sent states in binary order. The next states will not be in
binary order.

### Programming Binary Counters in VHDL

9.22 Write the VHDL code for a behavioral description of a 6-
bit binary counter with asynchronous clear.

9.23 Create a simulation file in MAX+PLUS II to verify the op-
eration of the counter in Problem 9.22. (Use a 40 ns clock,
which approximates the clock period of the oscillator on the
Altera UP-1 board.) Note: To make a useful simulation, you
must include the recycle point, which may be beyond the de-
fault end time of the simulation (1 μs). To change the end
time, select End Time from the MAX+PLUS II File menu in
the Simulator menu. To change the clock period, select Grid
Size from the MAX+PLUS II Options menu in the Simula-
tor window. The default clock period is two grid spaces.

9.24 Write a VHDL file that instantiates a counter from the
Library of Parameterized Modules to make a 12-bit bi-
ary counter. Create a MAX+PLUS II simulation to verify
the operation of the counter. (Refer to the note after
Problem 9.23.)

### Control Options for Synchronous Counters

9.25 Briefly explain the difference between asynchronous and
 synchronous parallel load in a synchronous counter. Draw a
partial timing diagram that illustrates both functions for
a 4-bit counter.

9.26 Refer to the 4-bit counter of Figure 9.26 (p. 391). The
graphic design files for the counter are found on the CD
accompanying this text as 4bit_sle.gdf and sl_count.gdf in
the folder drive:\Student Files\Chapter09. Copy these files
to a new folder and use the MAX+PLUS II graphic editor to expand the counter of Figure 9.26 to a 5-bit
counter with synchronous load and asynchronous reset.
Save and compile the file to make sure that there are no
design errors.

9.27 Create a MAX+PLUS II simulation to verify the func-
tions of the counter in Problem 9.26. The simulation must
include the recycle point of the counter and show that the
load is really synchronous and that the reset is really
asynchronous.

9.28 Refer to the 4-bit counter of Figure 9.33 (p. 396). The
graphic design files for the counter are found on the ac-
companying CD as 4bit_sle.gdf and sl_count.gdf in the
folder drive:\Student Files\Chapter09. Copy these files
to a new folder and modify the synchronous count ele-
ment sl_count.gdf so that it implements an active-HIGH
synchronous load and an active-LOW synchronous clear
function, as well as the binary count function. Create a
default symbol for the new element and substitute it in
4bit_sle.gdf for the existing counter elements sl_count.
The load function should have priority over count enable,
and clear (reset) should have priority over both. Save and
compile the new file. Hints: (1) The clear function makes
Q = 0 after a clock pulse. (2) Q follows D.

9.29 Create a MAX+PLUS II simulation to verify the func-
tions of the counter in Problem 9.28. The simulation must
include the recycle point of the counter and show that the
load and clear really are synchronous and that load has
priority over count enable and clear has priority over both.

9.30 Derive the Boolean equations for the synchronous
DOWN-counter in Figure 9.35.

9.31 Write the Boolean equations for the count logic of the 4-
bit bidirectional counter in Figure 9.38. Briefly explain
how the logic works.

9.32 Draw a MAX+PLUS II Graphic Design File for a bidi-
rectional counter, using T flip-flops. Create a simulation
of the counter to verify its function.

9.33 Use MAX+PLUS II to create a synchronous bidirec-
tional counter with synchronous load, asynchronous reset,
and count enable. The count enable should not affect the
operation of the load and reset functions. The functions
should have the following priority: (1) clear; (2) load; and
(3) count. Create a MAX+PLUS II simulation to verify
the operation of your design.

### Programming Presettable and Bidirectional
Counters in VHDL

9.34 Write the VHDL code for a counter that uses a behavioral
description of the following functions: 12-bit binary UP
count; active-LOW asynchronous clear, active-LOW syn-
chronous load, active-LOW count enable, terminal count
decoder. The clear function should have the highest pri-
ority, followed by load, then count enable. Create a simu-
lation in MAX+PLUS II that verifies the functions of this
counter.

9.35 Write the VHDL code for a behavioral description of a bidi-
rectional counter with a modulus of 24. The counter should
also have an active-LOW synchronous clear function that
has priority over the count. Create a MAX+PLUS II simu-
lation file to verify the counter operation.

9.36 Write the VHDL code for a 4-bit counter with two decoding
outputs called eq8 and eq12. Out eq8 goes HIGH
when the count equals 8 and eq12 goes HIGH when the
count equals 12 (decimal). The counter should also have
an active-LOW asynchronous clear function that has pri-

Table 9.20  Biquinary
Sequence

<table>
<thead>
<tr>
<th>Q3 Q2 Q1 Q0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
</tr>
<tr>
<td>0001</td>
</tr>
<tr>
<td>0010</td>
</tr>
<tr>
<td>0011</td>
</tr>
<tr>
<td>0100</td>
</tr>
<tr>
<td>1000</td>
</tr>
<tr>
<td>1001</td>
</tr>
<tr>
<td>1010</td>
</tr>
<tr>
<td>1011</td>
</tr>
<tr>
<td>1100</td>
</tr>
</tbody>
</table>

9.28 Refer to the 4-bit counter of Figure 9.33 (p. 396). The
graphic design files for the counter are found on the ac-
companying CD as 4bit_sle.gdf and sl_count.gdf in the
folder drive:\Student Files\Chapter09. Copy these files
to a new folder and modify the synchronous count ele-
ment sl_count.gdf so that it implements an active-HIGH
synchronous load and an active-LOW synchronous clear
function, as well as the binary count function. Create a
default symbol for the new element and substitute it in
4bit_sle.gdf for the existing counter elements sl_count.
The load function should have priority over count enable,
and clear (reset) should have priority over both. Save and
compile the new file. Hints: (1) The clear function makes
Q = 0 after a clock pulse. (2) Q follows D.

9.29 Create a MAX+PLUS II simulation to verify the func-
tions of the counter in Problem 9.28. The simulation must
include the recycle point of the counter and show that the
load and clear really are synchronous and that load has
priority over count enable and clear has priority over both.

9.30 Derive the Boolean equations for the synchronous
DOWN-counter in Figure 9.35.

9.31 Write the Boolean equations for the count logic of the 4-
bit bidirectional counter in Figure 9.38. Briefly explain
how the logic works.

9.32 Draw a MAX+PLUS II Graphic Design File for a bidi-
rectional counter, using T flip-flops. Create a simulation
of the counter to verify its function.

9.33 Use MAX+PLUS II to create a synchronous bidirec-
tional counter with synchronous load, asynchronous reset,
and count enable. The count enable should not affect the
operation of the load and reset functions. The functions
should have the following priority: (1) clear; (2) load; and
(3) count. Create a MAX+PLUS II simulation to verify
the operation of your design.

9.6 Programming Presettable and Bidirectional
Counters in VHDL

9.34 Write the VHDL code for a counter that uses a behavioral
description of the following functions: 12-bit binary UP
count; active-LOW asynchronous clear, active-LOW syn-
chronous load, active-LOW count enable, terminal count
decoder. The clear function should have the highest pri-
ority, followed by load, then count enable. Create a simu-
lation in MAX+PLUS II that verifies the functions of this
counter.

9.35 Write the VHDL code for a behavioral description of a bidi-
rectional counter with a modulus of 24. The counter should
also have an active-LOW synchronous clear function that
has priority over the count. Create a MAX+PLUS II simu-
lation file to verify the counter operation.

9.36 Write the VHDL code for a 4-bit counter with two decoding
outputs called eq8 and eq12. Out eq8 goes HIGH
when the count equals 8 and eq12 goes HIGH when the
count equals 12 (decimal). The counter should also have
an active-LOW asynchronous clear function that has pri-
ority over the count. Create a MAX+PLUS II simulation file to verify the counter operation.

9.37 Modify the VHDL code in Example 9.10 (p. 412) so that the counter synchronously sets to all 1s (H11001), rather than to 2047. Do not use SV ALUE H110014095. Create a simulation in MAX+PLUS II that verifies the operation of the counter. State the main difference between the code for Example 9.10 and the solution to this problem.

9.38 Use a counter from the Library of Parameterized Modules to implement the counter described in Problem 9.35. Create a MAX+PLUS II simulation file to verify the operation of the counter.

9.39 Write a VHDL file that instantiates an 8-bit LPM count with synchronous load and clear, count enable, and directional control. Also include a terminal count decoder. (The LPM counter has no port for the terminal count function, so it must be done separately.) Create a MAX+PLUS II simulation to verify the operation of the counter.

9.40 Use the MAX+PLUS II Graphic Editor to draw the circuit of a serial shift register constructed from JK flip-flops. Create a simulation to verify the operation of the shift register.

9.41 Use the MAX+PLUS II Graphic Editor to create the logic diagram of the 4-bit serial shift register based on JK flip-flops that shifts left, rather than right. Create a simulation to verify the operation of the shift register.

9.42 The following bits are applied in sequence to the input of a 6-bit serial right-shift register: 01111111 (0 is applied first). Draw the timing diagram.

9.43 After the data in Problem 9.42 are applied to the 6-bit shift register, the serial input goes to 0 for the next 8 clock pulses and then returns to 1. Write the internal states, Q_5 through Q_0, of the shift register flip-flops after the first 2 clock pulses. Write the states after 6, 8, and 10 clock pulses.

9.44 Complete the timing diagram of Figure 9.90, which is for a serial shift register (right-shift). Assume the shift register is initially cleared. What happens to the state of the circuit if D_7 stays HIGH beyond the end of the diagram and the CLK input continues to pulse?

9.45 An 8-bit right-shift serial-in-serial-out shift register is initially cleared and has the following data clocked into its serial input: 1011001110. Draw a timing diagram of the circuit showing the CLK, Serial Input, and Serial Output. (Assume the individual flip-flop outputs are not accessible.)
9.46 Complete the logic circuit shown in Figure 9.91 to make a bidirectional shift register.

9.47 Complete the logic circuit shown in Figure 9.92 to make a parallel-in-serial-out shift register.

9.8 Programming Shift Registers in VHDL

9.48 Write the VHDL code for an 8-bit serial shift register using a structural design procedure. Use JK flip-flops. (MAX+PLUS II primitive: JKFF.) Create a MAX+PLUS II simulation file to verify the operation of your design.

9.49 Repeat Problem 9.48 using a dataflow design procedure.

9.50 Modify the VHDL code for the behaviorally designed shift register srg4behv.vhd so that the shift register moves the data left, not right. Hint: The statement q (3 downto 0) <= serial_in & q(3 downto 1); is equivalent to the following two statements:

q(3) <= serial_in;
q(2 downto 0) <= q(3 downto 1);

Create a simulation file to verify the operation of this device.

9.51 Modify the VHDL code for the left-shift register Problem 9.50 to make a shift register of generic width. Use this component in another VHDL file to make a 32-bit shift register that shifts left. Create a simulation file to verify the operation of this design.

9.52 Write the code for a VHDL design entity that implements a 4-bit universal shift register with asynchronous clear. Create a simulation that verifies the design function.

9.53 Use MAX+PLUS II to create simulations for the generic-width and the 16-bit universal shift registers in Example 9.16 (p. 432). What is the difference in width between the default value of the generic shift register and the instantiated component in the 16-bit file? Given this difference, why can the generic-width shift register be correctly used as a component in the 16-bit design entity?

9.54 Use an LPM shift register in a VHDL file to instantiate a 48-bit shift register with the following functions: serial input, parallel output, synchronous clear.

9.55 Use an LPM shift register in a VHDL file to instantiate a 10-bit shift register with the following functions: serial input and output whose internal value can be synchro-
nously set to 960. Create a MAX+PLUS II simulation to verify the operation of the design.

9.9 Shift Register Counters

9.56 Write the VHDL code for a ring counter of generic width and instantiate it as an 8-bit ring counter. List the sequence of states in a table, assuming the counter is initially cleared, and create a simulation to verify the circuit’s operation. Include a clear input (synchronous).

9.57 Construct the count sequence table of a 5-bit Johnson counter, assuming the counter is initially cleared. What changes must be made to the decoder part of the circuit in Figure 9.84 (p. 446) if it is to decode the 5-bit Johnson counter?

A N S W E R S T O S E C T I O N R E V I E W P R O B L E M S

Section 9.1

9.1 A mod-24 UP counter goes from 00000 to 10111 (0 to 23). This requires 5 outputs. The counter is a truncated sequence since its modulus is less than \(2^5 = 32\).

Section 9.2

9.2 1001, 0000

Section 9.3

9.3 JK flip-flops: \(J_3K_3 = X0, J_2K_2 = 1X, J_1K_1 = X1, J_0K_0 = X1\)
D flip-flops: \(D_3 = 1, D_2 = 1, D_1 = 0, D_0 = 0\)

Section 9.4

9.4 If (clock’EVENT AND clock = ‘0’) THEN
  count := count + 1;
END IF;

Section 9.5

9.5 The completed timing diagram is shown in Figure 9.93.

Section 9.6

9.6 Asynchronous clear: PROCESS (clock, clear); Synchronous clear: PROCESS (clock)

Section 9.7

9.7 JK flip-flops can be used in the shift register of Figure 9.58. The \(Q\) output of any stage connects to the \(J\) input of the next stage and the \(\bar{Q}\) output of any stage connects to the \(K\) input of the next. The \(\text{serial} \_\text{in}\) input connects directly to the \(J\) input of the first flip-flop. \(\text{Serial} \_\text{in}\) is applied to \(K\) of the first flip-flop through an inverter (NOT gate).

Section 9.8

9.8 A shift register output is defined as a port of mode BUFFER because this mode allows a signal to be fed back into the PLD matrix and reused as an input to another part of the circuit.

Section 9.9

Binary: 5 flip-flops, 24 5-inputs NANDs; Ring: 24 flip-flops, no

FIGURE 9.93
Answer to Section Review Problem 9.5
CHAPTER 10
State Machine Design

CHAPTER OBJECTIVES
Upon successful completion of this chapter you will be able to:
• Describe the components of a state machine.
• Distinguish between Moore and Mealy implementations of state machines.
• Draw the state diagram of a state machine from a verbal description.
• Use the “classical” (state table) method of state machine design to determine the Boolean equations of the state machine.
• Translate the Boolean equations of a state machine into a Graphic Design File in Altera’s MAX+PLUS II software.
• Write VHDL code to implement state machines.
• Create simulations in MAX+PLUS II to verify the function of a state machine design.
• Determine whether the output of a state machine is vulnerable to asynchronous changes of input.
• Design state machine applications, such as a switch debouncer, a single-pulse generator, and a traffic light controller.

10.1 State Machines

KEY TERMS
State machine A synchronous sequential circuit, consisting of a sequential logic section and a combinational logic section, whose outputs and internal flip-flops progress through a predictable sequence of states in response to a clock and other input signals.
Moore machine A state machine whose output is determined only by the sequential logic of the machine.
Mealy machine A state machine whose output is determined by both the sequential logic and the combinational logic of the machine.
State variables The variables held in the flip-flops of a state machine that determine its present state. The number of state variables in a machine is equivalent to the number of flip-flops.
The synchronous counters and shift registers we examined in Chapter 9 are examples of a larger class of circuits known as state machines. As described for synchronous counters in Section 9.2, a state machine consists of a memory section that holds the present state of the machine and a control section that determines the machine’s next state. These sections communicate via a series of command and status lines. Depending on the type of machine, the outputs will either be functions of the present state only or of the present and next states.

Figure 10.1 shows the block diagram of a Moore machine. The outputs of a Moore machine are determined solely by the present state of the machine’s memory section. The output may be directly connected to the \( Q \) outputs of the internal flip-flops, or the \( Q \) outputs might pass through a decoder circuit. The output of a Moore machine is synchronous to the system clock, since the output can only change when the machine’s internal state variables change.

The block diagram of a Mealy machine is shown in Figure 10.2. The outputs of the Mealy machine are derived from the combinational (control) section of the machine, as
well as the sequential (memory) part of the machine. Therefore, the outputs can change asynchronously when the combinational circuit inputs change out of phase with the clock. (When we say that the outputs change asynchronously, we generally do not mean a change via a function such as asynchronous reset that directly affects the machine’s flip-flops.)

SECTION 10.1 REVIEW PROBLEM
10.1 What is the main difference between a Moore-type state machine and a Mealy-type state machine?

10.2 State Machines with No Control Inputs

**KEY TERMS**

*Bubble* A circle in a state diagram containing the state name and values of the state variables.

A state machine can be designed using a classical technique, similar to that used to design a synchronous counter. We can also use a VHDL design method. We will design several state machines, using both classical and VHDL techniques.

As an example of these techniques, we will design a state machine whose output depends only on the clock input: a 3-bit counter with a Gray code count sequence. A 3-bit Gray code, shown in Table 10.1, changes only one bit between adjacent codes and is therefore not a binary-weighted sequence.

<table>
<thead>
<tr>
<th>Table 10.1</th>
<th>3-bit Gray Code Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q2Q1Q0</strong></td>
<td></td>
</tr>
<tr>
<td>000</td>
<td></td>
</tr>
<tr>
<td>001</td>
<td></td>
</tr>
<tr>
<td>011</td>
<td></td>
</tr>
<tr>
<td>010</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td></td>
</tr>
<tr>
<td>111</td>
<td></td>
</tr>
<tr>
<td>101</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Gray code is often used in situations where it is important to minimize the effect of single-bit errors. For example, suppose the angle of a motor shaft is measured by a detected code on a Gray-coded shaft encoder, shown in Figure 10.3. The encoder indicates a 3-bit number for each of eight angular positions by having three concentric circular segments for each code. A dark band indicates a 1 and a transparent band indicates a 0, with the MSB as the outermost band. The dark or transparent bands are detected by three sensors that detect
light shining through a transparent band. (A real shaft encoder has more bits to indicate an angle more precisely. For example, a shaft encoder that measures an angle of one degree would require nine bits, since there are 360 degrees in a circle and $2^9 \approx 360 \approx 2^6$.)

For most positions on the encoder, the error of a single bit results in a positional error of only one eighth of the circle. This is not true with binary coding, where single bit errors can give larger positional errors. For example if the positional decoder reads 100 instead of 000, this is a difference of 4 in binary. The same codes differ by only one position in Gray code.

**Classical Design Techniques**

We can summarize the classical design technique for a state machine, as follows:

1. Define the problem.
2. Draw a state diagram.
3. Make a state table that lists all possible present states and inputs and the next state and output state for each present state/input combination. **List the present states and inputs in binary order.**
4. Use flip-flop excitation tables to determine at what states the flip-flop synchronous inputs must be to make the circuit go from each present state to its next state. **The next state variables are functions of the inputs and present state variables.**
5. Write the output value for each present state/input combination. **The output variables are functions of the inputs and present state variables.**
6. Simplify the Boolean expression for each output and synchronous input.
7. Use the Boolean expressions found in step 6 to draw the required logic circuit.

Let us follow this procedure to design a 3-bit Gray code counter. We will modify the procedure to account for the fact that there are no inputs other than the clock and no outputs that must be designed apart from the counter itself.

1. **Define the problem.** Design a counter whose outputs progress in the sequence defined in Table 10.1.
2. **Draw a state diagram.** The state diagram is shown in Figure 10.4. In addition to the values of state variables shown in each circle (or bubble), we also indicate a state name, such as s0, s1, s2, and so on. This name is independent of the value of state variables. We use numbered states (s0, s1, . . .) for convenience, but we could use any names we wanted to.

**FIGURE 10.4**
State Diagram for a 3-bit Gray Code Counter

3. **Make a state table.** The state table, based on D flip-flops, is shown in Table 10.2. Since there are eight unique states in the state diagram, we require three state variables ($2^3 = 8$), and hence three flip-flops. Note that the present states are in binary-weighted order, even though the count does not progress in this order. In such a case, it is essential to have an accurate state diagram, from which we derive each next state. For example, if
The K-maps yield three Boolean equations:

\[ D_2 = Q_1 \bar{Q}_0 + Q_2 Q_0 \]
\[ D_1 = Q_1 \bar{Q}_0 + Q_2 Q_0 \]
\[ D_0 = Q_2 \bar{Q}_1 + Q_2 Q_1 \]

FIGURE 10.5
Karnaugh Maps for 3-bit Gray Code Counter

The K-maps yield three Boolean equations:

\[ D_2 = Q_1 \bar{Q}_0 + Q_2 Q_0 \]
\[ D_1 = Q_1 \bar{Q}_0 + Q_2 Q_0 \]
\[ D_0 = Q_2 \bar{Q}_1 + Q_2 Q_1 \]

6. Draw the logic circuit for the state machine. Figure 10.6 shows the circuit for a 3-bit Gray code counter, drawn as a Graphic Design File in MAX+PLUS II. A simulation for this circuit is shown in Figure 10.7, with the outputs shown as individual waveforms and as a group with a binary value.
FIGURE 10.6
Logic Diagram of a 3-bit Gray Code Counter
VHDL Design of State Machines

**Key Terms**

**Enumerated type**  A user-defined type in VHDL in which all possible values of a named identifier are listed in a type definition statement.

State machines can be defined in VHDL within a CASE statement. The VHDL code below illustrates the principle, using the 3-bit Gray code counter as an example.

```vhdl
-- gray_ct1.vhd
-- 3-bit Gray code counter
-- (state machine with decoded outputs)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY gray_ct1 IS
    PORT(
        clk : IN STD_LOGIC;
        q   : OUT STD_LOGIC_VECTOR(2 downto 0));
END gray_ct1;

ARCHITECTURE a OF gray_ct1 IS
    TYPE STATE_TYPE IS (s0, s1, s2, s3, s4, s5, s6, s7);
    SIGNAL state: STATE_TYPE;
BEGIN
    PROCESS (clk)
    BEGIN
        IF clk'EVENT AND clk = '1' THEN
            CASE state IS
                WHEN s0 =>
                    state <= s1;
                WHEN s1 =>
                    state <= s2;
                WHEN s2 =>
                    state <= s3;
                WHEN s3 =>
                    state <= s4;
                WHEN s4 =>
                    state <= s5;
            END CASE;
        END IF;
    END PROCESS;
END gray_ct1;
```

**Figure 10.7**
Simulation of a 3-bit Gray Code Counter (from Graphic Design File)
WHEN s5 =>
    state <= s6;
WHEN s6 =>
    state <= s7;
WHEN s7 =>
    state <= s0;
END CASE;
END IF;
END PROCESS;

WITH state SELECT
    q <= "000" WHEN s0,
        "001" WHEN s1,
        "011" WHEN s2,
        "010" WHEN s3,
        "110" WHEN s4,
        "111" WHEN s5,
        "101" WHEN s6,
        "100" WHEN s7;
END a;

Recall that the format of a CASE statement is:

```
CASE __expression IS
    WHEN __constant_value =>
        __statement;
        __statement;
    WHEN __constant_value =>
        __statement;
        __statement;
    WHEN OTHERS =>
        __statement;
        __statement;
END CASE;
```

The keyword _expression_ in the CASE statement refers to a signal called _state_ that we define to represent the state variables within the machine. For each possible value of _state_, we make an assignment indicating the next state of the machine. For example, the clause `(WHEN s0 => (state <= s1))` indicates a transition from state s0 to state s1. The actual output values of the counter are assigned in a selected signal assignment statement after the PROCESS statement.

Notice that the signal _state_ can have one of eight different values, from s0 to s7. Until now, we have seen signals with values such as ‘1’ (BIT or STD_LOGIC types), “011” (BIT_VECTOR or STD_LOGIC_VECTOR types), or 7 (INTEGER types). The signal _state_ is of type STATE_TYPE, which is a user-defined enumerated type. An enumerated type is simply a list of all values a signal, variable, or port of that type is allowed to have.

For example, we could define a type called DIRECTION with four values, with the statement:

```
TYPE DIRECTION IS (up, down, left, right);
```

We could then define a signal called _position_ of type DIRECTION:

```
SIGNAL position: DIRECTION;
```

An IF statement or other construct could then assign one of the four defined values of type DIRECTION to the signal called _position:_
IF (x='0' and y='0') THEN
    position <= down;
ELSIF (x='0' and y='1') THEN
    position <= left;
ELSIF (x='1' and y='0') THEN
    position <= up;
ELSE
    position <= right;
END IF;

Thus the named identifier position of type DIRECTION can take on only the four values specified in the enumerated type definition.

An alternative way to encode the 3-bit counter is to include output assignments within the body of the CASE statement. Each case then has more than one statement, as indicated in the following VHDL code.

-- gray_ct2.vhd
-- 3-bit Gray code counter
-- (outputs defined within states)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY gray_ct2 IS
    PORT(
        clk : IN   STD_LOGIC;
        q   : OUT  STD_LOGIC_VECTOR(2 downto 0));
END gray_ct2;
ARCHITECTURE a OF gray_ct2 IS
    TYPE STATE_TYPE IS (s0, s1, s2, s3, s4, s5, s6, s7);
    SIGNAL state: STATE_TYPE;
BEGIN
    PROCESS (clk)
    BEGIN
        IF clk'EVENT AND clk = '1' THEN
            CASE state IS
                WHEN s0 =>
                    state <= s1;
                    q <= "001";
                WHEN s1 =>
                    state <= s2;
                    q <= "011";
                WHEN s2 =>
                    state <= s3;
                    q <= "010";
                WHEN s3 =>
                    state <= s4;
                    q <= "110";
                WHEN s4 =>
                    state <= s5;
                    q <= "111";
                WHEN s5 =>
                    state <= s6;
                    q <= "101";
                WHEN s6 =>
                    state <= s7;
                    q <= "100";
                WHEN s7 =>
            END CASE;
        END IF;
    END PROCESS;
END a;

gray_ct2.vhd
The above VHDL code is identical to that of the previous example, except for the way the outputs are assigned.

**SECTION 10.2 REVIEW PROBLEM**

10.2 Write the Boolean equations for the \( J \) and \( K \) inputs of the flip-flops in a 3-bit Gray code counter based on JK flip-flops.

### 10.3 State Machines with Control Inputs

**KEY TERMS**

- **Control input** A state machine input that directs the machine from state to state.
- **Conditional transition** A transition between states of a state machine that occurs only under specific conditions of one or more control inputs.
- **Unconditional transition** A transition between states of a state machine that occurs regardless of the status of any control inputs.

As an extension of the techniques used in the previous section, we will examine the design of state machines that use **control inputs**, as well as the clock, to direct their operation. Outputs of these state machines will not necessarily be the same as the states of the machine’s flip-flops. As a result, this type of state machine requires a more detailed state diagram notation, such as that shown in Figure 10.8.

The state machine represented by the diagram in Figure 10.8 has two states, and thus requires only one state variable. Each state is represented by a bubble (circle) containing the state name and the value of the state variable. For example, the bubble containing the notation **start** \( 0 \) indicates that the state called **start** corresponds to a state variable with a value of 0. Each state must have a unique value for the state variable(s).

Transitions between states are marked with a combination of input and output values.
corresponding to the transition. The inputs and outputs are labeled in1, in2, . . . , inx/out1, out2, . . . , outx. The inputs and outputs are sometimes simply indicated by the value of each variable for each transition. In this case, a legend indicates which variable corresponds to which position in the label.

For example, the legend in the state diagram of Figure 10.8 indicates that the inputs and outputs are labeled in the order in1/out1, out2. Thus if the machine is in the start state and the input in1 goes to 0, there is a transition to the state continue. During this transition, out1 goes to 1 and out2 goes to 0. This is indicated by the notation 0/10 beside the transitional arrow. This is called a conditional transition because the transition depends on the state of in1. The other possibility from the start state is a no-change transition, with both outputs at 0, if in1 = 1. This is shown as 1/00.

If the machine is in the state named continue, the notation X/01 indicates that the machine makes a transition back to the start state, regardless of the value of in1, and that out1 = 0 and out2 = 1 upon this transition. Since the transition always happens, it is called an unconditional transition.

What does this state machine do? We can determine its function by analyzing the state diagram, as follows.

1. There are two states, called start and continue. The machine begins in the start state and waits for a LOW input on in1. As long as in1 is HIGH, the machine waits and the outputs out1 and out2 are both LOW.
2. When in1 goes LOW, the machine makes a transition to continue in one clock pulse. Output out1 goes HIGH.
3. On the next clock pulse, the machine goes back to start. The output out2 goes HIGH and out1 goes back LOW.
4. If in1 is HIGH, the machine waits for a new LOW on in1. Both outputs are LOW again. If in1 is LOW, the cycle repeats.

In summary, the machine waits for a LOW input on in1, then generates a pulse of one clock cycle duration on out1, then on out2. A timing diagram describing this operation is shown in Figure 10.9.

**FIGURE 10.9**
Ideal Operation of State Machine in Figure 10.8

Classical Design of State Machines with Control Inputs

We can use the classical design technique of the previous section to design a circuit that implements the state diagram of Figure 10.8.

1. Define the problem. Implement a digital circuit that generates a pulse on each of two outputs, as described above. For this implementation, let us use JK flip-flops for the state logic. If we so chose, we could also use D or T flip-flops.
2. Draw a state diagram. The state diagram is shown in Figure 10.8.
3. Make a state table. The state table is shown in Table 10.3. The combination of present state and input are listed in binary order, thus making Table 10.3 into a truth table for the next state and output functions. Since there are two states, we require one state variable, Q. The next state of Q, a function of the present state and the input in1, is determined by examining the state diagram. (Thus, if you are in state 0, the next state is 1 if in1 = 0 and 0 if in1 = 1. If you are in state 1, the next state is always 0.)

4. Use flip-flop excitation tables to determine at what states the flip-flop synchronous inputs must be to make the circuit go from each present state to its next state. Table 10.4 shows the flip-flop excitation table for a JK flip-flop. The synchronous inputs are derived from the present-to-next state transitions in Table 10.4 and entered into Table 10.3. (Refer to the synchronous counter design process in Chapter 9 for more detail about using flip-flop excitation tables.)

5. Write the output values for each present state/input combination. These can be determined from the state diagram and are entered in the last two columns of Table 10.3.

6. Simplify the Boolean expression for each output and synchronous input. The following equations represent the next state and output logic of the state machine:

   \[
   J = \overline{Q} \cdot \overline{\text{in1}} + Q \cdot \text{in1} = \overline{\text{in1}} \\
   K = 1 \\
   \text{out1} = \overline{Q} \cdot \overline{\text{in1}} \\
   \text{out2} = Q \cdot \overline{\text{in1}} + Q \cdot \text{in1} = Q
   \]

7. Use the Boolean expressions found in step 6 to draw the required logic circuit.

   Figure 10.10 shows the circuit of the state machine drawn as a MAX+PLUS II Graphic Design File. Since out1 is a function of the control section and the memory section of the machine, we can categorize the circuit as a Mealy machine. (All counter circuits that we have previously examined have been Moore machines since their outputs are derived solely from the flip-flop outputs of the circuit.)

   Since the circuit is a Mealy machine, it is vulnerable to asynchronous changes of output due to asynchronous input changes. This is shown in the simulation waveforms of Figure 10.11.

![Figure 10.10](state_x2a.gdf) Implementation of State Machine of Figure 10.8
The state variable is stored as the state of the JK flip-flop. This state is clocked through a D flip-flop to generate \( \text{out2} \) and combined with \( \text{in1} \) to generate \( \text{out1} \) via another flip-flop. The simulation for this circuit, shown in Figure 10.13, indicates that the two outputs are synchronous with the clock, but delayed by one clock cycle after the state change.

Ideally, \( \text{out1} \) should not change until the first positive clock edge after \( \text{in1} \) goes LOW. However, since \( \text{out1} \) is derived from a combinational output, it will change as soon as \( \text{in1} \) goes LOW, after allowing for a short propagation delay. Also, since \( \text{out2} \) is derived directly from a flip-flop and \( \text{out1} \) is derived from the same flip-flop via a gate, \( \text{out1} \) stays HIGH for a short time after \( \text{out2} \) goes HIGH. (The extra time represents the propagation delay of the gate.)

If output synchronization is a problem (and it may not be), it can be fixed by adding a synchronizing D flip-flop to each output, as shown in Figure 10.12.

The state variable is stored as the state of the JK flip-flop. This state is clocked through a D flip-flop to generate \( \text{out2} \) and combined with \( \text{in1} \) to generate \( \text{out1} \) via another flip-flop. The simulation for this circuit, shown in Figure 10.13, indicates that the two outputs are synchronous with the clock, but delayed by one clock cycle after the state change.

### VHDL Implementation of State Machines with Control Inputs

The VHDL code for a state machine with one or more control inputs is similar to that for a machine with no control inputs. The machine states are still defined using a CASE statement, but a case representing a conditional transition will contain an IF statement.
The VHDL code for the state machine implemented above is as follows.

```
-- state_x1.vhd
-- state machine example 1
-- Two states, one input, two outputs
-- Generates a pulse on one output, then the next
-- after receiving a LOW on the input

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY state_x1 IS
    PORT(
        clk, in1   : IN   STD_LOGIC;
        out1, out2 : OUT  STD_LOGIC);
END state_x1;

ARCHITECTURE a OF state_x1 IS
    TYPE PULSER IS (start, continue);
    SIGNAL sequence: PULSER;

BEGIN
    PROCESS (clk)
    BEGIN
        IF clk'EVENT AND clk = '1' THEN
            CASE sequence IS
                WHEN start =>
                    IF in1 = '1' THEN
                        sequence <= start; -- no change if in1
                        out1 <= '0';
                        out2 <= '0';
                    ELSE
                        sequence <= continue; -- proceed if in1
                        out1 <= '1';          -- pulse on out1
                        out2 <= '0';
                    END IF;
                WHEN continue =>
                    sequence <= start;
                    out1 <= '0';
                    out2 <= '1';           -- pulse on out2
            END CASE;
        END IF;
    END PROCESS;
END a;
```
The transition from start is conditional, so the case for start contains an IF statement that defines the possible state transitions and their associated output states. The transition from continue is unconditional, so no IF statement is needed in the corresponding case.

Figure 10.14 shows the simulation for the VHDL design entity, state_x1.vhd. The values of the state variable, sequence, are also shown in the simulation. This gives us a ready indication of the machine’s state (start or continue).

The design of the state machine is such that if the input in1 is held LOW beyond the end of one pulse cycle, the cycle will repeat, as shown in the simulation of Figure 10.15.

EXAMPLE 10.1

A state machine called a single-pulse generator operates as follows:

1. The circuit has two states: seek and find, an input called sync and an output called pulse.
2. The state machine resets to the state seek. If sync = 1, the machine remains in seek and the output, pulse, remains LOW.
3. When sync = 0, the machine makes a transition to find. In this transition, pulse goes HIGH.
4. When the machine is in state find and sync = 0, the machine remains in find and pulse goes LOW.
5. When the machine is in find and sync = 1, the machine goes back to seek and pulse remains LOW.

Use classical state machine design techniques to design the circuit for the single-pulse generator, using D flip-flops for the state logic. Use MAX+PLUS II to draw the state
machine circuit. Create a simulation to verify the design operation. Briefly describe what this state machine does.

**Solution**  Figure 10.16 shows the state diagram derived from the description of the state machine. The state table is shown in Table 10.5. Since \( Q \) follows \( D \), the \( D \) input is the same as the next state of \( Q \).

**FIGURE 10.16**
Example 10.1
State Diagram for a Single-pulse Generator

![State Diagram](image)

**Table 10.5** State Table for Single-Pulse Generator

<table>
<thead>
<tr>
<th>Present State</th>
<th>Input</th>
<th>Next State</th>
<th>Sync. Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>sync</td>
<td>( Q )</td>
<td>( D )</td>
<td>pulse</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The next-state and output equations are:

\[
D = Q \cdot \overline{\text{sync}} + Q \cdot \overline{\text{sync}} = \overline{\text{sync}}
\]

\[
\text{pulse} = Q \cdot \text{sync}
\]

Figure 10.17 shows the state machine circuit derived from the above Boolean equations. The simulation for this circuit is shown in Figure 10.18. The simulation shows that the circuit generates one pulse when the input \( \text{sync} \) goes LOW, regardless of the length of time that \( \text{sync} \) is LOW. The circuit could be used in conjunction with a debounced pushbutton to produce exactly one pulse, regardless of how long the pushbutton was held down. Figure 10.19 shows such a circuit.

**FIGURE 10.17**
Example 10.1
Single-pulse Generator
EXAMPLE 10.2

The state machine of Example 10.1 is vulnerable to asynchronous input changes. How do we know this from the circuit schematic and from the simulation waveform? Modify the circuit to eliminate the asynchronous behavior and show the effect of the change on a simulation of the design. How does this change improve the design?

Solution

The output, pulse, in the state machine of Figure 10.17 is derived from the state flip-flop and the combinational logic of the circuit. The output can be affected by a change that is purely combinational, thus making the output asynchronous. This is demonstrated on the first pulse of the simulation in Figure 10.18, where pulse momentarily goes HIGH between clock edges. Since no clock edge was present when either the input, sync, changed or when pulse changed, the output pulse must be due entirely to changes in the combinational part of the circuit.

The circuit output can be synchronized to the clock by adding an output flip-flop, as in Figure 10.20. A simulation of this circuit is shown in Figure 10.21. With the synchronized output, the output pulse is always the same width: one clock period. This gives a more predictable operation of the circuit.
EXAMPLE 10.3

Write the VHDL code for a design entity that implements the single-pulse generator, as described in Example 10.1. Create a simulation that verifies the operation of the design.

Solution

The required VHDL code is given here in the design entity `sngl_pls`.

```
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY sngl_pls IS
  PORT(
    clk, sync : IN STD_LOGIC;
    pulse : OUT STD_LOGIC);
END sngl_pls;

ARCHITECTURE pulser OF sngl_pls IS
  TYPE PULSE_STATE IS (seek, find);
  SIGNAL status: PULSE_STATE;
  BEGIN
    PROCESS (clk, sync)
    BEGIN
      IF (clk'EVENT and clk = '1') THEN
        CASE status IS
        WHEN seek =>   IF (sync = '1') THEN
            status <= seek;
            pulse <= '0';
            ELSE
            status <= find;
            pulse <= '1';
            END CASE;
        WHEN find =>    IF (sync = '1') THEN
            status <= seek;
            pulse <= '0';
            ELSE
            status <= find;
            pulse <= '0';
            END CASE;
        END IF;
    END PROCESS;
  END pulser;
```
SECTION 10.3 REVIEW PROBLEM

10.3  Briefly explain why the single-pulse circuit in Figure 10.20 has a flip-flop on its output.

10.4  Switch Debouncer for a Normally Open Pushbutton Switch

**KEY TERMS**

Form A contact  A normally open contact on a switch or relay.

Form B contact  A normally closed contact on a switch or relay.

Form C contact  A pair of contacts, one normally open and one normally closed, that operate with a single action of a switch or relay.

A useful interface function is implemented by a digital circuit that removes the mechanical bounce from a pushbutton switch. The easiest way to debounce a pushbutton switch is with a NAND latch, as shown in Figure 10.23.

![NAND Latch as a Switch Debouncer](image)

The latch eliminates switch bounce by setting or resetting on the first bounce of a switch contact and ignoring further bounces. The limitation of this circuit is that the input switch must have **Form C contacts**. That is, the switch has normally open, normally closed, and common contacts. This is so that the switch resets the latch when pressed (i.e.,
when the normally open contact closes) and sets the latch when released (normally closed contact recloses). Each switch position activates an opposite latch function.

If the only available switch has a single set of contacts, such as the normally open (Form A) pushbuttons on the Altera UP-1 Education Board, a different debouncer circuit must be used. We will look at two solutions using VHDL: one based on an existing device (the Motorola MC14490 Contact Bounce Eliminator) and another that implements a state machine solution to the contact bounce problem.

**Switch Debouncer Based on a 4-bit Shift Register**

The circuit in Figure 10.24 is based on the same principle as the Motorola MC14490 Contact Bounce Eliminator, adapted for use in an Altera CPLD, such as the EPM7128S or the EPF10K20 on the Altera UP-1 Education Board.

![FIGURE 10.24](image)

Switch Debouncer Based on a 4-bit Shift Register

The heart of the debouncer circuit in Figure 10.24 is a 2-bit comparator (an Exclusive NOR gate) and a 4-bit serial shift register, with active-HIGH synchronous LOAD. The XNOR gate compares the shift register serial input and output. When the shift register input and output are different, the input data are serially shifted through the register. When input and output of the shift register are the same, the binary value at the serial output is parallel-loaded back into all bits of the shift register.

Figure 10.25 shows the timing of the debouncer circuit with switch bounces on both make and break phases of the switch contact. The line labeled 4-bit delay refers to the shift register flip-flop outputs. Pushbutton input is pb_in, debounced output is pb_out and clk is the UP-1 system clock, divided by 2^16. (Time values in Figure 10.25 are not to scale and should be disregarded.)

![FIGURE 10.25](image)

Simulation of the Shift Register-Based Debouncer
Assume the shift register is initially filled with 0s. The pushbutton rest state is HIGH. As shown in Figure 10.24, the pushbutton input value is inverted and applied to the shift register input. Therefore, before the switch is pressed, both input and output of the shift register are LOW. Since they are the same, the XNOR output is HIGH, which keeps the shift register in LOAD mode and the LOW at pb_out is reloaded to the register on every positive clock edge.

When the switch is pressed, it will bounce, as shown above the second, third, and fourth clock pulses on Figure 10.25. Just before the second clock pulse, pb_in is LOW. This makes the shift register input and output different, so a 1 is shifted in. (Recall that pb_in is at the opposite logic level to the shift register input.) On the next clock pulse, pb_in has bounced HIGH again. The shift register input and output are now the same, so the output value, 0, is loaded in parallel to all flip-flops of the shift register. On the fifth pulse, pb_in is stable at logic LOW. Since the shift register input is now HIGH and the output is LOW, the HIGH is shifted through the register. We see this by 4-bit delay increasing in value: 0, 1, 3, 7, F, which in binary is equivalent to 0000, 0001, 0111, 0111, 1111. At this point, the input and output are now the same and the output value, 1, is parallel-loaded into the register on each clock pulse.

A similar process occurs when the waveform goes back to the HIGH state. When the input goes HIGH, a LOW is shifted into the shift register. If the input bounces back LOW, the shift register is parallel-loaded with HIGHs and the process starts over. When pb_in is stable at a HIGH level, a LOW is shifted through the register, resulting in the hexadecimal sequence F, E, C, 8, 0, which is equivalent to the binary values 1111, 1110, 1100, 1000, 0000.

To produce an output change, the shift register input and output must remain different for at least four clock pulses. This implies that the input is stable for that period of time. If the input and output are the same, this could mean one of two things. Either the input is stable and the shift register flip-flops should be kept at a constant state or the input has bounced back to its previous level and the shift register should be reinitialized. In either case, the output value should be parallel loaded back into the shift register. Serial shifting should only occur if there has been an input change.

The debouncer in Figure 10.24 is effective for removing bounce that lasts for no more than 4 clock periods. Since switch bounce is typically about 10 ms in duration, the clock should have a period of about 2.5 ms. At 25.175 MHz (a clock period of about 40 ns), the Altera UP-1 system clock is much too fast.

If we divide the oscillator frequency by 65536 ($= 2^{16}$) using a 16-bit counter, we obtain a clock waveform for the debouncer with a period of 2.6 ms. Four clock periods (10.2 ms) are sufficient to take care of switch bounce.

We can use VHDL to synthesize the switch debouncer by instantiating a counter and shift register from the Altera Library of Parameterized Modules and connecting them together with internal signals. The VHDL code is as follows.

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;
```

```vhdl
-- debounce.vhd
-- Switch Debouncer for a Form A contact, based on a 4-bit shift register. Function is similar to a Motorola MC14490 Contact Bounce Eliminator.
-- Use modules from Library of Parameterized Modules (LPM):
-- LPM_SHIFTREG (Shift Register)
-- LPM_COUNTER (16-bit counter)

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;
```
ENTITY debounce IS
    PORT(
        clk : IN STD_LOGIC;
        pb_in : IN STD_LOGIC;
        pb_out : OUT STD_LOGIC);
END debounce;

ARCHITECTURE debouncer OF debounce IS
    -- Internal signals required to interconnect counter and shift register
    SIGNAL srg_ser_out, srg_ser_in, srg_clk, srg_load : STD_LOGIC;
    SIGNAL srg_data   : STD_LOGIC_VECTOR(3 DOWNTO 0);
    SIGNAL ctr_q      : STD_LOGIC_VECTOR (15 DOWNTO 0);
BEGIN
    -- Instantiate 16-bit counter
    clock_divider: lpm_counter
        GENERIC MAP (LPM_WIDTH    => 16)
        PORT MAP (clock      => clk,
                   q          => ctr_q(15 DOWNTO 0));
    -- Instantiate 4-bit shift register
    four_bit_delay: lpm_shiftreg
        GENERIC MAP (LPM_WIDTH     => 4)
        PORT MAP (shiftin      => srg_ser_in,
                   clock       => srg_clk,
                   load        => srg_load,
                   data        => srg_data(3 downto 0),
                   shiftout     => srg_ser_out);
    -- Shift register is clocked by counter output
    -- (divides system clock by 2^16)
    srg_clk   <= ctr_q(15);
    -- Undebounced pushbutton input to shift register
    srg_ser_in <=  not pb_in;
    -- Shift register is parallel-loaded with output data if
    -- shift register input and output are the same.
    -- If input and output are different,
    -- data are serial-shifted.
    srg_data(3)   <= srg_ser_out;
    srg_data(2)   <= srg_ser_out;
    srg_data(1)   <= srg_ser_out;
    srg_data(0)   <= srg_ser_out;
    pb_out        <= srg_ser_out;
    srg_load      <= not((not pb_in) xor srg_ser_out);
END debouncer;

Figure 10.26 shows a fairly easy way to test the switch debouncer. The debouncer output is used to clock an 8-bit counter whose outputs are decoded by two seven-segment decoders. (The decoders are VHDL files developed in a similar way to the seven-segment decoders in Chapter 5.)

Pin numbers are given for the EPM7128S CPLD on the Altera UP-1 circuit board. Since the clock and seven segment displays are hardwired on the Altera board, the only external connections required for the circuit are wires for the two pushbutton inputs, reset and pb_in.
FIGURE 10.26
Test Circuit for a Switch Debouncer
If the debouncer is working properly, the seven-segment display should advance by one each time pb_in is pressed. If the debouncer is not working, the display will change by an unpredictable number with each switch press.

The component source files for the debouncer and test circuit components are supplied on the CD accompanying this book in the folder `drive\Student Files\Chapter 10`. To use these files, create a symbol for each one (File menu; Project; Set Project to Current File; then File menu; Create Default Symbol) and draw the Graphic Design File of Figure 10.26.

Alternatively, you can instantiate each file as a component in a VHDL design entity (all components are designed in VHDL) and connect them together with internal signals.

**Behaviorally Designed Switch Debouncer**

We can also design a switch debouncer by using a behavioral state machine description in VHDL. In order to do so, we need to define the operation of the circuit with a state diagram, as in Figure 10.27.

![State Diagram for a Behaviorally Designed Switch Debouncer](image)

Transitions between states are determined by comparing pb_in and pb_out. If they are the same (00 or 11), the machine advances to the next state; if they are different (01 or 10), the machine reverts to the initial state, s0. At any point in the state diagram (including state s3, the last state), the machine will reset if pb_in and pb_out are different, indicating a bounce on the input.

If pb_in and pb_out are the same for four clock pulses, the input is deemed to be stable. Only at this point will the output change to its opposite state.

**NOTE**

In the shift register–based debouncer, the circuit advanced to the next state if the shift register input and output were different and reset if they were the same. This might appear to be opposite to our behavioral description, but it is not if you look carefully. The shift register debouncer circuit inverts pb_in before applying the signal to the serial input of the shift register. Therefore, viewed from the circuit input and output terminals, rather than at the shift register input and output, the description is the same in both cases.
The VHDL code corresponding to the behavioral description of the switch debouncer is given next. The only output change is specified on the transition from state s3 to s0 when \( pb_{in} = pb_{out} \). Since no change is allowed at any other time, no other output state needs to be specified.

```vhdl
-- dbc_behv.vhd
-- Behavioral definition of a switch debouncer
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY dbc_behv IS
PORT(
   clk, pb_in : IN STD_LOGIC;
   pb_out : BUFFER STD_LOGIC);
END dbc_behv;

ARCHITECTURE debounce of dbc_behv IS
    TYPE sequence IS (s0, s1, s2, s3);
    SIGNAL state: sequence;
BEGIN
    PROCESS (clk, pb_in)
    BEGIN
        IF (clk'EVENT and clk='1') THEN
            CASE state IS
                WHEN s0=> IF (pb_in = pb_out) THEN
                    state <= s1;
                ELSE
                    state <= s0;
                END IF;
                WHEN s1=> IF (pb_in = pb_out) THEN
                    state <= s2;
                ELSE
                    state <= s0;
                END IF;
                WHEN S2=> IF (pb_in = pb_out) THEN
                    state <= s3;
                ELSE
                    state <= s0;
                END IF;
                WHEN s3=> IF (pb_in = pb_out) THEN
                    state <= s0;
                    pb_out <= not pb_out;
                ELSE
                    state <= s0;
                END IF;
                WHEN others => state <= s0;
            END CASE;
        END IF;
    END PROCESS;
END debounce;
```

Figure 10.28 shows a simulation of the behaviorally-designed switch debouncer. State s1 through s3 are of too short a duration to show properly on the simulation, so further details of the simulation are shown in Figures 10.29 and 10.30.
Note that the behaviorally designed switch debouncer does not have a built-in clock divider. If we were to use the circuit on the Altera UP-1 board, we would need to include a divide-by-$2^{16}$ counter to the circuit, as shown in Figure 10.31.

**SECTION 10.4 REVIEW PROBLEM**

10.4 What is the fastest acceptable clock rate for the shift register portion of the debouncer in Figure 10.24 if the pushbutton switch bounces for 15ms?
FIGURE 10.31
Using a Behaviorally Designed Debouncer with a 16-bit Clock Divider
10.5 Unused States in State Machines

In our study of counter circuits in Chapter 9, we found that when a counter modulus is not equal to a power of two there were unused states in the counter’s sequence. For example, a mod-10 counter has six unused states, as the counter requires four bits to express ten states and the maximum number of 4-bit states is sixteen. The unused states (1010, 1011, 1100, 1101, 1110, and 1111) have to be accounted for in the design of a mod-10 counter.

The same is true of state machines whose number of states does not equal a power of two. For instance, a machine with five states requires three state variables. There are up to eight states available in a machine with three state variables, leaving three unused states. Figure 10.32 shows the state diagram of such a machine.

Unused states can be dealt with in two ways: they can be treated as don’t care states, or they can be assigned specific destinations in the state diagram. In the latter case, the safest destination is the first state, in this case the state called start.

EXAMPLE 10.4

Redraw the state diagram of Figure 10.32 to include the unused states of the machine’s state variables. Set the unused states to have a destination state of start. Briefly describe the intended operation of the state machine.

Solution

Figure 10.33 shows the revised state diagram.

The machine begins in state start and waits for a HIGH on in1. The machine then makes a transition to wait1 and stays there until in1 goes LOW again. The machine goes to wait2 and stays there until in1 goes HIGH and then makes an unconditional transition to pulse1 on the next clock pulse. Until this point, there is no change in either output.

The machine makes an unconditional transition to pulse2 and makes out1 go HIGH. The next transition, also unconditional, is to start, when out1 goes LOW and out2 goes HIGH. If in1 is LOW, the machine stays in start. Otherwise, the cycle continues as above. In either case, out2 goes LOW again.
Thus the machine waits for a HIGH-LOW-HIGH input sequence and generates a pulse sequence on two outputs.

**EXAMPLE 10.5**

Use classical state machine design techniques to implement the state machine described in the modified state diagram of Figure 10.33. Draw the state machine as a Graphic Design File in Max+PLUS II and create a simulation to verify its function.

**Solution**

Table 10.6 shows the state table of the state machine represented by Figure 10.33.

<table>
<thead>
<tr>
<th>Present State $Q_2Q_1Q_0$</th>
<th>Input $in1$</th>
<th>Next State $Q_2Q_1Q_0$</th>
<th>Outputs $out1$, $out2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0</td>
<td>000</td>
<td>0 0</td>
</tr>
<tr>
<td>000</td>
<td>1</td>
<td>001</td>
<td>0 0</td>
</tr>
<tr>
<td>001</td>
<td>0</td>
<td>010</td>
<td>0 0</td>
</tr>
<tr>
<td>001</td>
<td>1</td>
<td>001</td>
<td>0 0</td>
</tr>
<tr>
<td>010</td>
<td>0</td>
<td>010</td>
<td>0 0</td>
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<tr>
<td>010</td>
<td>1</td>
<td>011</td>
<td>0 0</td>
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<tr>
<td>011</td>
<td>0</td>
<td>100</td>
<td>1 0</td>
</tr>
<tr>
<td>011</td>
<td>1</td>
<td>100</td>
<td>1 0</td>
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<tr>
<td>100</td>
<td>0</td>
<td>000</td>
<td>0 1</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>000</td>
<td>0 1</td>
</tr>
<tr>
<td>101</td>
<td>0</td>
<td>000</td>
<td>0 0</td>
</tr>
<tr>
<td>101</td>
<td>1</td>
<td>000</td>
<td>0 0</td>
</tr>
<tr>
<td>110</td>
<td>0</td>
<td>000</td>
<td>0 0</td>
</tr>
<tr>
<td>110</td>
<td>1</td>
<td>000</td>
<td>0 0</td>
</tr>
<tr>
<td>111</td>
<td>0</td>
<td>000</td>
<td>0 0</td>
</tr>
<tr>
<td>111</td>
<td>1</td>
<td>000</td>
<td>0 0</td>
</tr>
</tbody>
</table>
Figure 10.34 shows the Karnaugh maps used to simplify the next-state equations for the state variable flip-flops. The output equations can be simplified by inspection.

The next-state and output equations for the state machine are:

\[
\begin{align*}
D_2 &= \overline{Q}_2Q_1Q_0 \\
D_1 &= \overline{Q}_2Q_1Q_0 + \overline{Q}_2Q_1Q_0\text{in}_1 \\
D_0 &= \overline{Q}_2Q_0\text{in}_1 + \overline{Q}_2Q_1\text{in}_1 \\
\text{out}_1 &= \overline{Q}_2Q_1Q_0 \\
\text{out}_2 &= Q_2\overline{Q}_1\overline{Q}_0
\end{align*}
\]

Figure 10.35 shows the Graphic Design File schematic for the state machine. Figure 10.36 shows the MAX+PLUS II simulation waveforms.

We can monitor the state variables in the MAX+PLUS II simulation file by adding a group of waveforms for the buried nodes q2, q1, and q0. These are shown on the simulation as q[2..0].Q, meaning the Q outputs of the flip-flops named q2, q1, q0.

To add the buried nodes, select **Enter Node from SNF** from the **Node** menu in the simulator window. In the dialog box shown in Figure 10.37, check the box that says **All**, and click on **List**. Select the nodes q2.Q, q1.Q, and q0.Q from the **Available Nodes and Groups** and transfer them to the **Selected Nodes and Groups**. Click **OK**. Select the three new waveforms and from the **Node** menu, select **Group**. Click **OK** in the resulting dialog box.
FIGURE 10.35
Example 10.5
Two-pulse Generator
EXAMPLE 10.6

Write the VHDL code required to implement the two-pulse generator described in Examples 10.4 and 10.5. Create a MAX+PLUS II simulation to verify the operation of the design. Based on your examination of the simulations for the VHDL design and the GDF design of the previous example, how do the two designs differ in their operation? What is the reason for the difference?

Solution

The VHDL code for the state machine in design entity two_pulse.vhd follows. The unused states are accounted for in the others clause.

```vhdl
-- two_pulse.vhd
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY two_pulse IS
  PORT(
    clk, in1 : IN    STD_LOGIC;
    output   : OUT   STD_LOGIC_VECTOR (1 to 2));
END two_pulse;

ARCHITECTURE a OF two_pulse IS
  TYPE SEQUENCE IS (start, wait1, wait2, pulse1, pulse2);
  SIGNAL pulse_state : SEQUENCE;
BEGIN
  PROCESS(clk)
  BEGIN
    IF (clk EVENT and clk = '1') THEN
      CASE pulse_state IS
```

FIGURE 10.36
Example 10.5
Simulation of a Two-pulse Generator (GDF)

FIGURE 10.37
Adding Buried Nodes to a Simulation
WHEN start =>
  IF in1 = '0' THEN
    pulse_state <= start;
    output    <= "00";
  ELSIF in1 = '1' THEN
    pulse_state <= wait1;
    output    <= "00";
  END IF;
WHEN wait1 =>
  IF in1 = '0' THEN
    pulse_state <= wait2;
    output    <= "00";
  ELSIF in1 = '1' THEN
    pulse_state <= wait1;
    output    <= "00";
  END IF;
WHEN wait2 =>
  IF in1 = '0' THEN
    pulse_state <= wait2;
    output    <= "00";
  ELSIF in1 = '1' THEN
    pulse_state <= pulse1;
    output    <= "00";
  END IF;
WHEN pulse1 =>
  pulse_state <= pulse2;
  output    <= "10";
WHEN pulse2 =>
  pulse_state <= start;
  output    <= "01";
WHEN others =>
  pulse_state <= start;
  output    <= "00";
END CASE;
END IF;
END PROCESS;
END a;

Figure 10.38 shows the MAX+PLUS II simulation of the state machine. 
If you closely examine the simulation waveforms in Figures 10.36 and 10.38, you will note that the pulse outputs in Figure 10.38 (VHDL design) occur one clock cycle later than they do in Figure 10.36 (graphical design). This is because the VHDL compiler has synthesized each output with a D flip-flop, as we did for the single-pulse circuit in Figure

**FIGURE 10.38**
Example 10.6
Simulation of a Two-pulse Generator (VHDL)
10.20, in order to ensure synchronous output operation. (We can verify this by examining the EQUATIONS section of the project report file, two_pulse.rpt.) Since the outputs are both derived entirely from flip-flop outputs, this synthesis step is not strictly necessary to ensure that the outputs are synchronous with the clock.

SECTION 10.5 REVIEW PROBLEM

10.5 Is the state machine designed in Example 10.5 a Moore machine or a Mealy machine? Why?

10.6 Traffic Light Controller

A simple traffic light controller can be implemented by a state machine with a state diagram such as the one shown in Figure 10.39.

The control scheme assumes control over a north-south road and an east-west road. The north-south lights are controlled by outputs called nsr, nsy, and nsg (north-south red, yellow, green). The east-west road is controlled by similar outputs called ewr, ewy, and ewg. A LOW controller output turns on a light. Thus an output 011110 corresponds to the north-south red and east-west green lights.

An input called TIMER controls the length of the two green-light cycles. When TIMER = 1, a transition from s0 to s1 or from s2 to s3 is possible (s0 represents the EW green; s2 the NS green). This transition accompanies a change from green to yellow on the active road. The light on the other road stays red. An unconditional transition follows, changing the yellow light to red on one road and the red light to green on the other.

The cycle can be set to any length by changing the signal on the TIMER input. (The yellow light will always be on for one clock pulse in this design.) For ease of observation, we will use a cycle of ten clock pulses. For either direction, the cycle consists of 4 clocks GREEN, 1 clock YELLOW, 5 clocks RED. This cycle can be generated by the MSB of a mod-5 counter, as shown in Figure 10.40. If we model the traffic controller using the Altera UP-1 board, we require a clock divider to slow down the 25.175 MHz clock to a rate of about 0.75 Hz, making it easy to observe the changes of lights. These blocks can all be instantiated in VHDL, which will be left as part of an exercise in the lab manual accompanying this book.
Figure 10.41 shows the simulation of the mod-5 counter that generates the *TIMER* control signal. The MSB goes HIGH for one clock period, then LOW for four. When applied to the *TIMER* input of the output controller, this signal directs the controller from state to state.

![Figure 10.41 Simulation of a Mod-5 Counter](image1)

Figure 10.42 shows a simulation of the mod-5 counter and output controller. The north-south lights are red for five clock pulses (shown by 011 in the *north_south* waveform). At the same time, the east-west lights are green for four clock pulses (*east_west* = 110), followed by yellow for one clock pulse (*east_west* = 101). The cycle continues with an east-west red and north-south green and yellow.

According to the state diagram, the yellow light should happen on the transition where *TIMER* = 1. This corresponds to the point on the simulation waveforms where *count* = 4.

![Figure 10.42 Simulation of a Traffic Light Controller](image2)
However, the yellow light does not come on until \texttt{count} = 0. This is because the MAX+PLUS II VHDL compiler synthesizes the controller outputs with synchronous outputs (flip-flops). As a result, the output states are delayed by one clock cycle. Since the relative lengths of the cycle proportions are preserved, this does not affect the operation of the controller.

### Summary

1. A state machine is a synchronous sequential circuit with a memory section (flip-flops) to hold the present state of the machine and a control section (gates) to determine the machine’s next state.
2. The number of flip-flops in a state machine’s memory section is the same as the number of state variables.
3. Two main types of state machine are the Moore machine and the Mealy machine.
4. The outputs of a Moore machine are entirely dependent on the states of the machine’s flip-flops. Output changes will always be synchronous with the system clock.
5. The outputs of a Mealy machine depend on the states of the machine’s flip-flops and the gates in the control section. A Mealy machine’s outputs can change asynchronously, relative to the system clock.
6. A state machine can be designed in a classical fashion using the same method as in designing a synchronous counter, as follows:
   a. Define the problem and draw a state diagram.
   b. Construct a table of present and next states.
   c. Use flip-flop excitation tables to determine the flip-flop inputs for each state transition.
   d. Use Boolean algebra or K-maps to find the simplest Boolean expression for flip-flop inputs \(D, T, \text{ or } JK\) in terms of outputs \(Q\).
   e. Draw the logic diagram of the state machine.
7. The state names in a state machine can be named numerically \(s_0, s_1, s_2, \ldots\) or literally (start, idle, read, write), depending on the machine function. State names are independent of the values of the state variables.
8. A state machine can be defined in VHDL by using a CASE statement within a PROCESS to define the progression of states. The output values can be defined by a separate decoder construct or they can be assigned within each case of the CASE statement.
9. The possible values of the state variables of a machine are defined within an enumerated type definition. An enumerated type is a list of possible values that a port, variable, or signal of that type is allowed to have.
10. Notation for a state diagram includes a series of bubbles (circles) containing state names and values of state variables in the form \(\text{state_name} = \text{state_variable(s)}\).
11. The inputs and outputs of a state machine are labeled \texttt{in1, in2, \ldots, inx/out1, out2, \ldots, outx}.
12. Transitions between states can be conditional or unconditional. A conditional transition happens only under certain conditions of a control input and is labeled with the relevant input condition. An unconditional transition happens under all conditions of input and is labeled with an X for each input variable.
13. Conditional transitions in a VHDL state machine are described by an IF statement within a particular case of the CASE statement that describes the machine.
14. Mealy machine outputs are susceptible to asynchronous output changes if a combinational input changes out of synchronization with the clock. This can be remedied by clocking each output through a separate synchronizing flip-flop.
15. A maximum of \(2^n\) states can be assigned to a state machine that has \(n\) state variables. If the number of states is less than \(2^n\), the unused states must be accounted for. Either they can be treated as don’t care states, or they can be assigned a specific destination state, usually the reset state.
16. In a VHDL implementation of a state machine, any unused states can be covered with an \texttt{others} clause in the CASE statement that defines the machine.

### Glossary

**Conditional transition** A transition between states of a state machine that occurs only under specific conditions of one or more control inputs.

**Control input** A state machine input that directs the operation of the machine from state to state.

**Enumerated type** A user-defined type in VHDL in which all possible values of a named identifier are listed in a type definition statement.

**Form A contact** A normally open contact on a switch or relay.

**Form B contact** A normally closed contact on a switch or relay.

**Form C contact** A pair of contacts, one normally open and one normally closed, that operate with a single action of a switch or relay.

**Mealy machine** A state machine whose output is determined by both the sequential logic and the combinational logic of the machine.
Moore machine  A state machine whose output is determined only by the sequential logic of the machine.

State machine  A synchronous sequential circuit, consisting of a sequential logic section and a combinational logic section, whose outputs and internal flip-flops progress through a predictable sequence of states in response to a clock and other input signals.

State variables  The variables held in the flip-flops of a state machine that determine its present state.

Unconditional transition  A transition between states of a state machine that occurs regardless of the status of any control inputs.

**Problems**

*Problem numbers set in color indicate more difficult problems: those with underlines indicate most difficult problems.*

**Section 10.1 State Machines**

10.1  Is the state machine in Figure 10.43 a Moore machine or a Mealy machine? Explain your answer.

10.2  Is the state machine in Figure 10.44 a Moore machine or a Mealy machine? Explain your answer.

**FIGURE 10.43**  
Problem 10.1  
State Machine Circuit

**FIGURE 10.44**  
Problem 10.2  
State Machine Circuit
10.4 Use classical state machine design techniques to design a counter whose output sequence is shown in Table 10.8. (This is a divide-by-twelve counter in which the MSB output has a duty cycle of 50%.) Draw the resulting circuit diagram in a MAX+PLUS II Graphic Design File. Create a simulation to verify the circuit operation.

Table 10.8 Counter Sequence for Problem 10.4

<table>
<thead>
<tr>
<th>Q3Q2Q1Q0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
</tr>
<tr>
<td>0001</td>
</tr>
<tr>
<td>0011</td>
</tr>
<tr>
<td>0010</td>
</tr>
<tr>
<td>0110</td>
</tr>
<tr>
<td>0111</td>
</tr>
<tr>
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<tr>
<td>1011</td>
</tr>
<tr>
<td>1001</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>

10.5 Write the VHDL code required to implement a 4-bit Gray code counter. Create a simulation in MAX+PLUS II to verify the operation of the circuit.

10.6 Write the VHDL code required to implement a counter with the sequence shown in Table 10.8. Create a simulation in MAX+PLUS II to verify the operation of the circuit.

Section 10.2 State Machines with No Control Inputs

10.3 A 4-bit Gray code sequence is shown in Table 10.7. Use classical design methods to design a counter with this sequence, using D flip-flops. Draw the resulting circuit diagram in a MAX+PLUS II Graphic Design File. Create a simulation to verify the circuit operation.

Table 10.7 4-bit Gray code sequence

<table>
<thead>
<tr>
<th>Q3Q2Q1Q0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000</td>
</tr>
<tr>
<td>0001</td>
</tr>
<tr>
<td>0011</td>
</tr>
<tr>
<td>0010</td>
</tr>
<tr>
<td>0110</td>
</tr>
<tr>
<td>0111</td>
</tr>
<tr>
<td>0101</td>
</tr>
<tr>
<td>0100</td>
</tr>
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<td>1100</td>
</tr>
<tr>
<td>1101</td>
</tr>
<tr>
<td>1111</td>
</tr>
<tr>
<td>1110</td>
</tr>
<tr>
<td>1010</td>
</tr>
<tr>
<td>1011</td>
</tr>
<tr>
<td>1001</td>
</tr>
<tr>
<td>1000</td>
</tr>
</tbody>
</table>

10.5 Write the VHDL code required to implement a 4-bit Gray code counter. Create a simulation in MAX+PLUS II to verify the operation of the circuit.

10.6 Write the VHDL code required to implement a counter with the sequence shown in Table 10.8. Create a simulation in MAX+PLUS II to verify the operation of the circuit.

Section 10.3 State Machines with Control Inputs

10.7 Use classical state machine design techniques to find the Boolean next state and output equations for the state machine represented by the state diagram in Figure 10.45. Draw the state machine circuit as a Graphic Design File in MAX+PLUS II. Create a simulation file to verify the operation of the circuit. Briefly explain the intended function of the state machine.

Figure 10.45 Problem 10.7 State Diagram

10.8 Referring to the simulation for the state machine in Problem 10.7, briefly explain why it is susceptible to asynchronous input changes. Modify the state machine circuit to eliminate the asynchronous behavior of the outputs. Create a MAX+PLUS II simulation to verify the function of the modified state machine.

10.9 Write the VHDL code required to implement the state machine in Problem 10.7. Create a simulation to verify the operation of the state machine.

10.10 A state machine is used to control an analog-to-digital converter, as shown in the block diagram of Figure 10.46.

Figure 10.46 Problem 10.10 Analog-to-Digital Converter and Controller

The controller has four states, defined by state variables Q1 and Q0 as follows: idle (00), start (01), waiting (11), and read (10). There are two outputs: sc (Start Conversion; active-HIGH) and oe (Output Enable; active LOW). There are four inputs: clock, go (active-LOW) oe (End of Conversion), and asynchronous reset (active LOW). The machine operates as follows:
Problems

a. In the idle state, the outputs are: \(sc = 0, oe = 1\). The machine defaults to the idle state when the machine is reset.
b. Upon detecting a 0 at the go input, the machine makes a transition to the start state. In this transition, \(sc = 1, oe = 1\).
c. The machine makes an unconditional transition to the waiting state; \(sc = 0, oe = 1\). It remains in this state, with no output change, until input eoc = 1.
d. When eoc = 1, the machine goes to the read state; \(sc = 0, oe = 0\).
e. The machine makes an unconditional transition to the idle state; \(sc = 0, oe = 1\).

Use classical state machine design techniques to design the controller. Draw the required circuit in MAX-PLUS II and create a simulation to verify its operation. Is this machine vulnerable to asynchronous input change?

10.11 Use VHDL to implement the controller circuit of Problem 10.10. Create a simulation to verify its operation.

10.12 Write a VHDL file for a state machine that selects a 3-bit binary or Gray code count, depending on the state of an input called gray. If gray = 1, count in Gray code. Otherwise count in binary. Create a simulation file that verifies the operation of the circuit, clearly showing the full Gray code count, binary count, and reset function.

Section 10.4 Switch Debouncer for a Normally Open Pushbutton Switch

10.13 Why is it not possible to debounce the pushbuttons on the Altera UP-1 board using a NAND latch?

10.14 Refer to the switch debouncer circuit in Figure 10.24 (p. 476). For how many clock periods must the input of the debouncer remain stable before the output can change?

10.15 What is the maximum switch bounce time that can be removed by the circuit of Figure 10.24 if the clock at the shift register is running at a rate of 480 Hz?

10.16 Briefly explain how the Exclusive NOR gate in the debounce circuit of Figure 10.24 determines if switch bounce has occurred.

10.17 Refer to the section on the behaviorally designed switch debouncer in Section 10.4. For how many clock periods must the input of the debouncer remain stable before the output can change? What is the maximum switch bounce time that can be removed by the circuit of Figure 10.24 if the state machine clock is running at a rate of 480 Hz?

10.18 Refer to the state diagram in Figure 10.47.

a. How many state variables are required to implement this state machine? Why?

b. How many unused states are there for this state machine? List the unused states.

c. Complete the partial timing diagram shown in Figure 10.48 to illustrate one complete cycle of the state machine represented by the state diagram of Figure 10.47.
10.19 Write the VHDL code required to implement the state machine described by the state diagram of Figure 10.47. Create a simulation file to verify the operation of the circuit.

10.20 Use classical state machine design techniques to design a state machine described by the state diagram of Figure 10.49. Briefly describe the intended operation of the circuit. Create a MAX+PLUS II simulation to verify the operation of the state machine design. Unused states may be treated as don’t care states, but unspecified outputs should always be assigned to 0.

FIGURE 10.49
Problem 10.20
State Diagram

10.21 Determine the next state for each of the unused states of the state machine designed in Problem 10.20. Use this analysis to redraw the state diagram of Figure 10.49 so that it properly includes the unused states. (There is more than one right answer, depending on the result of the Boolean simplification process used in Problem 10.20.)

ANSWERS TO SECTION REVIEW PROBLEMS

Section 10.1

10.1 A Moore state machine has outputs that depend only on the states of the flip-flops in the machine. A Mealy machine’s outputs depend on the states of its flip-flops as well as the gates of the machine’s control section. This can result in asynchronous output changes in the Mealy machine outputs.

Section 10.2

10.2

\[ J_2 = Q_1 \bar{Q}_0 \]
\[ K_2 = Q_1 \bar{Q}_0 \]
\[ J_1 = Q_2 Q_0 \]
\[ K_1 = Q_2 Q_0 \]
\[ J_0 = Q_2 Q_1 + Q_2 Q_1 = Q_2 + Q_1 \]
\[ K_0 = Q_2 Q_1 + Q_2 Q_1 = Q_2 + Q_1 \]

Section 10.3

10.3 The output flip-flop synchronizes the output to the system clock, yielding the following advantages: (1) the output is always a known width of one clock cycle; and (2) the output is not vulnerable to change due to asynchronous changes of input.

Section 10.4

10.4 \( T_c = 3.75 \text{ ms}; f_c = 267 \text{ Hz} \)

Section 10.5

10.5 Moore machine. The outputs are derived entirely from the output states of the state machine and are not vulnerable to asynchronous changes of input.
CHAPTER 11

Logic Gate Circuitry

OUTLINE

11.1 Electrical Characteristics of Logic Gates
11.2 Propagation Delay
11.3 Fanout
11.4 Power Dissipation
11.5 Noise Margin
11.6 Interfacing TTL and CMOS Gates
11.7 Internal Circuitry of TTL Gates
11.8 Internal Circuitry of CMOS Gates
11.9 TTL and CMOS Variations

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

- Name the various logic families most commonly in use today and state several advantages and disadvantages of each.
- Define propagation delay.
- Calculate propagation delay of simple circuits, using data sheets.
- Define fanout and calculate its value, using data sheets.
- Calculate power dissipation of TTL and CMOS circuits.
- Calculate noise margin of a logic gate from data sheets.
- Draw circuits that will interface various CMOS and TTL gates.
- Explain how a bipolar junction transistor can be used as a logic inverter.
- Describe the function of a TTL input transistor in all possible input states: HIGH, LOW, and open-circuit.
- Explain the operation of a totem pole output.
- Illustrate how a totem pole output generates power line noise and describe how to remedy this problem.
- Illustrate why totem pole outputs cannot be tied together.
- Explain the difference between open-collector and totem pole outputs of a TTL gate.
- Illustrate the operation of TTL open-collector inverter, NAND, and NOR gates.
- Write the Boolean expression of a wired-AND circuit.
- Design a circuit that uses an open-collector gate to drive a high-current load.
- Calculate the value of a pull-up resistor at the output of an open-collector gate.
- Explain the operation of a tristate gate and name several of its advantages.
- Design a circuit using a tristate bus driver to direct the flow of data from one device to another.
- Describe the basic structure of a MOSFET and state its bias voltage requirements.
- Draw the circuit of an CMOS inverter and show how it works.
• Draw the circuits of CMOS NAND and NOR gates and explain the operation of each.
• Design a circuit using a CMOS transmission gate to enable and inhibit digital and analog signals.
• Interpret TTL data sheets to distinguish between the various TTL families.
• Describe the use of the Schottky barrier diode in TTL gates.
• Calculate speed-power products from data sheets.

Our study of logic gates and flip-flops in previous chapters has concentrated on digital logic and has largely ignored digital electronics. Digital logic devices are electronic circuits with their own characteristic voltages and currents. No serious study of digital circuitry is complete without some examination of this topic.

It is particularly important to understand the inputs and outputs of logic devices as electronic circuits. Knowing the input and output voltages and currents of these circuits is essential, since gate loading, power dissipation, noise voltages, and interfacing between logic families depend on them. The switching speed of device outputs is also fundamental and may be a consideration when choosing the logic family for a circuit design.

Input and output voltages of logic devices are specified in manufacturers’ data sheets, which allows us to take a “black box” approach initially.

Later in the chapter, we will examine some basic digital circuits at a transistor level, since digital logic is based on transistor switching. Two major types of transistors, the bipolar junction transistor and the metal-oxide-semiconductor field effect transistor (MOSFET), form the basis of the major logic families in use today. Transistor-transistor logic (TTL) is based on the bipolar transistor. Complementary MOS (CMOS) is based on the MOSFET.

We will briefly study the operating characteristics of both bipolar transistors and MOSFETs and then see how these devices give rise to the electrical characteristics of simple logic gates.

11.1 Electrical Characteristics of Logic Gates

**KEY TERMS**

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTL</td>
<td>Transistor-transistor logic. A logic family based on bipolar transistors.</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary metal-oxide semiconductor. A logic family based on metal-oxide-semiconductor field effect transistors (MOSFETs).</td>
</tr>
<tr>
<td>ECL</td>
<td>Emitter coupled logic. A high-speed logic family based on bipolar transistors.</td>
</tr>
</tbody>
</table>

When we examine the electrical characteristics of logic circuits, we see them as practical, rather than ideal devices. We look at properties such as switching speed, power dissipation, noise immunity, and current-driving capability. There are several commonly available logic families in use today, each having a unique set of electrical characteristics that differentiates it from all the others. Each logic family gives superior performance in one or more of its electrical properties.

**CMOS** consumes very little power, has excellent noise immunity, and can be used with a wide range of power supply voltages.

**TTL** has a larger current-driving capability than CMOS. Its power consumption is higher than that of CMOS, and its power supply requirements are more rigid.

**ECL** is fast, making it the choice for high-speed applications. It is inferior to CMOS and TTL in terms of noise immunity and power consumption.

TTL and CMOS gates come in a wide range of subfamilies. Table 11.1 lists some of the TTL and CMOS variations of the quadruple 2-input NAND gate. All gates listed have
the same logic function but different electrical characteristics. Other gates would be similarly designated, with the last two or three digits indicating the gate function (e.g., a quadruple 2-input NOR gate would be designated 74LS02, 74ALS02, 74F02, etc.).

We will examine four electrical characteristics of TTL and CMOS circuits: propagation delay, fanout, noise margin, and power dissipation. The first of these has to do with speed of output response to a change of input. The last three have to do with input and output voltages and currents. All four properties can be read directly from specifications given in a manufacturer’s data sheet or derived from these specifications.

Figures 11.1 and 11.2 show how the input and output voltages and currents are defined in a 74XX00 NAND gate. This designation can be generalized to any logic gate input or output.

<table>
<thead>
<tr>
<th>Part Number</th>
<th>Logic Family</th>
</tr>
</thead>
<tbody>
<tr>
<td>TTL</td>
<td>74LS00 Low-power Schottky TTL</td>
</tr>
<tr>
<td></td>
<td>74ALS00 Advanced low-power Schottky TTL</td>
</tr>
<tr>
<td></td>
<td>74F00 Fast TTL</td>
</tr>
<tr>
<td>CMOS</td>
<td>74HC00 High-speed CMOS</td>
</tr>
<tr>
<td></td>
<td>74HCT00 High-speed CMOS (TTL-compatible inputs)</td>
</tr>
<tr>
<td></td>
<td>74LVX00 Low-voltage CMOS</td>
</tr>
</tbody>
</table>

The voltages and currents are designated with two subscripts, one that designates an input or output and another that indicates the logic level. For example, $V_{OL}$ is the voltage at the gate output when the output is in the logic LOW state. $I_{IL}$ is the input current when the input is in the LOW state.

These voltages and currents are specified in manufacturers’ published data sheets, which are usually available in print form in a data book or in an electronic format, such as Portable Document Format (pdf) on a CD or internet site.

Figure 11.3 shows a data sheet for a 74LS00 NAND gate, which also shows parameter values for a 54LS00 device. A 54-series device is manufactured to military specifications, which require a high range of environmental operating conditions. A 74-series device is suitable for general or commercial use. We will limit ourselves to the 74-series devices.

The voltage and current parameters indicated in Figures 11.1 and 11.2 are all shown in the 74LS00 data sheet. Some parameters are shown as typical values, as well as maximum or minimum. Typical values should be considered “information only” as device manufacturers
**QUAD 2-INPUT NAND GATE**

- ESD > 3500 Volts

**GUARANTEED OPERATING RANGES**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>Supply Voltage</td>
<td>54</td>
<td>4.5</td>
<td>5.0</td>
<td>5.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Ambient Temperature Range</td>
<td>54</td>
<td>-55</td>
<td>25</td>
<td>125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td>0</td>
<td>25</td>
<td>70</td>
</tr>
<tr>
<td>IOH</td>
<td>Output Current — High</td>
<td>54</td>
<td></td>
<td>-0.4</td>
<td>mA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td></td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>IOL</td>
<td>Output Current — Low</td>
<td>54</td>
<td></td>
<td>4.0</td>
<td>mA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td></td>
<td>8.0</td>
<td>mA</td>
</tr>
</tbody>
</table>

**FIGURE 11.3**

74LS00 Data (1 of 2) Reprinted with permission of Motorola.
do not guarantee these values. An exception to this would be the supply voltage, $V_{CC}$, whose typical value is simply indicated as the average of maximum and minimum values.

Note that $I_{IH}$ and $I_{IL}$ are shown in Figure 11.2 as flowing in opposite directions, as are $I_{OH}$ and $I_{OL}$. On a data sheet, a current entering a gate is indicated as positive and a current leaving the gate is shown as having a negative value. The reason for these current directions will become apparent when we examine the internal circuits of the gates later in the chapter.

---

**EXAMPLE 11.1**

What is the maximum value of $V_{OL}$ for a 74LS00 NAND gate when the output current is at its maximum value?

**Solution** When the output is in the LOW state, the output current is given by $I_{OL}$, which has a maximum value of 8 mA. The output voltage, $V_{OL}$, is specified for a value of 4 mA and for 8 mA. Since the output condition is specified for maximum $I_{OL}$ (8 mA), then $V_{OL} = 0.5$ V.
The 74XX00 NAND gate data is sufficient to represent any logic functions having “normal” output current within its particular logic family. This data can be used for most gate or flip-flop circuits within the family. Some specialized devices with higher-current outputs (e.g., 74XX244 octal tristate buffers) have a different set of electrical characteristics within their family.

In the following sections of the chapter, we will use a NAND gate from each of three device families (74LS00, 74HC00A, and 74HCT00A) for illustrating the general principles of the various electrical characteristics. Devices from other families will also be used in examples and problems. Data sheets for the various devices are included in Appendix C.

SECTION 11.1 REVIEW PROBLEM

11.1 What are the maximum values of voltage and current we can expect at the output of a 74LS00 NAND gate when both inputs are LOW?

11.2 Propagation Delay

**KEY TERMS**

\( t_{pHL} \) Propagation delay when the device output is changing from HIGH to LOW.

\( t_{pLH} \) Propagation delay when the device output is changing from LOW to HIGH.

Propagation delay occurs because the output of a logic gate or flip-flop cannot respond instantaneously to changes at its input. There is a short delay, on the order of several nanoseconds, between input change and output response. This is largely due to the charging and discharging of capacitances inherent in the switching transistors of the gate or flip-flop.

Figure 11.4 shows propagation delay in two gates: a 74XX00 NAND gate and a 74XX08 AND gate. Each gate has an identical input waveform, a LOW-HIGH-LOW pulse. After each input transition, the output changes after a short delay, \( t_p \).

![Propagation Delay in NAND and AND Gates](image)

**FIGURE 11.4**

Propagation Delay in NAND and AND Gates

Two delays are shown for each gate: \( t_{pLH} \) and \( t_{pHL} \). The \( LH \) and \( HL \) subscripts show the direction of change at the gate output; \( LH \) indicates that the output goes from LOW to HIGH, and \( HL \) shows the output changing from HIGH to LOW.

Propagation delay is the time between input and output voltages passing through a standard reference value. The reference voltage for standard TTL is 1.5 V. LSTTL and CMOS have different reference voltages, as follows.
Example 11.2

Use the data sheet in Figure 11.3, as well as those in Appendix C, to find the maximum propagation delays for each of the following gates: 74LS00 (quadruple 2-input NAND), 74LS02 (quadruple 2-input NOR), 74LS08 (quadruple 2-input AND), and 74LS32 (quadruple 2-input OR).

Solution

Table 11.2 Propagation Delays of 74LS Gates

<table>
<thead>
<tr>
<th></th>
<th>74LS00</th>
<th>74LS02</th>
<th>74LS08</th>
<th>74LS32</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{plH}$</td>
<td>15 ns</td>
<td>15 ns</td>
<td>15 ns</td>
<td>22 ns</td>
</tr>
<tr>
<td>$t_{pHL}$</td>
<td>15 ns</td>
<td>15 ns</td>
<td>20 ns</td>
<td>22 ns</td>
</tr>
</tbody>
</table>

Table 11.2 shows the variation of propagation delay among logic gates of the same family (74LS TTL). Since each logic function has a different circuit, its propagation delay will differ from those of gates with different functions.

Example 11.3

Use data sheets to find the maximum propagation delays for each of the following logic gates: 74F00, 74AS00, 74ALS00, 74HC00, and 74HCT00.

Solution

Table 11.3 Propagation Delays of 74LS Gates

<table>
<thead>
<tr>
<th></th>
<th>74F00*</th>
<th>74AS00</th>
<th>74ALS00</th>
<th>74HC00**</th>
<th>74HCT00***</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{plH}$</td>
<td>6 ns</td>
<td>4.5 ns</td>
<td>11 ns</td>
<td>15 ns</td>
<td>19 ns</td>
</tr>
<tr>
<td>$t_{pHL}$</td>
<td>5.3 ns</td>
<td>4 ns</td>
<td>8 ns</td>
<td>15 ns</td>
<td>19 ns</td>
</tr>
</tbody>
</table>

*Temperature range (74F00): 0°C to 70°C.

**$V_{CC} = 4.5$ V, temperature range (74HC00): $-55^\circ C$ to $25^\circ C$.

***$V_{CC} = 5$ V, temperature range (74HCT00): $-55^\circ C$ to $25^\circ C$.

As indicated by the notes for Table 11.3, propagation delay (and other parameters) vary with certain operating conditions, such as ambient temperature and power supply voltage. Always make sure that the operating conditions are correctly specified when looking up a data sheet parameter.

All gates in Example 11.3 have the same logic function (2-input NAND), but different propagation delay times. We might ask, “Why not always use the advanced Schottky TTL gate (74AS00), since it is the fastest?” The main reason is that it has the highest power dissipation of the gates shown. We wouldn’t know this without looking up other specs on the data sheet. (We will learn how to do this later in the chapter.) Thus, it is important to make design decisions based on complete information, not just one parameter.
Propagation Delay in Logic Circuits

A circuit consisting of two or more gates or flip-flops has a propagation delay that is the sum of delays in the input-to-output path. Delays in gates that do not affect the circuit output are disregarded. Figure 11.5 shows how propagation delay works in a simple logic circuit consisting of a 74HC08 AND gate and a 74HC32 OR gate. Changes at inputs $A$ and $B$ must propagate through both gates to affect the output. The total delay in such a case is the sum of $t_{p1}$ and $t_{p2}$. A change at input $C$ must pass only through gate 2. The circuit delay resulting from this change is only $t_{p2}$.

![Figure 11.5](image)

The timing diagram in Figure 11.5 shows the changes at inputs $A$, $B$, and $C$ and the resulting transitions at all gate outputs.

Assume $V_{CC} = 4.5$ V and temperature range is $-55°C$ to $25°C$.

1. When $A$ goes LOW, $AB$, the output of gate 1, also goes LOW after a maximum delay of $t_{pHL} = 15$ ns. This makes $Y$ go LOW after a further delay of up to $t_{pHL} = 15$ ns. Total delay: $t_p = t_{pHL1} + t_{pHL2} = 15\text{ ns} + 15\text{ ns} = 30\text{ ns}$, max.

2. The HIGH-to-LOW transition at input $B$ has no effect, since there is no difference between 0 and 0. $AB$ is already LOW.

3. The LOW-to-HIGH transition at input $C$ makes $Y$ go HIGH after a maximum delay of $t_{pLH2} = 15$ ns.

**SECTION 11.2 REVIEW PROBLEM**

11.2 Assume the gates in Figure 11.5 are replaced by a 74LS08 AND gate and a 74LS32 OR gate. Repeat the calculations of for the propagation delays if the waveforms of Figure 11.5 are applied to the circuit. The data sheets for the 74LS08 and 74LS32 are found in Appendix C.
11.3 Fanout

**KEY TERMS**

**Fanout**  The number of load gates that a logic gate output is capable of driving without possible logic errors.

**Driving gate**  A gate whose output supplies current to the inputs of other gates.

**Load gate**  A gate whose input current is supplied by the output of another gate.

**Sourcing**  A terminal on a gate or flip-flop is sourcing current when the current flows out of the terminal.

**Sinking**  A terminal on a gate or flip-flop is sinking current when the current flows into the terminal.

- **$I_{OL}$**  Current measured at a device output when the output is LOW.
- **$I_{OH}$**  Current measured at a device output when the output is HIGH.
- **$I_{IL}$**  Current measured at a device input when the input is LOW.
- **$I_{IH}$**  Current measured at a device input when the input is HIGH.

We have assumed that logic gates are able to drive any number of other logic gates. Since gates are electrical devices with finite current-driving capabilities, this is obviously not the case. The number of gates ("loads") a logic gate can drive is referred to as its **fanout**.

**NOTE**

Fanout is simply an application of Kirchhoff’s current law: The algebraic sum of currents at a node must be zero. Thus, the fanout of a logic gate is limited by:

a. The maximum current its output can supply safely in a given logic state ($I_{OH}$ or $I_{OL}$), and
b. The current requirements of the load to which it is connected ($I_{IH}$ or $I_{IL}$).

Figure 11.6 shows the fanout of an AND gate when its output is in the HIGH and LOW states. The AND gate, or **driving gate**, supplies current to the inputs of the other four gates, which are called the **load gates**.

Each load gate requires a fixed amount of input current, depending on which state it is in. The sum of these input currents equals the current supplied by the driving gate. The

**FIGURE 11.6**

Driving Gates and Load Gates
fanout is determined by the amount of current the driving gate can supply without damaging its output circuit.

The input and output currents of a gate are established by its internal circuitry. These values are usually the same for two gates in the same family, since the input and output circuitry of a gate is common to all members of the family. Exceptions may occur when the output of a particular gate, such as the 74XX244 octal three-state buffer, has additional output buffering or an input of a gate such as a 74LS86 Exclusive OR is equivalent to more than one input load.

### Example 11.4

The gates in Figure 11.7a and b are 74LS00 NAND gates. Determine the output current of the driving gate in each figure.

#### FIGURE 11.7

**Example 11.4**

**Output Current due to One Load Gate**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Symbol</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low output on driving gate</td>
<td>$I_{OL}$</td>
<td>$0.4$ mA</td>
</tr>
<tr>
<td>High output on driving gate</td>
<td>$I_{OH}$</td>
<td>$-20$ μA</td>
</tr>
</tbody>
</table>

**Solution**

From the 74LS00 data sheet, $I_{IL} = -0.4$ mA and $I_{IH} = 20$ μA. (There are two values of $I_{IH}$ given in the data sheet. Choose the one for the condition $V_{IN} = 2.7$ V, which is the minimum output voltage of a driving gate in the HIGH state ($V_{OH}$). The other value is not appropriate since a gate will never have a 7 V output, as specified in the condition, if its supply voltage is 5 V.)

Since the driving gate is driving one load, its output current is the same as the input current of the load gate. Therefore, the driving gate output currents are given by $I_{OL} = 0.4$ mA (positive, since it is entering the driving gate output) and $I_{OH} = -20$ μA (negative, since it is leaving the driving gate output).

### Example 11.5

Determine the output current of the driving gate in each of Figures 11.8a and b if the gates are all 74LS00 NAND gates.

#### FIGURE 11.8

**Example 11.5**

**Output Current due to Two Load Gates**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Symbol</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low output on driving gate</td>
<td>$I_{OL}$</td>
<td>$0.4$ mA</td>
</tr>
<tr>
<td>High output on driving gate</td>
<td>$I_{OH}$</td>
<td>$-20$ μA</td>
</tr>
</tbody>
</table>
**Solution**  Since there are two identical load gates in the circuits of Figure 11.8, the driving gate output current will be twice the load gate input current.

\[
\begin{align*}
I_{OL} &= 2 \times 0.4 \text{ mA} = 0.8 \text{ mA}. \\
I_{OH} &= 2 \times (-20 \text{ \mu A}) = -40 \text{ \mu A}.
\end{align*}
\]

Figure 11.9 shows the extension of the circuits in Figures 11.7 and 11.8, where the number of load gates is the maximum that can be driven by the driving gate. This is the condition used to calculate fanout.

**FIGURE 11.9**
Output Current to Fanout Calculation

If the load gates each represent the same load, then by Kirchhoff’s current law (KCL):

\[
\begin{align*}
I_{OL} &= I_{IL1} + I_{IL2} + \cdots + I_{ILnL} = n_L I_{IL} \\
I_{OH} &= I_{IH1} + I_{IH2} + \cdots + I_{IHnH} = n_H I_{IH}
\end{align*}
\]

The fanout of the driving gate in the LOW and HIGH states can be calculated as:

\[
\begin{align*}
\frac{n_L}{I_{IL}} &= \frac{I_{OL}}{I_{IL}} \\
\frac{n_H}{I_{IH}} &= \frac{I_{OH}}{I_{IH}}
\end{align*}
\]

By convention, current entering a gate \((I_{IL}, I_{OL})\) is denoted as positive, and current leaving a gate \((I_{IH}, I_{OH})\) is denoted as negative. When current is leaving a gate, we say the gate is **sourcing** current. When current is entering a gate, we say the gate is **sinking** current.

Note that the output of a gate does not always source current, nor does an input always sink current. The current direction changes for the HIGH and LOW states at the same terminal. The reason for this will become apparent when we study the circuitry of logic gate inputs and outputs.
EXAMPLE 11.6
How many 74LS00 inputs can a 74LS00 NAND gate drive? (that is, what is the fanout of a 74LS00 NAND gate?)

Solution  We must consider the following cases:

a. When the output of the driving gate is LOW
b. When the output of the driving gate is HIGH

Output LOW:

\[
I_{OL} = 8 \text{ mA (sinking)} \\
I_{IH} = -0.4 \text{ mA (sourcing)} \\
n_L = \frac{8 \text{ mA}}{0.4 \text{ mA}} = 20
\]

Output HIGH:

\[
I_{OH} = -0.4 \text{ mA (sourcing)} \\
I_{IH} = 20 \mu\text{A (sinking)} \\
n_H = \frac{0.4 \text{ mA}}{20 \mu\text{A}} = 20
\]

Since \( n_L = n_H \), fanout is 20.

We disregard the negative sign in our calculations, since the input current of the load gate and output current of the driving gate are actually in the same direction. For example, even though \( I_{OH} \) is leaving the driving gate (negative), \( I_{IH} \) is entering the load gates (positive). These currents flow in the same direction. If we include the minus sign in our calculation, we get a negative value of fanout, which is meaningless.

The fanout in both HIGH and LOW states is the same in this case, but that is not always so. If the values of HIGH- and LOW-state fanout are different, the smallest value must be used. For example, if a gate can drive four loads in the HIGH state or eight in the LOW state, the fanout of the driving gate is four loads. If we attempt to drive eight loads, we can’t guarantee enough driving current to supply all loads in both states.

If a gate from one logic family is used to drive gates from another logic family, we must use the output parameters \( (I_{OL}, I_{OH}) \) for the driving gate and the input parameters \( (I_{IL}, I_{IH}) \) for the load gates.

EXAMPLE 11.7
Calculate the maximum number of Schottky TTL loads (74SXX series) that a 74LS86 XOR gate can drive.

Solution

Driving gate: 74LS86  \( I_{OH} = -0.4 \text{ mA,} \)  \( I_{OL} = 8 \text{ mA} \)

Load gates: 74SXX  \( I_{IH} = 50 \mu\text{A,} \)  \( I_{IL} = -2 \text{ mA} \)

Output LOW:

\[
I_{OL} = 8 \text{ mA (sinking)} \\
I_{IH} = -2 \text{ mA (sourcing)} \\
n_L = \frac{8 \text{ mA}}{2 \text{ mA}} = 4
\]

Output HIGH:

\[
I_{OH} = -0.4 \text{ mA (sourcing)} \\
I_{IH} = 50 \mu\text{A (sinking)} \\
n_H = \frac{0.4 \text{ mA}}{50 \mu\text{A}} = 8
\]

Since \( n_L < n_H \), fanout = \( n_L = 4 \).
What happens if we load a gate output beyond its rated fanout? Adding more load gates will do this by increasing the value of $I_{OL}$ beyond its maximum rating. If enough load is added, the output of the driving gate might be destroyed by the heat generated by the excess current. More likely, the performance of the driving gate will be degraded.

Figure 11.10 shows the relationship between output voltage and current for a 74LS00 and a 74F00 NAND gate. Figure 11.10a shows that the output voltage (LOW state) increases with increasing sink current. Figure 11.10b indicates a decrease in HIGH state output voltage with an increase of source current.

In other words, a greater load in either state takes the output voltage further away from its nominal value. This has an effect on other performance factors, such as noise margin, which we will examine in a later section of the chapter.

The output voltage of a logic gate is defined in a datasheet for a particular value of output current.

We will examine the fanout of CMOS devices in a later section on interfacing between CMOS and TTL.
SECTION 11.3 REVIEW PROBLEM

11.3 The input and output currents $I_{OH}$, $I_{OL}$, $I_{IH}$, and $I_{IL}$ of a TTL device may be classified as source currents or sink currents. List each input or output current as a source or sink current.

14.4 Power Dissipation

**KEY TERMS**

- **Power dissipation** The electrical energy used by a logic circuit in a specified period of time. Abbreviation: $P_D$
- $V_{CC}$ TTL or high-speed CMOS supply voltage.
- $I_{CC}$ Total TTL or high-speed CMOS supply current.
- $I_{CCH}$ TTL supply current with all outputs HIGH.
- $I_{CCL}$ TTL supply current with all outputs LOW.
- $I_T$ When referring to CMOS supply current, the sum of static and dynamic supply currents.
- $C_{PD}$ Internal capacitance of a high-speed CMOS device used to calculate its power dissipation.

Electronic logic gates require a certain amount of electrical energy to operate. The measure of the energy used over time is called power dissipation. Each of the different families of logic has a characteristic range of values for the power it consumes.

For TTL and CMOS, the power dissipation is calculated as follows:

- **TTL:** $P_D = V_{CC} I_{CC}$
- **High-Speed CMOS:** $P_D = V_{CC} I_T$ ($I_T = \text{quiescent + dynamic supply current}$)

Figure 11.11 shows the supply voltage and current in a 74XX00 NAND gate.

**FIGURE 11.11**

Power Supply Voltage and Current in a 74XX00 NAND gate.

The main difference between the two families is the calculation of supply current. The supply current in a TTL device is different when its outputs are HIGH than when they are LOW. Thus, supply current, $I_{CC}$, and therefore power dissipation, depends on the states of the device outputs. If the outputs are switching, $I_{CC}$ is proportional to output duty cycle.

In a CMOS device, very little power is consumed when the device outputs are static. Much more current is drawn from the supply when the outputs switch from one state to another. Thus, the power dissipation of a device depends on the switching frequency of its outputs.
Power Dissipation in TTL Devices

Two values are given for supply current in a TTL data sheet. \( I_{CCL} \) is the current drawn from the power supply when all gate outputs are LOW. \( I_{CCH} \) is the current drawn from the supply when all outputs are HIGH. If the gate outputs are not all at the same level, the supply current is the sum of currents given by:

\[
I_{CC} = \frac{n_H}{n} I_{CCH} + \frac{n_L}{n} I_{CCL}
\]

where

- \( n \) is the total number of gates in the package
- \( n_H \) is the number of gates whose output is HIGH
- \( n_L \) is the number of gates whose output is LOW

The power dissipation of a TTL chip also depends on the duty cycle of the gate outputs. That is, it depends on the fraction of time that the chip’s outputs are HIGH.

If we assume that, on average, the outputs of a chip are switching with a duty cycle of 50%, the supply current can be calculated as follows:

\[
I_{CC} = (I_{CCH} + I_{CCL})/2
\]

If the output duty cycle is other than 50%, the supply current is given by:

\[
I_{CC} = DC \cdot I_{CCH} + (1 - DC) \cdot I_{CCL}
\]

where \( DC = \) duty cycle.

### Example 11.8

Figure 11.12 shows a circuit constructed from the gates in a 74XX00 quadruple 2-input NAND gate package. Use the data sheet shown in Figure 11.3 to determine the maximum power dissipation of the circuit if the input is \( DCBA = 1001 \) and the gates are 74LS00 NANDs. Refer to the data sheets in Appendix C and repeat the calculation for 74ALS00 and 74AS00 gates.

**Solution**

Gate 1: \( \overline{AB} = 1 \)
Gate 2: \( \overline{CD} = 1 \)
Gate 3: \( AB + CD = 0 \)
Gate 4: \( AB + CD = 1 \)

Since three outputs are HIGH and one is LOW, the supply current is given by:

\[
I_{CC} = \frac{n_H}{n} I_{CCH} + \frac{n_L}{n} I_{CCL}
\]

\[
= \frac{3}{4} I_{CCH} + \frac{1}{4} I_{CCL}
\]
Maximum supply current for each device is:

- **74LS00**: \( I_{CC} = 0.75(1.6 \text{ mA}) + 0.25(4.4 \text{ mA}) = 2.3 \text{ mA} \)
- **74ALS00**: \( I_{CC} = 0.75(0.85 \text{ mA}) + 0.25(3 \text{ mA}) = 1.3875 \text{ mA} \)
- **74AS00**: \( I_{CC} = 0.75(3.2 \text{ mA}) + 0.25(17.4 \text{ mA}) = 6.75 \text{ mA} \)

Maximum power dissipation for each device is:

- **74LS00**: \( P_D = V_{CC} I_{CC} = (5\text{ V})(2.3 \text{ mA}) = 11.5 \text{ mW} \)
- **74ALS00**: \( P_D = V_{CC} I_{CC} = (5\text{ V})(1.3875 \text{ mA}) = 6.94 \text{ mW} \)
- **74AS00**: \( P_D = V_{CC} I_{CC} = (5\text{ V})(6.75 \text{ mA}) = 33.75 \text{ mW} \)

\( (1 \text{ mW} = 1 \text{ milliwatt} = 10^{-3} \text{ W}) \)

---

**EXAMPLE 11.9**

Find the maximum power dissipation of the circuit in Figure 11.12 if the gates are 74LS00 and the gate outputs are switching with an average duty cycle of 30%.

**Solution**

\[
I_{CC} = 0.3 I_{CCH} + 0.7 I_{CCL} \\
= 0.3(1.6 \text{ mA}) + 0.7(4.4 \text{ mA}) \\
= 3.56 \text{ mA} \\
P_D = V_{CC} I_{CC} = (5\text{ V})(3.56 \text{ mA}) = 17.8 \text{ mW}
\]

---

**Power Dissipation in High-Speed CMOS Devices**

CMOS gates draw the most power when their outputs are switching from one logic state to the other. When the outputs are static (not switching), the large internal impedances of the gate limit the supply current. A change of state requires the charging and discharging of internal gate capacitances, resulting in a greater demand on the power supply current. Thus, the faster a CMOS gate switches, the more current, and hence more power, it requires.

CMOS supply current has two components: a quiescent current that flows when the gate is in a steady state and a dynamic component that depends on frequency. For relatively high frequencies (about 1 MHz and up), the quiescent component is small compared to the dynamic component and can be neglected.

The quiescent current is usually specified for an entire chip package, regardless of the number of gates. It is given by \( I_{CC} V_{CC} \). For a 74HC00A NAND gate, \( I_{CC} = 1 \mu\text{A} \) at room temperature for a supply voltage of \( V_{CC} = 6.0 \text{ V} \). The dynamic component calculation accounts for internal and load capacitance and is given, per gate, by:

\[
(C_L + C_{PD}) V_{CC}^2 f
\]

where

- \( C_L \) is the gate load capacitance
- \( C_{PD} \) is the gate internal capacitance
- \( V_{CC} \) is the supply voltage
- \( f \) is the switching frequency of the gate output

---

**EXAMPLE 11.10**

The circuit in Figure 11.12 is constructed from 74HC00A high-speed CMOS NAND gates. Calculate the power dissipation of the circuit:

a. When the gate inputs are steady at the state \( DCBA = 1010 \)
b. When the outputs are switching at an average frequency of 10 kHz
c. When the outputs are switching at an average frequency of 1 MHz

Supply voltage is 5 V. Temperature range is 25°C to −55°C.
Solution Refer to the 74HC00A data sheet in Appendix C.

a. \( P_D = V_{CC} I_{CC} = (5 \text{ V})(1 \text{ } \mu\text{A}) = 5 \text{ } \mu\text{W} \). This is the quiescent power dissipation of the circuit.

b. The 74HC00A data sheet indicates that each gate has a maximum input capacitance, \( C_{\text{in}} \) of 10 pF. Assume that this value represents the load capacitance of gates 1, 2, and 3 of the circuit in Figure 11.12. Further assume that gate 4 has a load capacitance of 0. The total power dissipation of the circuit is given by:

\[
\begin{align*}
P_D &= 3(22 \text{ pF} + 10 \text{ pF})(5 \text{ V})^2 (0.01 \text{ MHz}) \\
& \quad + (22 \text{ pF})(5 \text{ V})^2 (0.01 \text{ MHz}) + 5 \text{ } \mu\text{W} \\
& = 3(8 \text{ } \mu\text{W}) + 5.5 \text{ } \mu\text{W} + 5 \text{ } \mu\text{W} \\
& = 34.5 \text{ } \mu\text{W}
\end{align*}
\]

For \( f = 1 \text{ MHz} \), total power dissipation is given by:

\[
\begin{align*}
P_D &= 3(22 \text{ pF} + 10 \text{ pF})(5 \text{ V})^2 (1 \text{ MHz}) \\
& \quad + (22 \text{ pF})(5 \text{ V})^2 (1 \text{ MHz}) + 5 \text{ } \mu\text{W} \\
& = 3(800 \text{ } \mu\text{W}) + 550 \text{ } \mu\text{W} + 5 \text{ } \mu\text{W} \\
& = 2955 \text{ } \mu\text{W} = 2.95 \text{ mW}
\end{align*}
\]

c. For \( f = 1 \text{ MHz} \), total power dissipation is given by:

\[
\begin{align*}
P_D &= (32 \text{ pF} + 22 \text{ pF})(5 \text{ V})^2 f \\
& = (96 \text{ pF} + 22 \text{ pF})(25 \text{ V}^2) f \\
& = (118 \text{ pF})(25 \text{ V}^2) f
\end{align*}
\]

For \( P_D = 15 \text{ mW} \):

\[
f = \frac{15\text{mW}}{(118\text{pF})(25\text{V}^2)} = 5.08\text{MHz}
\]

The power dissipation of the 74HC00 circuit exceeds that of the 74LS00 circuit at 5.08 MHz.

The circuit in Figure 11.12 is constructed using a 74LS00 quad 2-in NAND gate and again with a 74HC00 quad 2-in NAND. Both circuits have identical waveforms applied to their inputs that make all gate outputs switch with a duty cycle of 50%. Calculate the frequency at which the power dissipation of the 74HC00 circuit exceeds that of the 74LS00 circuit.

Assume \( V_{CC} = 5 \text{ V} \) and temperature = 25°C for both circuits.

Solution The power dissipation of the LSTTL circuit is:

\[
P_D = V_{CC} I_{CC} = (V_{CC})(I_{CCH} + I_{CCL})/2 = (5 \text{ V}) (1.6 \text{ mA} + 4.4 \text{ mA})/2 \\
= (5 \text{ V}) (3.0 \text{ mA}) = 15 \text{ mW}
\]

Neglect the quiescent current of the high-speed CMOS circuit.

Per gate: \( P_D = (C_L + C_{PD})V_{CC}^2 f \)

\( C_{PD} = 22 \text{ pF} \) per gate

\( C_L = 10 \text{ pF} \) for 3 gates and 0 pF for 1 gate

Total:

\[
P_D = (3)(10 \text{ pF} + 22 \text{ pF}) (25 \text{ V}^2) f \\
= (3)(32 \text{ pF} + 22 \text{ pF})(25 \text{ V}^2) f \\
= (96 \text{ pF} + 22 \text{ pF})(25 \text{ V}^2) f \\
= (118 \text{ pF})(25 \text{ V}^2) f
\]

For \( P_D = 15 \text{ mW} \):

\[
f = \frac{15\text{mW}}{(118\text{pF})(25\text{V}^2)} = 5.08\text{MHz}
\]

The power dissipation of the 74HC00 circuit exceeds that of the 74LS00 circuit at 5.08 MHz.

NOTE

The power saving in a high-speed CMOS circuit generally results from the fact that most device outputs are not switching at any given time. The power dissipation of a TTL circuit is independent of frequency and therefore draws some power at all times. This is not the case for CMOS, which draws the majority of its power when switching.
SECTION 11.4 REVIEW PROBLEM

11.4 Why does CMOS power dissipation increase with frequency?

11.5 Noise Margin

**KEY TERMS**

<table>
<thead>
<tr>
<th>Noise</th>
<th>Unwanted electrical signal, often resulting from electromagnetic radiation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise margin</td>
<td>A measure of the ability of a logic circuit to tolerate noise.</td>
</tr>
<tr>
<td>( V_{IH} )</td>
<td>Voltage level required to make the input of a logic circuit HIGH.</td>
</tr>
<tr>
<td>( V_{IL} )</td>
<td>Voltage level required to make the input of a logic circuit LOW.</td>
</tr>
<tr>
<td>( V_{OH} )</td>
<td>Voltage measured at a device output when the output is HIGH.</td>
</tr>
<tr>
<td>( V_{OL} )</td>
<td>Voltage measured at a device output when the output is LOW.</td>
</tr>
</tbody>
</table>

Electrical circuits are susceptible to noise, or unwanted electrical signals. Such signals are often induced by electromagnetic fields of motors, fluorescent lighting, high-frequency electronic circuits, and cosmic rays. They can cause erroneous operation of a digital circuit. Since it is impossible to eliminate all noise from a circuit, it is desirable to build a certain amount of tolerance, or noise margin, into digital devices used in the circuit.

In all circuits studied so far, we have assumed that logic HIGH is +5 volts and logic LOW is 0 volts in devices with a 5-volt supply. In practice, there is a certain amount of tolerance on both the logic HIGH and LOW voltages; for TTL devices, a HIGH at a device input is anything above about +2 volts, and a LOW is any voltage below about +0.8 volts. Due to internal voltage drops, the HIGH output of a TTL gate is typically about +3.5 volts.

Figure 11.13 shows one inverter driving another. In Figure 11.13a, the output of the first inverter and the input of the second have the same logic threshold. That is, the input of the second gate recognizes any voltage above 2.7 volts as HIGH (\( V_{IH} = 2.7 \) V) and any voltage below 0.5 volts as LOW (\( V_{IL} = 0.5 \) V). The output of the first inverter produces at least 2.7 volts when HIGH (\( V_{OH} = 2.7 \) V) and no more than 0.5 volts as LOW (\( V_{OL} = 0.5 \) V).

If there is noise on the line connecting the two gates, it will likely cause the voltage of the second gate input to penetrate into the forbidden region between logic HIGH and LOW levels. This is shown on the graph of the waveform in Figure 11.13a. When the voltage enters the forbidden region, the gate will not operate reliably. Its output may switch states when it is not supposed to.

Figure 11.13b shows the same circuit with different logic thresholds at input and output. The output of the first inverter is guaranteed to be at least 2.7 volts when HIGH (\( V_{OH} = 2.7 \) V) and no more than 0.5 volts when LOW (\( V_{OL} = 0.5 \) V). The second gate recognizes any input voltage greater than 2 volts as a HIGH (\( V_{IH} = 2 \) V) and any input voltage less than 0.8 volts (\( V_{IL} = 0.8 \) V) a LOW.

The difference between logic thresholds allows for a small noise voltage, equal to or less than the difference, to be superimposed on the desired signal. It will not cause the input voltage of the second inverter to penetrate the forbidden region. This ensures reliable operation even in the presence of some noise.

For the 74LS04 inverter, the HIGH-state and LOW-state noise margins, \( V_{NH} \) and \( V_{NL} \), are:

\[
V_{NH} = V_{OH} - V_{IH} = 2.7 \text{ V} - 2.0 \text{ V} = 0.7 \text{ V}
\]

\[
V_{NL} = V_{IL} - V_{OL} = 0.8 \text{ V} - 0.5 \text{ V} = 0.3 \text{ V}
\]

A device with these values of \( V_{IH} \) and \( V_{IL} \) is deemed to be TTL compatible.
EXAMPLE 11.12

Use the 74HC00A data sheet in Appendix C to calculate the noise margins for this gate. Assume $V_{CC} = 4.5$ V, ambient temperature ($T_A$) is 25°C, and the driving gate is fully loaded ($I_{OUT} = \pm 4$ mA).

**Solution**

- $V_{NH} = V_{OH} - V_{IH} = 3.98 \text{ V} - 3.15 \text{ V} = 0.83 \text{ V}$
- $V_{NL} = V_{IL} - V_{OL} = 1.35 \text{ V} - 0.26 \text{ V} = 1.09 \text{ V}$
11.5 Calculate the noise margins of a 74HCT00A NAND gate from the data sheet in Appendix C. \( V_{CC} = 4.5 \text{ V}, T_A = 25^\circ \text{C}, I_{OUT} = \pm 4 \text{ mA} \)

### 11.6 Interfacing TTL and CMOS Gates

**KEY TERM**

**TTL Compatible** Able to be driven directly by a TTL output. Usually implies voltage compatibility with TTL.

Interfacing different logic families is just an extension of the fanout and noise margin problems; you have to know what the load gates of a circuit require and what the driving gates can supply. In practice, this means you must know the specified values of input and output voltages and currents for the gates in question. Table 11.4, which is derived from the manufacturers’ data sheets included in Appendix C, gives an overview of input and output parameters for a variety of TTL and CMOS families. Ambient temperature is assumed to be 25°C.

<table>
<thead>
<tr>
<th></th>
<th>74LS</th>
<th>74F</th>
<th>74AS</th>
<th>74ALS</th>
<th>74HC</th>
<th>74HCT</th>
<th>74VHC</th>
<th>74VHCT</th>
<th>74LVX</th>
<th>74LCX</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{CC} ) (V)</td>
<td>5.0</td>
<td>5.0</td>
<td>5.5</td>
<td>5.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>( V_{OH} ) (V)</td>
<td>2.7</td>
<td>2.7</td>
<td>3.0</td>
<td>3.0</td>
<td>3.98</td>
<td>3.98</td>
<td>3.94</td>
<td>3.94</td>
<td>2.58</td>
<td>2.2</td>
</tr>
<tr>
<td>( V_{OL} ) (V)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.26</td>
<td>0.26</td>
<td>0.36</td>
<td>0.36</td>
<td>0.36</td>
<td>0.55</td>
</tr>
<tr>
<td>( I_{OH} ) (mA)</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>3.15</td>
<td>2.0</td>
<td>3.15</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>( I_{IL} ) (mA)</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>1.35</td>
<td>0.8</td>
<td>1.35</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>( I_{OL} ) (mA)</td>
<td>-0.4</td>
<td>-1.0</td>
<td>-2.0</td>
<td>-0.4</td>
<td>-4.0</td>
<td>-4.0</td>
<td>-8.0</td>
<td>-8.0</td>
<td>-4.0</td>
<td>-24.0</td>
</tr>
<tr>
<td>( I_{OH} ) (mA)</td>
<td>8.0</td>
<td>20.0</td>
<td>20.0</td>
<td>8.0</td>
<td>4.0</td>
<td>4.0</td>
<td>8.0</td>
<td>8.0</td>
<td>4.0</td>
<td>24.0</td>
</tr>
<tr>
<td>( I_{IL} ) (mA)</td>
<td>0.02</td>
<td>0.1</td>
<td>0.02</td>
<td>0.02</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>( I_{OH} ) (mA)</td>
<td>-0.4</td>
<td>-0.6</td>
<td>-0.5</td>
<td>-0.1</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 11.4 is useful for comparison of logic families, but it is not a substitute for reading data sheets, as it gives parameters only under a restricted set of conditions. We can, however, make some observations based on the data in Table 11.4.

1. Input currents in a CMOS gate are very low, due to its high input impedance. As a result fanout is generally not a problem with CMOS loads. Interface problems to CMOS loads have to do with input voltage, not current.

2. CMOS devices, such as 74HCT, that have the same values of \( V_{IH} \) and \( V_{IL} \) as the TTL families in Table 11.4, are considered to be **TTL compatible**, since they can be driven directly by TTL drivers.

3. LSTTL is usually regarded as the benchmark for measuring TTL loading of a CMOS circuit. For example, a data sheet will claim that a device can drive 10 LSTTL loads. This claim depends on the values of \( I_{OH} \) and \( I_{OL} \) for the driving gate, which are not listed directly in CMOS data sheets, except as absolute maximum ratings. The values in Table 11.4 are the values of current for which the output voltages, \( V_{OH} \) and \( V_{OL} \), are defined. (Recall from the section on fanout in this chapter that increasing output current causes output voltages to migrate away from their nominal values, thus reducing device noise margins.)

Let us examine four interfacing problems: high-speed CMOS driving 74LS, 74LS driving 74HC, 74LS driving 74HCT, and 74LS driving low-voltage CMOS.
High-Speed CMOS driving 74LS

To design an interface between any two logic families, we must examine the output voltages and currents of the driving gate and the input voltages and currents of the load gates.

Assume a 74HC00 NAND gate drives one or more 74LS00 NAND gates. From the 74HC00 data sheet, we determine that $V_{OH} = 3.98\, \text{V}$ and $V_{OL} = 0.26\, \text{V}$ for $V_{CC} = 4.5\, \text{V}$. The 74LS00 requires at least 2.0 V at its input in the HIGH state and no more than 0.8 V in the LOW state. The 74HC00 therefore satisfies the input voltage requirement of the 74LS00.

For the defined output voltages, the 74HC00 gate can source or sink 4 mA. The fanout for the circuit is therefore calculated as follows:

$$n_H = \frac{I_{OH}}{I_{IH}} = \frac{4\, \text{mA}}{20\, \mu\text{A}} = 200$$

$$n_L = \frac{I_{OL}}{I_{IL}} = \frac{4\, \text{mA}}{0.4\, \text{A}} = 10$$

$$n = 10$$

Therefore a 74HC00 NAND can drive a 74LS00 directly, with a fanout of 10.

74LS Driving 74HC

As mentioned earlier, CMOS has a very small input current and therefore does not present a fanout problem to a 74LS driving gate. However, we must also examine the interface for voltage compatibility.

From data sheets, we see that a 74LS00 gate is guaranteed to provide at least 2.7 V in the HIGH state and no more than 0.5 V in the LOW state. A 74HC00 gate will recognize anything less than 1.35 V as a logic LOW and anything more than 3.15 V as a logic HIGH. The 74LS00 meets the LOW-state criterion, but it cannot guarantee sufficient output voltage in the HIGH state.

In order to properly drive a 74HC input with a 74LS output, we must provide a pull-up resistor to ensure sufficient HIGH-state voltage at the 74HC input. The circuit is illustrated in Figure 11.14. The pull-up resistor should be between 1 kΩ and 10 kΩ.

![FIGURE 11.14](image)

LSTTL driving 74HC CMOS

74LS Driving 74HCT

74HCT inputs are designed to be compatible with TTL outputs. As with 74HC devices, input currents are sufficiently low that fanout is not a problem with the 74LS-to-74HCT interface. 74HCT input voltages are the same as those for TTL ($V_{IH} = 2.0\, \text{V}$ and $V_{IL} = 0.8\, \text{V}$). Therefore, 74HCT inputs can be driven directly by LSTTL outputs.

74LS Driving Low-voltage CMOS

CMOS families with supply voltages less than 5 V are rapidly becoming popular in new applications. Two of the reasons for their increasing prominence are reduced power dissipation (inversely proportional to the square of the supply voltage) and smaller feature size
(i.e., size of the internal transistors) that allows more efficient packaging and faster operation. Low-voltage logic is particularly popular for battery-powered applications such as laptop computing or cell phones. Low voltage families typically operate at $V_{CC} = 3.3\, \text{V}$ or $2.5\, \text{V}$. Newer devices are available for $V_{CC} = 1.8\, \text{V}$ or $1.65\, \text{V}$.

Low-voltage CMOS families such as 74LVX or 74LCX can interface directly with TTL outputs if they are operated with a $3.0\, \text{V}$ to $3.3\, \text{V}$ power supply. These families are not really suitable for driving 5-volt TTL, as their noise margins are too small when they use a $3.0\, \text{V}$ supply voltage.

If we wish to use a 74LS device to drive a 74HC device operating at a power supply voltage of less than $4.5\, \text{V}$, we can use a 74HC4049 or 74HC4050 buffer to translate the TTL logic level down to an appropriate value. The 74HC4049 is a package of six inverting buffers. The 74HC4050 has six noninverting buffers. These buffers can tolerate up to $15\, \text{V}$ on their inputs. Their output voltages are determined by the value of their supply voltage.

Figure 11.15 shows an LSTTL-to-74HC interface circuit with a 74HC4050 buffer. Note that the interface buffer has the same power supply voltage as the load gate. Both sides of the interface are referenced to the same ground.

![FIGURE 11.15](image-url)

74LS-to-74HC Interface Using a 74HC4050 Buffer

**SECTION 11.6 REVIEW PROBLEM**

11.6 A 74LS00 driving gate is to be interfaced to a 74HC00 load using a 74HC4050 non-inverting buffer. The 74HC00 has a power supply voltage of $2.5\, \text{V}$. What supply voltage should the 74HC4050 buffer have? Why?

11.7 Internal Circuitry of TTL Gates

**KEY TERMS**

- **Cutoff mode**  The operating mode of a bipolar transistor when there is no collector current flowing and the path from collector to emitter is effectively an open circuit. In a digital application, a transistor in cutoff mode is considered OFF.

- **Saturation mode**  The operating mode of a bipolar transistor when an increase in base current will not cause a further increase in the collector current and the path from collector to emitter is very nearly (but not quite) a short circuit. This is the ON state of a transistor in a digital circuit.

TTL has been around for a long time. The first transistor-transistor logic ICs were developed by Texas Instruments around 1965. Since then, there have been many improvements in the speed and power consumption of these devices, but the basic logic principles remain largely unchanged. Even though they are seldom used in modern designs, it makes sense to examine the internal circuitry of standard TTL gates such as the 7400 NAND, 7402 NOR, and 7404 inverter because the internal logic concepts are similar to the more advanced types of TTL.

The most important parts of the circuit, as far as a designer or technician is concerned, are the input and output circuits, because they are the only parts of the chip to which we have access. It is to these points that we interface other circuits and where we make diag-
nagnostic measurements. A basic understanding of the inputs and outputs of logic gate circuitry is helpful when we design or troubleshoot a digital circuit.

Bipolar Transistors as Logic Devices

The basic element of a TTL device is the bipolar junction transistor, illustrated in Figure 11.16. This is not the place to give a detailed analysis of the operation of a bipolar transistor, but a simplified summary of operating modes will be useful.

The bipolar transistor is a current amplifier having three terminals called the collector, emitter, and base. Current flowing into the base controls the amount of current flowing from the collector to the emitter. If base current is below a certain threshold, the transistor is in **cutoff mode** and no current flows in the collector. In this state, the base-emitter voltage is less than 0.6 V and the collector-emitter path acts like an open circuit. We can treat the collector-emitter path as an open switch, as shown in the lefthand diagram in Figure 11.17.

![FIGURE 11.16](image)

*FIGURE 11.16*  
Currents and Voltages in an NPN Bipolar Transistor

If the base current increases, the transistor enters the “active region,” where the collector current is proportional to the base current by a current gain factor, $\beta$. This is the linear, or amplification, region of operation, used by analog amplifiers.

If the base current increases still further, collector current reaches a maximum value and will no longer increase with base current. This is called the **saturation mode** of the transistor. The saturated value of collector current, $I_{CS}$, is determined by (1) the resistance in the collector-emitter current path, (2) the voltage drop across the collector and emitter, $V_{CE}$, and (3) the collector supply voltage, $V_{CC}$. Base-emitter voltage is about 0.7 V and will not increase significantly with increasing base current. The voltage between collector and emitter is in the range from 0.2 V to 0.5 V. In this mode, we can treat the transistor as a closed switch, as shown in the righthand diagram of Figure 11.17.

![FIGURE 11.17](image)

*FIGURE 11.17*  
NPN Bipolar Transistor as a Switch

Table 11.5 summarizes the voltages and currents in the cutoff, active, and saturation regions.

<table>
<thead>
<tr>
<th>Table 11.5</th>
<th>Bipolar Transistor Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Cutoff</strong></td>
</tr>
<tr>
<td>$I_C$</td>
<td>0</td>
</tr>
<tr>
<td>$V_{CE}$</td>
<td>Open cct.</td>
</tr>
<tr>
<td>$V_{BE}$</td>
<td>$&lt; 0.6$ V</td>
</tr>
</tbody>
</table>
EXAMPLE 11.13

Figure 11.18 shows an NPN bipolar transistor connected in a common-emitter configuration. With the right choice of input voltages, this circuit acts as a digital inverter.

FIGURE 11.18
Example 11.13
Transistor as Inverter

Analyze the circuit to show that it acts as an inverter if a logic HIGH is defined as \( \geq 3 \text{ V} \) and a logic LOW is defined as \( \leq 0.5 \text{ V} \). Assume that \( \beta = 100 \), and assume that \( V_{BE} = 0.7 \text{ V} \) and \( V_{CE} = 0.2 \text{ V} \) in saturation.

Solution

We will analyze the circuit with two input voltages: 3 V (logic HIGH) and 0.5 V (logic LOW). These two conditions are shown in Figure 11.19.

FIGURE 11.19
Example 11.13
Voltage and Current Analysis of Inverter

High input. We must prove that \( V_I = 3 \text{ V} \) is sufficient to saturate the transistor. Let us assume that this is true and find out if calculations confirm our assumption.

Figure 11.19a shows the circuit with \( V_I = 3 \text{ V} \). By Kirchhoff’s voltage law (KVL):

\[
V_I = I_B R_B + V_{BE}, \text{ or } I_B = (V_I - V_{BE})/R_B
\]

If we assume that \( I_B \) is sufficient to saturate the transistor, then:

\[
I_B = (3 \text{ V} - 0.7 \text{ V})/22 \text{ k}\Omega \\
= 105 \text{ µA} \\
\beta I_B = (100)(105 \text{ µA}) = 10.5 \text{ mA}
\]

Collector current won’t increase beyond its saturated value, even if base current increases. Therefore, if the transistor is saturated, \( \beta I_B \) will be larger than the current actually flowing in the collector-emitter path.
In saturation, the collector current can be calculated by KVL:

\[ V_{CC} = I_C R_C + V_{CE}, \text{ or} \]
\[ I_C = (V_{CC} - V_{CE})/R_C \]
\[ I_C = (5 \text{ V} - 0.2 \text{ V})/470 \text{ } \Omega \]
\[ = 10.2 \text{ mA} \]

Since \( \beta I_B > I_C \), the transistor is saturated. Thus, an input voltage of 3 V will produce sufficient base current to saturate the transistor. The output is given by \( V_O = V_{CE} = 0.2 \text{ V} \), which is within the defined range of a logic LOW.

LOW input. Figure 11.19b shows the circuit with \( V_i = 0.5 \text{ V} \). By KVL:

\[ V_i = I_B R_B + V_{BE} \]
\[ V_{BE} = 0.5 \text{ V} - I_B R_B \]

Since \( V_{BE} \) must be \(<0.6 \text{ V} \), the transistor is in cutoff mode. Thus, in the collector circuit:

\[ V_{CC} = I_C R_C + V_{CE} \]
\[ 5 \text{ V} = (0)(470 \text{ } \Omega) + V_{CE} \]
\[ V_O = V_{CE} = 5 \text{ V} \text{ (logic HIGH)} \]

Table 11.6 summarizes the operation of the circuit as an inverter.

<table>
<thead>
<tr>
<th>Input</th>
<th>Logic Level</th>
<th>Output</th>
<th>Logic Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0.5 \text{ V} )</td>
<td>LOW</td>
<td>( 5 \text{ V} )</td>
<td>HIGH</td>
</tr>
<tr>
<td>( 3 \text{ V} )</td>
<td>HIGH</td>
<td>( 0.2 \text{ V} )</td>
<td>LOW</td>
</tr>
</tbody>
</table>

**TTL Open-Collector Inverter and NAND Gate**

**KEY TERM**

**Open-collector output** A TTL output where the collector of the LOW-state output transistor is brought out directly to the output pin. There is no built-in HIGH-state output circuitry, which allows two or more open-collector outputs to be connected without possible damage.

**NOTE**

The TTL gates (7405, 7401, 7404, 7400, and 7402) used in the following sections to illustrate TTL circuit principles are no longer in general use. They are from the original (“standard”) TTL family, which has been superceded by faster and more efficient devices. However, the standard TTL devices are easier to understand than devices from the newer TTL subfamilies, since their circuit structure is simpler. The operating principles are similar in both the standard and newer families, so we will use the standard devices to illustrate the general principles of TTL operation.

Figure 11.20 shows the circuit of the simplest TTL gate: a 7405 inverter with open-collector outputs. This circuit performs the same function as the single-transistor inverter we examined in Example 11.13. These circuits differ most obviously in their input circuitry. The inverter circuit in Example 11.13 has a resistor as its input; the 7405 inverter has a transistor,
$Q_1$, as its input. The input transistor allows faster switching of input states. This configuration is common to all standard TTL gates and will be examined in detail later in this section.

The logic function of the 7405 is performed by transistors $Q_2$ and $Q_3$. Output transistor $Q_3$ is switched ON and OFF by current flowing in the collector-emitter path of $Q_2$. When $Q_3$ is ON, $Y$ is LOW. However, when $Q_3$ is OFF, $Y$ is floating. There is a high impedance between $Y$ and ground, so the output is not LOW. But there is no connection to $V_{CC}$ to make the output HIGH. In this condition, $Y$ is neither HIGH nor LOW.

To enable the output to produce a HIGH state, we need to add an external pull-up resistor. The value of this resistor depends on the current sinking capability of $Q_3$, specified in the data sheet as $I_{OL}$. We will do such calculations in a later example.

**TTL Inputs**

Transistor $Q_1$ and diode $D_1$ make up the input circuit of the TTL inverter of Figure 11.20. The diode protects the input against small negative voltages. If the input goes more negative than about $-0.7$ V, the diode will conduct, effectively short-circuiting the input to ground plus one diode drop. This clamps the input to $-0.7$ V. $D_1$ has no logic function.

$Q_1$ can be treated as two back-to-back diodes, as shown in Figure 11.21. Figure 11.22 shows how the input responds to logic HIGH and LOW voltages.

**LOW Input.** When a TTL input is made LOW, the base-emitter junction of $Q_1$ acts as a forward-biased diode, creating a current path from $V_{CC}$ to ground via the input pin. This
current makes up the majority of current $I_{IL}$, which has a maximum value of 1.6 mA in standard TTL (0.4mA in LSTTL).

At the moment the input is made LOW, the transistor action of $Q_1$ transports charge away from the base of $Q_2$, pulling it LOW and keeping it in cutoff mode. This current dies out when the base charge of $Q_2$ has been depleted, shortly after the LOW is applied to the input pin. The diode formed by the base-collector junction of $Q_1$ does not carry sufficient current to turn on $Q_2$, since the base-emitter path is of much lower impedance.

**HIGH Input.** A HIGH at a TTL input reverse-biases the base-emitter junction of $Q_1$. Only a small leakage current, $I_{IH}$, flows. The maximum value of $I_{IH}$ is 40 $\mu$A for standard TTL (20 $\mu$A for LSTTL).

Since the low-impedance current path to the input pin has not been established, current flows to the base of $Q_2$ via the forward-biased base-collector junction of $Q_1$. This current is sufficient to saturate $Q_2$.

**Open (Floating) TTL Input.** An open-circuit TTL input acts as a logic HIGH, as illustrated by Figure 11.23. A TTL input relies on a logic LOW to establish a low-impedance current path from $V_{CC}$ to the input pin. If the input is open, this LOW is not present and current flows in the base-collector junction of the transistor, by default. This is the same current that flows under the HIGH-input condition.

This HIGH is not stable; it can be converted to logic LOW by induced noise at the input pin. To avoid this uncertainty, an unused input should always be wired to a logic HIGH or LOW state.
LOW Input. As was described above, a LOW input establishes a low-impedance path to ground, which draws current through the base-emitter junction of $Q_1$. This action also prevents base current from flowing in transistor $Q_2$, causing it to be in cutoff mode and making $I_{C2} = 0$.

Since $I_{B3}$ is derived from $I_{C2}$, $I_{B3} = 0$ and $Q_3$ is cut off, making a high-impedance path between the collector and emitter of $Q_3$. As was the case with the single-transistor inverter in Example 11.13, when $I_{C3} = 0$, then $V_O = V_{CE} = V_{CC}$. (Since no current flows through the pull-up resistor, the voltage must be the same at both ends.) Output $Y$ is HIGH.

HIGH Input. When input $A$ is HIGH, the base-emitter junction of $Q_1$ does not have sufficient voltage across it to be forward-biased. Current flows through the base-collector junction of $Q_1$, saturating $Q_2$.

Since $Q_2$ is ON, current flows to the $Q_2$ emitter and splits through the 1-kΩ resistor and the base of $Q_3$. The output transistor, $Q_3$, turns ON, establishing a low-impedance current path from output $Y$ to ground. Current is limited by the external pull-up resistor, which must be chosen to keep $I_{OL}$ at or under its rated value of 16 mA. $V_{CE3}$ is about 0.2 V to 0.4 V. Output $Y$ is LOW.

TTL Open-Collector NAND

Figure 11.25 shows one gate of a 7401 quadruple 2-input NAND gate with open-collector outputs. The circuit is the same as that of the 7405 inverter, except that the input transistor has a second emitter. Multiple-emitter transistors of this type are common in TTL circuits and can be modeled by the diode equivalent in Figure 11.25b. Figure 11.26 shows the response of the multiple-emitter input transistor to various combinations of logic levels.
If both inputs are LOW, the NAND acts exactly the same as the 7405 inverter with a LOW input. (A low-impedance path is created through a base-emitter junction.) Output $Y$ is HIGH, provided an external pull-up resistor is connected to output. A partial truth table for this condition is:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

If one input is LOW, the input acts the same as the inverter with a LOW input. The low-impedance current path through the one grounded emitter prevents sufficient base-collector current from flowing to forward-bias that junction. Output $Y$ is HIGH if a pull-up resistor is connected to the output. A partial truth table is as follows:

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
If both inputs are HIGH, the NAND circuit acts like the 7405 when its input is HIGH. (There is no base-emitter current path. A collector-emitter path is established by default.) Output Y is LOW. This condition can be represented by:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Combining all these conditions, we get the standard NAND truth table:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**NOTE**

If one or more emitters of a TTL multiple-emitter input transistor is LOW, the input is a LOW equivalent. All emitters must be HIGH to make the transistor input a HIGH equivalent.

These statements lead to the familiar NAND-gate descriptive sentences, illustrated by the gate symbols in Figure 11.27.

a. At least one input LOW makes the output HIGH.
b. Both inputs HIGH make the output LOW.

![Diagram of NAND gate](image)

**FIGURE 11.27**

DeMorgan Equivalent Forms of a NAND Gate

**SECTION 11.7A REVIEW PROBLEM**

11.7 What are the two main functions of the pull-up resistor on the output of an open-collector gate?

**Open-Collector Applications**

**KEY TERM**

**Wired-AND** A connection where open-collector outputs of logic gates are wired together. The logical effect is the ANDing of connected functions.

A more common TTL output than the open collector is the totem pole output, which we will study later in this chapter. The totem pole output has its own internal pull-up circuit for HIGH outputs.
Gates with totem pole outputs cannot be used in all digital circuits. For example, open-collector gates are required when several outputs must be tied together, a connection called **wired-AND**. Totem pole outputs would be damaged by such a connection, since there is the possibility of conflict between an output HIGH and LOW state.

Open-collector outputs can also be used for applications requiring high current drive and for interfacing to circuits having supply voltages other than TTL levels.

A special symbol defined by IEEE/ANSI Standard 91-1984, an underlined square diamond, is shown in Figure 11.28. This symbol is added to a logic gate symbol to indicate that it has an open-collector output. Other symbols, such as a star (*), a dot (●), or the initials OC are also used.

### Wired-AND

*NOTE*

A wired-AND connection combines the outputs of the connected gates in an AND function.

![Open-Collector Symbols](image)

**FIGURE 11.28**

Open-Collector Symbols Shown for a NAND Gate (e.g., 7401)

![Three Inverters](image)

**FIGURE 11.29**

Three Inverters in a Wired-AND Connection

Figure 11.29 shows three open-collector inverters connected in a wired-AND configuration. The output transistors of the inverters are shown in Figure 11.30, with different possible ON and OFF states. The only way output $Y$ can remain HIGH is if all the transistors are in their OFF states, as in Figure 11.30c. This can happen only if the outputs of the inverters are all HIGH. This is the same as saying the outputs are ANDed together at $Y$.

The Boolean expression for $Y$ is:

$$Y = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

$$= A + B + C$$

By DeMorgan’s theorem, the wired-AND connection of inverter outputs is equivalent to a NOR function. Because of this DeMorgan equivalence, the connection is sometimes called “wired-OR.”

Figure 11.31 shows three NAND gates in a wired-AND connection. Since the output functions are ANDed, the Boolean expression for $Y$ is:

$$Y = \overline{AB} \cdot \overline{CD} \cdot EF$$

$$= AB + CD + EF$$
The resulting function is called AND-OR-INVERT. Normally this requires at least two types of logic gate—AND and NOR. The wired-AND configuration can synthesize any size of AND-OR-INVERT network using only NAND gates.

The wired-AND function is sometimes shown as an AND symbol around a soldered connection, as shown in Figure 11.31b.
High-CURRENT Driver

Standard TTL outputs have higher current ratings in the LOW state than in the HIGH state. Thus, open-collector outputs are useful for driving loads that need more current than a standard TTL output can provide in the HIGH state. There are special TTL gates with higher ratings of \(I_{\text{OL}}\) to allow even larger loads to be driven. Typical loads would be LEDs, incandescent lamps, and relay coils, all of which require currents in the tens of milliamperes.

**EXAMPLE 11.14**

A 74LS07 hex buffer/driver contains six noninverting buffers whose outputs are open-collector, rated for \(I_{\text{OL\text{max}}} = 40\, \text{mA}\) and \(V_{\text{OH\text{max}}} = 30\, \text{V}\). That is, even though there is no internal circuit to provide a logic HIGH at the output, the output transistor can withstand a voltage of up to 30 V without damage.

Figure 11.32 shows a 74LS07 buffer driving an incandescent lamp rated at 24 V, with a resistance of 690 \(\Omega\). Calculate the current that flows when the lamp is illuminated. What logic level at \(A\) turns the lamp on? Could the lamp be driven by a 74LS05 inverter? Why or why not?

**Solution**  From the 74LS07 data sheet in Appendix C, we see that \(V_{\text{OL}} = 0.4\, \text{V}\) for \(I_{\text{OL}} = 16\, \text{mA}\) and \(V_{\text{OL}} = 0.7\, \text{V}\) for \(I_{\text{OL}} = 40\, \text{mA}\). Assume the latter value.

By KVL: \(24\, \text{V} - (I_{\text{OL}})(690\, \Omega) - V_{\text{OL}} = 0\)

Thus, \(I_{\text{OL}} = (24\, \text{V} - 0.7\, \text{V})/690\, \Omega = 33.8\, \text{mA}\)

Since the buffer is noninverting, and current flows when the output of the 74LS07 sinks current to ground (LOW), the lamp is on when \(A\) is LOW.

A 74LS05 open-collector inverter would not be a suitable driver for the circuit for two reasons: its output is only designed to withstand 5.5 V and it can only sink a maximum of 8 mA.

**Value of External Pull-up Resistor**

The value of the pull-up resistor required by an open-collector circuit is calculated using manufacturer’s specifications and the basic principles of circuit theory: Kirchhoff’s voltage and current laws (KVL and KCL) and Ohm’s law.

Figure 11.33 shows the circuit model for calculating the value of \(R_{\text{ext}}\). It accounts for the current requirements of the loads, the LOW-state output voltage, and current-sinking capacity of the open-collector gate.
EXAMPLE 11.15

Calculate the minimum value of the pull-up resistor for a 74LS05 inverter if the circuit drives ten 74LS00 NAND gate inputs.

Solution

From 74LS00 specs:

\[ I_{IL} = 0.4 \text{ mA} \]

For 10 gates:

\[ nI_{IL} = 10I_{IL} = 4 \text{ mA} \]

From 74LS05 specs:

\[ I_{OL} = 8 \text{ mA} \]

\[ I_{R} = I_{OL} - nI_{IL} \]

\[ = 8 \text{ mA} - 4 \text{ mA} \]

\[ = 4 \text{ mA} \]

For \( I_{OL} = 8 \text{ mA}, V_{OL} = 0.5 \text{ V} \)

\[ R_{ext} = \frac{(V_{CC} - V_{OL})}{I_{R}} \]

\[ = \frac{(5 \text{ V} - 0.5 \text{ V})}{4 \text{ mA}} \]

\[ = 4.5 \text{ V/4 mA} = 1.125 \text{ kΩ} \]

Use a 1.2-kΩ or 1.5-kΩ standard value resistor.

SECTION 11.7B REVIEW PROBLEM

11.8 Calculate the minimum value of pull-up resistor required for a 74LS05 inverter if it drives one input of a 74LS00 NAND gate. What is the minimum standard value of this resistor?
11.7 • Internal Circuitry of TTL Gates

Totem Pole Outputs

**KEY TERMS**

**Totem pole output**  A type of TTL output with a HIGH and a LOW output transistor, only one of which is active at any time.

**Phase splitter**  A transistor in a TTL circuit that ensures that the LOW- and HIGH-state output transistors of a totem pole output are always in opposite phase (i.e., one ON, one OFF).

Figure 11.34 shows one gate of a 7400 quadruple 2-input NAND with totem pole outputs. The circuit is the same as that for a 7401 open-collector NAND except for a transistor, resistor, and diode, which make up the HIGH-state output circuitry of the NAND gate.

![Diagram of 7400 NAND gate with totem pole output](image)

**FIGURE 11.34**

NAND Gate With Totem Pole Output

The totem pole output, shown in Figure 11.34b, has separate transistors to switch the output to the HIGH state ($Q_4$) and the LOW state ($Q_3$). These transistors are switched by $Q_2$, the phase splitter. Only one of them is ON at a time; the currents $I_{OH}$ and $I_{OL}$ never flow simultaneously.

The portion of the circuit consisting of $Q_4$, $D_3$, and the 130-$\Omega$ resistor replaces the external pull-up resistor required by the open-collector TTL output. Since the HIGH state is switched by its own transistor, we say that the circuit has an active pull-up.

The main advantage of the totem pole output over the open collector is that it can change states faster. The external pull-up resistance needed in an open-collector circuit slows down the output switching by contributing to the $RC$ time constant of the output. The HIGH-state transistor circuit, with its relatively low output impedance, reduces this time constant and thus improves switching speed.
Figure 11.35 shows the operation of the 7400 NAND gate for HIGH and LOW input conditions.

**HIGH Input.** When both inputs are HIGH, there is no low-impedance base-emitter current path in $Q_1$. The base-collector junction of $Q_1$ acts as a forward-biased diode. Base current flows in $Q_2$, saturating the transistor. Sufficient current flows to $Q_3$ to saturate it. $Y$ is connected to ground, via the collector-emitter path of $Q_3$. *The output is LOW.*

**LOW Input.** Figure 11.35b shows input $B$ of a 7400 NAND gate pulled LOW. The circuit operates the same way if $A$ or both $A$ and $B$ are LOW.

In this condition, a low-impedance path to ground is established through one of the base-emitter junctions of $Q_1$. This pulls the base of $Q_3$ LOW, causing it to be in cutoff.
mode. No current flows through the collector-emitter path of $Q_2$, so no base current flows in $Q_2$; it is also cut off.

Current flows through the 1.6-kΩ resistor to the base of $Q_4$, turning it ON. This connects the output, via $Q_4$, $D_3$, and the 130-Ω resistor, to $V_{CC}$. The output is HIGH.

$Q_4$ will not turn ON when $Q_3$ is ON. We can find out why by calculating $V_{BE4} + V_{D3}$. For $Q_4$ to conduct, two pn junctions ($D_3$ and the base-emitter junction of $Q_4$) must be forward-biased. Thus, $(V_{BE4} + V_{D3})$ must be greater than $0.6 \text{ V} + 0.6 \text{ V} = 1.2 \text{ V}$.

$$V_{BE4} + V_{D3} = V_{B4} - V_{CE3}$$

We can calculate $V_{B4}$ by adding up voltage drops, as follows:

$$V_{B4} = V_{CE2} + V_{BE3} = 0.2 \text{ V} + 0.7 \text{ V} = 0.9 \text{ V}$$

$Q_3$ is saturated, thus:

$$V_{CE3} = 0.2 \text{ V}$$

The difference between these voltages is:

$$V_{B4} - V_{CE3} = 0.9 \text{ V} - 0.2 \text{ V} = 0.7 \text{ V}$$

This is insufficient to forward-bias $BE_4$ and $D_3$. $Q_4$ stays OFF.

**NOTE**

Without $D_3$ in the circuit,

$$V_{BE4} = (V_{CE2} + V_{BE3}) - V_{CE3}$$

$$= (0.2 \text{ V} + 0.7 \text{ V}) - 0.2 \text{ V}$$

$$= 0.7 \text{ V}$$

This is sufficient to saturate $Q_4$, even when $Q_3$ is ON. The diode is therefore necessary to keep $Q_4$ OFF when $Q_3$ is ON.

### Switching Noise

**KEY TERM**

**Storage time**  Time required to transport stored charge away from the base region of a bipolar transistor before it can turn off.

A totem pole output is an inherently noisy circuit. Noise is generated on the supply voltage line when the output switches from LOW to HIGH.

When the output is in a steady HIGH or LOW state, $Q_3$ and $Q_4$ are always in opposite phase. The design of the totem pole output is such that when $Q_3$ is ON, it is saturated, but when $Q_4$ is ON, it operates in the transistor’s active, or linear, region.

A saturated transistor takes longer to shut off than an unsaturated one due to **storage time**, the time required to transport stored charge away from the base region of the transistor. Thus, $Q_3$ takes longer to turn off than $Q_4$.

When a totem pole output is LOW, $Q_1$ is ON and $Q_2$ is OFF. When the output changes state, $Q_1$ turns ON before $Q_3$ can turn OFF, due to the storage time of $Q_3$. For a few nanoseconds, both transistors are ON. This condition momentarily shorts $V_{CC}$ to ground, causing a surge of supply current, as shown in Figure 11.36.

The inductance of the power line produces a corresponding spike proportional to the instantaneous rate of change of the supply current ($v = L \frac{di}{dt}$, where $L$ is the power line inductance and $\frac{di}{dt}$ is the instantaneous rate of change of supply current).

These spikes on the supply voltage line can cause real problems, especially in synchronous circuits. They often cause erroneous switching that is nearly impossible to troubleshoot. The best cure for such problems is prevention.
Figure 11.37 shows the addition of a decoupling capacitor to a totem pole output to eliminate switching spikes. A low-inductance capacitor of about 0.1 μF is placed between the $V_{CC}$ and ground pins of the chip to be decoupled. This capacitor offsets the power line inductance and acts as a low-impedance path to ground for high-frequency noise (i.e., spikes). Since a capacitor is an open circuit for low frequencies, the normal DC supply voltage is not shorted out.

FIGURE 11.36
Spikes on Power Line During LOW-to-HIGH Transition of Totem Pole Output

Figure 11.37 shows the addition of a decoupling capacitor to a totem pole output to eliminate switching spikes. A low-inductance capacitor of about 0.1 μF is placed between the $V_{CC}$ and ground pins of the chip to be decoupled. This capacitor offsets the power line inductance and acts as a low-impedance path to ground for high-frequency noise (i.e., spikes). Since a capacitor is an open circuit for low frequencies, the normal DC supply voltage is not shorted out.

FIGURE 11.37
Decoupling the Power Supply
It is important that the capacitor be placed physically close to the decoupled chip. Inductance of the power line accumulates with distance, and if the capacitor is far away from the chip (say, at the end of the circuit board), the decoupling effect of the capacitor is lost.

**Connection of Totem Pole Outputs**

Totem pole outputs must never be connected together. As shown in Figure 11.40, the problem occurs when two connected outputs are in opposite states.

The active pull-up consisting of $Q_A$, $D_3$, and the 130-$\Omega$ resistor is designed to supply current to about 10 TTL inputs, each having a large input impedance. It will not withstand the current that flows when the output is forced to ground through the LOW output transistor of another gate.

Under this condition about 30 to 55 mA will flow through $Q_{4A}$ and $Q_{3B}$. This exceeds the ratings of the outputs in both the HIGH and LOW state and will cause damage to the outputs over time. The outputs will probably withstand this sort of abuse for several minutes, but eventually will be damaged.
SECTION 11.7C REVIEW PROBLEM

11.9 A totem pole output is likely to be damaged when shorted to ground. Why?

**Tristate Gates**

*KEY TERM*

**Tristate output** An output having three possible states: logic HIGH and LOW, and a high-impedance state, in which the output acts as an open circuit.

Figure 11.41 shows the circuits of two TTL inverters with tristate outputs. In addition to the usual binary states of HIGH and LOW, the output of the tristate inverter can also be in a “high-impedance” (Hi-Z) state. This state occurs when both $Q_3$ and $Q_4$ are OFF. The electrical effect is to produce an open circuit at the output, which is neither HIGH nor LOW.

The output of a tristate gate combines advantages of a totem pole output and an open-collector output. Like the totem pole output, it has an active pull-up with lower output impedance and faster switching than an open collector. Like the open collector, we can connect several outputs together, provided only one output is active at a time.

Input $G$, the “gating” or “enable” input, controls the gate. When $G$ is active, the gate acts as an ordinary inverter. When inactive, the gate is in the high-impedance state. Table 11.6 summarizes the operation of the tristate inverters in Figure 11.41.

The tristate inverter in Figure 11.41a is enabled by a HIGH at the $G$ input. The circuit is the same as a 7400 NAND gate with two exceptions: (1) an extra diode goes from the base of $Q_4$ to $G$, and (2) $G$ connects directly to one of the emitters of $Q_1$.

When $G = 0$, $Q_1$ acts as though there was a LOW at a NAND gate input. In a 7400 NAND circuit, this causes $Q_2$ and $Q_3$ to be in cutoff mode.

Due to the opposite states of the emitter and collector in $Q_2$, $Q_4$ would normally be ON. Instead, the LOW at $G$ pulls the base of $Q_4$ LOW through the extra diode. Thus, both $Q_3$ and $Q_4$ are OFF.

When $G = 1$, the $G$ emitter of $Q_1$ acts like a HIGH NAND input. By the enable/inhibit rules of a NAND gate, $Y = \overline{A}$. The additional diode prevents the HIGH at $G$ from activating $Q_4$.

The circuit in Figure 11.41b works the same way, except for the opposite sense of the activating input. This opposite active level is achieved by using an open-collector inverter, consisting of $Q_5$, $Q_6$, and $Q_7$, at input $\overline{G}$. 
FIGURE 11.41
Tristate Inverters

Table 11.6  Truth Tables of Tristate Inverters

<table>
<thead>
<tr>
<th>$G$</th>
<th>$A$</th>
<th>$Y$</th>
<th>$\bar{G}$</th>
<th>$A$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Hi-Z</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>Hi-Z</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Hi-Z</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>Hi-Z</td>
</tr>
</tbody>
</table>
SECTION 11.7D REVIEW PROBLEM

11.10 Why is the diode from the base of $Q_4$ necessary in the tristate inverters in Figure 11.41?

Other Basic TTL Gates

Other TTL gates are similar to the NAND and inverter gates we have already examined. A significant variation is the OR/NOR circuit, which has a different input configuration than the AND/NAND/inverter type gates.

7402 NOR Gate

Figure 11.42 shows one gate of a 7402 quadruple 2-input NOR gate package. The difference between this gate and the 7400 NAND gate is the structure of the inputs. The NOR gate does not use the multiple-emitter transistor, but rather an individual transistor ($Q_1$ or $Q_2$) for each input. There are two phase splitters ($Q_3$ and $Q_4$), which are paralleled, emitter-to-emitter and collector-to-collector.

If either $Q_3$ or $Q_4$ is enabled by a HIGH at its corresponding input, it will turn on $Q_5$, making the output LOW.

If both gate inputs are LOW, both $Q_3$ and $Q_4$ are in cutoff mode, and so is $Q_5$. The output is HIGH through $Q_6$.

Table 11.7 shows the truth table and the states of the transistors for this gate. It is not strictly correct to refer to $Q_1$ and $Q_2$ as being ON or OFF, since there is current flowing in these transistors regardless of whether the inputs are HIGH or LOW. Let us define the ON
state of an input transistor as the condition where the base-emitter junction is conducting (LOW input). If the base-collector junction conducts, we will consider the transistor OFF (HIGH input).

7408 AND Gate and 7432 OR Gate

It may not be obvious why we would choose to study NAND and NOR gates before AND and OR. After all, AND and OR are the more basic logic functions.

Electrically, it works the other way around. The simplest TTL circuit is the NAND/inverter, followed by the NOR. AND and OR gates are more complex since they are based on the NAND and NOR and require an extra inverter stage.

Figure 11.43 shows the circuit of a 7408 AND gate, and Figure 11.44 shows a 7432 TTL OR gate circuit. Each of these gates is like its NAND/NOR counterpart, except for an additional inverter, implemented by $Q_3$ in the AND gate and $Q_5$ in the OR gate.

Tables 11.8 and 11.9 show the transistor function and truth table for each gate. In keeping with the convention established for the NOR function table, an input transistor with a conducting base-emitter junction is considered ON.
SECTION 11.7E REVIEW PROBLEM

11.11 Why are noninverting gates more complex than inverting gates?

11.8 Internal Circuitry of MOS Gates

**KEY TERMS**

**MOSFET**  Metal-oxide-semiconductor field effect transistor. A MOSFET has three terminals—gate, source, and drain—which are analogous to the base, emitter, and collector of a bipolar junction transistor.
**Enhancement-mode MOSFET**  
A MOSFET that creates a conduction path (a channel) between its drain and source terminals when the voltage between gate and source exceeds a specified threshold level.

**Substrate**  
The foundation of $n$- or $p$-type silicon on which an integrated circuit is built.

**$n$-channel enhancement-mode MOSFET**  
A MOSFET built on a $p$-type substrate with $n$-type drain and source regions. An $n$-type channel is created in the $p$-substrate during conduction.

**$p$-channel enhancement-mode MOSFET**  
A MOSFET built on an $n$-type substrate with $p$-type drain and source regions. During conduction, a $p$-type channel is created in the $n$-substrate.

**CMOS**  
A logic family based on the switching of $n$- and $p$-channel ("complementary") enhancement-mode MOSFETs.

All the logic circuits we have examined so far have been based on the switching of bipolar junction transistors. Another major logic family, **CMOS**, is based on the switching of metal-oxide-semiconductor field effect transistors, or **MOSFETS**.

There are two major types of MOSFETs, called depletion-mode and **enhancement-mode MOSFETs**. We will concentrate on the enhancement-mode devices, as they are the type used in the manufacture of digital ICs. Details of the differences between depletion- and enhancement-mode transistors can be found in any good textbook on electronic devices.

MOSFETs can be categorized in another way: as **$n$-channel** and **$p$-channel** devices, much as bipolar transistors are classified as NPN or PNP.

CMOS logic is constructed from both $n$- and $p$-channel MOSFETs. CMOS ("Complementary MOS") refers to the opposite, or complementary, operation of $n$- and $p$-channel transistors.

### MOSFET Structure

Figure 11.45 shows the structure and symbol of an $n$-channel enhancement-mode MOSFET in an integrated circuit. The device is built on a **substrate** of $p$-type silicon, which has a deficiency of electrons in its structure. The drain and source regions are “wells” of $n$-type silicon, which has an excess of electrons. The drain and source are roughly equivalent to the emitter and collector of a bipolar transistor.

**FIGURE 11.45**  
n-Channel MOSFET

The substrate is shown as a terminal with an arrow. The arrow points in for an $n$-channel device and out for a $p$-channel device. In nearly all cases, the substrate is shorted to the source terminal. (Some exceptions to this general rule will be examined when we look at circuits of CMOS gates.)
The gate terminal is similar to the base of a bipolar transistor in that it controls the flow of current between the drain and source. The difference is that a MOSFET uses gate voltage to control drain current, whereas a bipolar transistor uses base current to control collector current.

The gate consists of an insulating layer of silicon dioxide (SiO₂) and a layer of metal over the substrate between the drain and source. This gate structure is what gives the MOSFET its name (metal-oxide-semiconductor field effect transistor).

**Note**

The oxide layer of the gate structure is subject to damage if excessive voltage (greater than about 100 V) is applied. This especially includes static electricity, or electrostatic discharge (ESD). There are standard precautions for working with MOS devices that should be followed carefully.

Most important are ensuring that MOS devices are stored in antistatic or conducting material, that work surfaces are not likely to generate static, that unused inputs are not left open or floating, that you avoid touching the pins of a MOS device, and that if you must handle a MOS IC, you discharge any static on your person before touching it.

A conductive wrist strap with a high series resistance to ground (about 1 MΩ) is often worn to reduce static. The high resistance protects the operator from shock injury in the event of a short circuit.

A list of handling precautions is included in Appendix D.

**SECTION 11.8A REVIEW PROBLEM**

11.12 Why are MOSFET circuits particularly susceptible to static damage?

**Bias Requirement for MOS Transistors**

**Key Terms**

- **Ohmic region** The MOSFET equivalent of saturation. When a MOSFET is biased ON, it acts like a relatively low resistance, or “ohmically.”

- **n-type inversion layer** The conducting layer formed between drain and source when an enhancement-mode n-channel MOSFET is biased ON. Also referred to as the channel.

- **Threshold voltage** $V_{\text{G(S)}}$ The minimum voltage between gate and source of a MOSFET for the formation of the conducting inversion layer (channel).

When we studied the operation of TTL gate circuits, we discovered that, for the most part, the bipolar transistors in the gates operated either in the saturation or the cutoff regions. In MOS-type gates, we make use of two similar operating regions in the constituent MOSFETs:

1. The cutoff region is the same as that for a bipolar transistor. Under this condition, there is a very high impedance between the drain and source terminals of the MOSFET.

2. The **ohmic region** is analogous to the saturation region of a bipolar transistor. In this state, there is a relatively low resistance between the MOSFET’s drain and source.

The MOSFET switches between cutoff and ohmic regions when the voltage between gate and source, $V_{\text{GS}}$, is less than or greater than a value called the **threshold voltage**. The abbreviation for this voltage is $V_{\text{G(S)}}$: its value is between 1 and 5 volts, typically 1.5 V.

Figure 11.46 shows an n-channel MOSFET operating in the cutoff region. The gate-source voltage, $V_{\text{GS}}$, is less than $V_{\text{G(S)}}$. There is no conduction between the drain and source. The resistance, $R_{\text{DS(OFF)}}$, between drain and source is very large, typically in the thousands of megohms.
When the value of $V_{GS}$ increases and exceeds the threshold voltage, the MOSFET enters the ohmic region. A conduction channel, called the \textit{n-type inversion layer}, is created in the $p$-substrate of the transistor, as shown in Figure 11.47. This layer is like an artificially created region of $n$-type silicon, which allows conduction between the drain and source, provided there is sufficient potential difference between them.

Figure 11.48 shows a MOSFET operating in the ohmic region. $R_{DS(ON)}$, the equivalent resistance of a MOSFET in the ohmic region, is typically around 500 Ω to 2 kΩ. The drain-source current, $I_{DS}$, is determined by Ohm's law: $I_{DS} = \frac{V_{CC}}{R_{DS(ON)}}$. 

![Diagram](image-url)
The operation of a $p$-channel MOSFET is similar, but with polarities reversed. If $V_{GS(Th)} = +1.5 \text{ V}$ for an $n$-channel device, an equivalent $p$-channel MOSFET has a threshold voltage of $-1.5 \text{ V}$. $V_{GS} > +1.5 \text{ V}$ turns ON an $n$-channel transistor; $V_{GS} < -1.5 \text{ V}$ turns ON a $p$-channel device.

Figure 11.49 summarizes the bias requirements for $n$- and $p$-channel enhancement-mode MOSFETs.

![FIGURE 11.49 Bias Requirements of n- and p-Channel MOSFETs](image)

**CMOS Inverter**

Figure 11.50 shows the circuit of a CMOS inverter, which consists of one $n$-channel and one $p$-channel MOSFET.

Recall the bias conditions of the two transistors:

- **$n$-channel**: threshold voltage, $V_{GS(Th)} \approx +1.5 \text{ V}$
  - ON when $V_{GS} > V_{GS(Th)}$ (e.g., $V_{GS} = V_{CC}$)
  - OFF when $V_{GS} < V_{GS(Th)}$ (e.g., $V_{GS} = 0 \text{ V}$)

- **$p$-channel**: threshold voltage, $V_{GS(Th)} \approx -1.5 \text{ V}$
  - ON when $V_{GS} < V_{GS(Th)}$ (e.g., $V_{GS} = -V_{CC}$)
  - OFF when $V_{GS} > V_{GS(Th)}$ (e.g., $V_{GS} = 0 \text{ V}$)
The operation of the CMOS inverter, and any other CMOS gate, depends on arranging the bias conditions of each complementary pair of transistors so that they are always in opposite states. Whenever \( Q_1 \) is ON, \( Q_2 \) is OFF, and vice versa. Figure 11.51 shows how this is accomplished.

Assume that a LOW input is at ground potential and that a HIGH input is equal to \( V_{CC} \).

**FIGURE 11.50**
CMOS Inverter

When input \( A \) is LOW, the gate voltage of \( Q_2 \) is the same as its source voltage; \( V_{GS2} = 0 \) and \( Q_2 \) is OFF. This places a high-impedance path between output \( Y \) and ground. At the same time, the gate voltage of \( Q_1 \) is 0 V and its source voltage is \( V_{CC} \); \( V_{GS1} = V_{G1} - V_{S1} = 0 - V_{CC} = -V_{CC} \). (The \( p \)-channel transistor, \( Q_1 \), is drawn “upside down” to make the complementary pair symmetrical.) \( Q_1 \) is ON, forming a low-impedance path from \( V_{DD} \) to the output \( Y \). **Output \( Y \) is HIGH.**

When input \( A \) is HIGH, the gate-source voltage of the \( n \)-channel transistor is \( V_{CC} \), causing \( Q_2 \) to turn ON. The gate of \( Q_1 \) is also at \( V_{CC} \). Since the source of the \( p \)-channel
transistor is at $V_{CC}$, $V_{GS1} = V_{G1} = V_{S1} = V_{CC} = 0$ V; $Q_1$ is OFF. This combination creates a high impedance between $V_{CC}$ and output $Y$ and a low-impedance path from output $Y$ to ground, as shown in Figure 15.51b. Output $Y$ is LOW.

**CMOS NAND/NOR Gates**

CMOS NAND and NOR gates are constructed from complementary pairs of MOSFETs. Each MOSFET pair has an $n$-channel transistor that is turned ON by a HIGH input and a $p$-channel transistor that is turned ON by a LOW input. The $n$-channel devices switch the output to ground; the $p$-channel ones switch the output to $V_{CC}$.

NAND and NOR functions are generated by arranging the MOSFET drain-source paths in series (AND) and parallel (OR) configurations.

In Figure 11.52, we see the DeMorgan equivalent forms of a NAND gate. Each form illustrates an aspect of NAND operation that can be described with a brief sentence and implemented by a MOSFET circuit. The combination of forms describes the complete operation of the device.

Figure 11.52a states that both NAND inputs must be HIGH to make the output LOW. A logic HIGH activates an $n$-channel MOSFET. The gate output is switched to ground by an $n$-channel MOSFET. Thus, the drain-source paths of two $n$-channel transistors must be connected in series to make the output LOW under the stated conditions.

Figure 11.52b shows that the NAND output is HIGH if either input is LOW. A $p$-channel transistor will turn ON when its gate is LOW and will switch a HIGH to the output. A parallel combination of $p$-channel MOSFETs will satisfy these conditions.

**FIGURE 11.52**
NAND Functions of MOSFETs

The stated conditions are combined in the CMOS NAND circuit shown in Figure 11.53. Transistors $Q_1$ and $Q_4$ form a complementary pair, as do $Q_2$ and $Q_3$. When $A$ and $B$ are both HIGH, $Q_1$ and $Q_2$ are both OFF, cutting off the connection between $V_{CC}$ and output $Y$. $Q_3$ and $Q_4$ are both ON, supplying a low-impedance path from output $Y$ to ground, making the output LOW. This is shown in the partial truth table in Table 11.10.

When $A$ is LOW and $B$ is HIGH, $Q_1$ is ON. This creates a path from $V_{CC}$ to output $Y$. At the same time, $Q_4$ is OFF. This cuts the $Y$-to-ground path through the $n$-channel
MOSFETs; the series path from output to ground is broken. One parallel path from $V_{CC}$ to output has been established. Output $Y$ is HIGH.

The remaining input combinations also make the output HIGH, as shown in Table 11.11. They do so by breaking the $n$-channel path from output to ground and enabling one or both $p$-channel paths from $V_{CC}$ to output.

Each MOSFET in a logic circuit must have its own independent substrate bias. This ensures that the transistor will operate as expected when a logic HIGH or LOW is applied to its gate.

Normally, the substrate of a MOSFET is shorted to its source terminal. If a MOSFET source terminal is isolated from $V_{CC}$ or ground, the substrate must be biased separately. For example, in the NAND gate in Figure 11.53, the substrate of $Q_3$ connects directly to ground.

NOR gates are similar to NANDs in construction. Figure 11.54 shows the DeMorgan equivalent forms of the NOR function and the MOSFET implementations of each aspect of the gate operation.

### Table 11.10 Partial CMOS NAND Function and Truth Table

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 11.11 Partial CMOS NAND Function and Truth Table

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>1</td>
</tr>
</tbody>
</table>
When either input is HIGH, the output is LOW. This function is implemented by two parallel n-channel MOSFETs. Both inputs must be LOW to make the output HIGH, which implies a series connection of two p-channel transistors. The complete NOR gate circuit is shown in Figure 11.55. (Note that the substrate of $Q_2$ is connected directly to $V_{CC}$ to ensure that it has its own bias voltage.)

---

**Table 11.12** Partial CMOS NOR Function and Truth Table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
<td>OFF</td>
<td>1</td>
</tr>
</tbody>
</table>

---

If either input is HIGH, one or both of the p-channel transistors will turn OFF. This action breaks the path from $V_{CC}$ to output $Y$. The complementary n-channel transistor will turn ON. This creates a low-impedance path from output $Y$ to ground. Output $Y$ is LOW. Table 11.13 summarizes the possible input conditions and MOSFET states when the NOR output is LOW.

---

**Table 11.13** Partial CMOS NOR Function and Truth Table

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>OFF</td>
<td>ON</td>
<td>OFF</td>
<td>ON</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>0</td>
</tr>
</tbody>
</table>
CMOS AND and OR Gates

Figures 11.56 and 11.57 show the circuits of CMOS AND and OR gates. The AND gate is the same as the NAND circuit, except for the output inverter section constructed from \( Q_5 \) and \( Q_6 \). The OR gate is the same as the NOR with an output inverter section.

SECTION 11.8B REVIEW PROBLEM

11.13 Why is the source of a \( p \)-channel MOSFET connected to \( V_{CC} \) in a CMOS gate?

CMOS Transmission Gate

Figure 11.58 shows the circuit of a CMOS transmission gate. A CMOS transmission gate, or analog switch, conducts in both directions. This makes it possible to enable or inhibit
CHAPTER 11 • Logic Gate Circuitry

time-varying analog signals having both positive and negative values. Conduction takes place between the input and output terminals through MOSFETs $Q_1$ and $Q_2$. Positive current (left to right in the diagram) flows through $Q_2$, and negative current (right to left) flows through $Q_1$. Two inverters, consisting of the $Q_3/Q_4$ and $Q_5/Q_6$ pairs of MOSFETs, control the ON/OFF state of the circuit.

When $CONTROL = 1$, the inverters bias both $Q_1$ and $Q_2$ ON, allowing them to conduct. When $CONTROL = 0$, the circuit inhibits conduction between input and output.

The substrate terminal of $Q_1$ is connected, not to the source terminal of that transistor, but directly to $V_{CC}$ thus providing the correct bias to $Q_1$ in the ON state.

A particular device with this function is the 74HC4066 quad analog switch, whose circuit symbol is shown in Figure 11.59. When the $CONTROL$ input is HIGH, analog and digital signals can pass between the bidirectional input terminals.

![FIGURE 11.58 CMOS Transmission Gate](image)

EXAMPLE 11.16

Figure 11.60 shows a circuit where the analog switches in a 74HC4066 package are used to control the selection and muting of two pairs of speakers in a stereophonic audio system. Briefly explain the circuit operation.

Solution The audio signal to each speaker is passed or blocked by a CMOS transmission gate. The speakers are paired into $A$ and $B$ groups. Each pair has a left and a right channel speaker. The same logic gate controls both speakers of each group.

The Select $A$ switch enables the $A$ speakers when it is open (logic HIGH). The Select $B$ switch enables the $B$ speakers when it is open. The Mute Toggle flip-flop mutes (disables) both sets of speakers when $Q$ is LOW. This action inhibits both AND gates, making all transmission gate $CONTROL$ inputs LOW. The mute function toggles ON and OFF with each push of the Mute ON/OFF switch.
Standard (74NN) TTL and CMOS represented the two main standards of logic design for many years, and their influence is still visible in other, more advanced types of logic. The changes that have been made in newer logic families are not fundamental changes in the working concepts, but improvements to the specifications, particularly switching speed and power dissipation.
TTL Logic Families

**KEY TERMS**

- **Schottky barrier diode**: A specialized diode with a forward drop of about +0.4 V.
- **Schottky transistor**: A bipolar transistor with a Schottky diode across its base-collector junction, which prevents the transistor from going into deep saturation.
- **Schottky TTL**: A series of unsaturated TTL logic families based on Schottky transistors. Schottky TTL switches faster than standard TTL due to decreased storage time in its transistors.
- **Speed-power product**: A measure of a logic circuit’s efficiency, calculated by multiplying its propagation delay by its power dissipation. Unit: picojoule (pJ)

Probably the most important development in TTL technology was the introduction, in the early 1970s, of the **Schottky barrier diode** into circuit designs. This made possible the first family of nonsaturated bipolar logic, with its resultant improvement in switching speed.

Figure 11.61 shows a bipolar transistor with a Schottky diode connected across its base and collector and the equivalent circuit symbol of this combination. We call this configuration a **Schottky transistor** and logic devices using such transistors **Schottky TTL**.

![Schottky Transistor](image)

**FIGURE 11.61**
Schottky Transistor

Normally the base-collector junction of a saturated bipolar transistor has a drop of about 0.5 volts, as shown in Figure 11.62. The Schottky diode clamps this junction voltage to about 0.4 volts. This keeps the transistor out of deep saturation in its ON state. The base region of the Schottky-clamped transistor holds less charge than does a standard bipolar transistor. Its storage time, the time required to dissipate base charge upon turn-off, is substantially reduced. The transistor can switch faster with the Schottky diode than without.

![ON-State Operating Voltages of Bipolar Transistors](image)

**FIGURE 11.62**
ON-State Operating Voltages of Bipolar Transistors

Figure 11.63 shows the circuits of the 74S00 Schottky and 74LS00 low-power Schottky NAND gates. Compare these circuits to each other and to the 7400 standard TTL NAND gate in Figure 11.33.
In the 74S00 circuit, $Q_1$ acts as the input and $Q_2$ as the phase splitter, as in the 7400 gate. The HIGH output circuit consists of $Q_3$ and $Q_4$ connected as a modified Darlington pair. When $Q_2$ is OFF (at least one input is LOW), enough base current flows in $Q_3$ to turn it on. Collector-emitter current in $Q_3$ turns on $Q_4$, making the output HIGH.

When $Q_2$ is ON (both inputs are HIGH), the base of $Q_3$ is pulled LOW, turning it OFF. Sufficient current flows in the base of $Q_5$ to turn it ON. The resultant current through $Q_5$
will turn on $Q_6$, making the output LOW. A similar analysis can be made for the 74LS00 gate.

One difference between the 74S00 and 74LS00 circuits is the size of the resistors; the LS device has larger resistors. Less current flows in the gate circuit. This reduces power dissipation of the chip. The larger resistor values also slow down the switching times of the various transistors by increasing the $RC$ time constants of the circuit elements.

### Speed-Power Product

One measure of logic circuit efficiency is its **speed-power product**, calculated by multiplying switching speed and power dissipation, usually expressed in picojoules ($\text{pJ}$). (The joule is the SI unit of energy. Power is the rate of energy used per unit time.) A major goal of logic circuit design is the reduction of a device’s speed-power product.

Table 11.14 shows the propagation delay, supply current, and speed-power product for a NAND gate in six TTL families: standard TTL (7400), Schottky (74S00), low-power Schottky (74LS00), fast TTL (74F00), advanced Schottky (74AS00), and advanced low-power Schottky (74ALS00).

**Table 11.14** TTL Speed and Power Specifications

<table>
<thead>
<tr>
<th></th>
<th>7400</th>
<th>74LS00</th>
<th>74S00</th>
<th>74F00</th>
<th>74ALS00</th>
<th>74AS00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{p_{LH}}$ (max)</td>
<td>22 ns</td>
<td>15 ns</td>
<td>4.5 ns</td>
<td>6 ns</td>
<td>11 ns</td>
<td>4.5 ns</td>
</tr>
<tr>
<td>$t_{p_{HL}}$ (max)</td>
<td>15 ns</td>
<td>15 ns</td>
<td>5 ns</td>
<td>5.3 ns</td>
<td>8 ns</td>
<td>4 ns</td>
</tr>
<tr>
<td>$I_{CC/4}$ (max)</td>
<td>2 mA</td>
<td>0.4 mA</td>
<td>4 mA</td>
<td>0.7 mA</td>
<td>0.21 mA</td>
<td>0.8 mA</td>
</tr>
<tr>
<td>$I_{CC/4}$ (max)</td>
<td>5.5 mA</td>
<td>1.1 mA</td>
<td>9 mA</td>
<td>2.6 mA</td>
<td>0.75 mA</td>
<td>4.35 mA</td>
</tr>
<tr>
<td>Speed-power product (per gate)</td>
<td>605 pJ</td>
<td>82.5 pJ</td>
<td>225 pJ</td>
<td>78.0 pJ</td>
<td>41.25 pJ</td>
<td>97.9 pJ</td>
</tr>
</tbody>
</table>

The speed-power product shown is the worst-case value. This is calculated by multiplying the largest value of $I_{CC/4}$ by the slowest switching speed by 5 volts for each family. We use $I_{CC/4}$ because $I_{CC}$ is specified per chip (four gates).

A faster switching speed results in an overall increase in speed-power product, other factors being equal. For example, the speed-power product of either advanced Schottky family is lower than that of the LS and S families. However, the ALS series (the slower advanced Schottky family) has a lower speed-power product than the AS series.

The smaller resistors used to speed up output switching imply a proportional drop in propagation delay (higher speed) but an increased supply current. Power dissipation increases in proportion to the square of the supply current, thus offsetting the effect of the increased switching speed.

### CMOS Logic Families

The CMOS gates we have looked at in this chapter are simpler than most gates actually in use. There are two main families of CMOS devices: metal-gate CMOS, and silicon-gate, or high-speed, CMOS.

#### Metal-Gate CMOS

There are two main variations on this type of circuit, designated B-series and UB-series CMOS. Most CMOS gates are B-series; UB-series is available in a limited number of inverting-type gates, such as inverters and 2-, 3-, and 4-input NAND and NOR gates. Figure 11.64 shows the difference in the two configurations.

Figure 11.64b shows one gate from a 4011UB quadruple 2-input NAND package. Its circuit is the same as the NAND configuration examined in Section 11.8. Power supply voltages in metal-gate CMOS are designated $V_{DD}$ (power) and $V_{SS}$ (ground). High-speed, or silicon-gate, CMOS uses the same power supply designations as TTL: $V_{CC}$ and ground.
11.9 • TTL and CMOS Variations

The B-series configuration of this circuit has two additional inverter outputs in cascade with the NAND logic. (The same gate becomes an AND when we add a third output inverter.) The inverter configuration is actually an amplifier; extra inverter stages provide additional gain and increase noise margin by allowing the circuit to accept smaller input signals.

CMOS gates are sometimes used in analog applications, such as oscillators. The UB-series gates, with their lower gain, are more desirable for such applications. Due to its low switching speed, metal-gate CMOS is rarely used in new designs.

High-Speed CMOS

**KEY TERM**

**High-speed (silicon-gate) CMOS**  A CMOS logic family with a smaller device structure and thus higher speed than standard (metal-gate) CMOS.

Metal-gate CMOS has been considered a nearly ideal family for logic designs, with its high noise immunity, low power consumption, and flexible power supply requirements. Unfortunately, its propagation delay times, typically 10 to 20 times greater than those of equivalent TTL devices, are just not fast enough for use in modern microprocessor-based systems.

**High-speed CMOS** was developed to address the problem of switching speed, while striving to keep the other advantages of CMOS. This is achieved by using MOSFETs with a polysilicon material for the gate, rather than metal, as in standard CMOS. Because of advantages gained in this manufacturing process, each transistor is physically smaller and has a lower gate capacitance than metal-gate MOSFETs. Both these factors contribute to a lower propagation delay for the logic gate circuit.

Several subfamilies of high-speed CMOS are available for various logic and linear applications, designated by the labels 74HCNN, 74HC4NNN, 74HCTNN, and 74HCUNN.

The 74HCNN series duplicates equivalent LSTTL functions in packages having identical pinouts to LSTTL. The 74HC4NNN replaces CMOS functions pin for pin. Both these series have CMOS-equivalent input and output levels, within the power supply limits (2.0 V to 6.0 V) of high-speed CMOS.
CHAPTER 11  • Logic Gate Circuitry

The 74HCTNN devices are designed to be directly compatible with LSTTL devices, and thus have LSTTL-equivalent inputs and CMOS-equivalent outputs. 74HCUNN devices have no output buffers, like the 4000 UB-series standard CMOS devices. The 74HCU devices are used, as are the 4000UB devices, for linear applications such as oscillators and multivibrators.

Table 11.15 shows the relative performance of the various CMOS families. As in TTL, the 2-input NAND gate is used as the standard, except for the HCU family, where this gate is not available. The quiescent speed-power product of all CMOS families is much smaller than that of any TTL family. The high-speed CMOS families have propagation delays comparable to those of LSTTL.

The power dissipation of a CMOS device increases directly with frequency. The speed-power product also goes up with higher frequencies.

Table 11.15 shows CMOS speed-power product for a switching speed of 1 MHz. At these speeds, B-series CMOS has no advantage over the common TTL families in terms of its efficiency. It still has the edge on TTL with respect to noise immunity and power supply flexibility.

### SECTION 11.9 REVIEW PROBLEM

11.14 Assuming that power dissipation of a 74HC00A NAND gate is directly proportional to its switching frequency, what is the speed-power product of the gate at 2 MHz, 5 MHz, and 10 MHz?

### SUMMARY

1. TTL (transistor-transistor logic) and CMOS (complementary metal-oxide semiconductor) are two major logic families in use today. TTL is constructed from bipolar junction transistors. CMOS is made from metal-oxide-semiconductor field effect transistors (MOSFETs).
2. The main CMOS advantages include low power consumption, high noise immunity, and a flexibility in choosing a power supply voltage.
3. The main advantages of TTL include relatively high switching speed and an ability to drive loads with relatively high current requirements.
4. TTL and high-speed CMOS logic families are alphabetically designated by a part number having the form 74XXNN, where XX is the family and NN is a numeric logic function designator. (For example, 74HC00 and 74LS00 have the same logic function, but are from different logic families.)
Devices from earlier CMOS families are designated by a part number of the form 4NNNB or 4NNNUB.

5. Devices of the same logic family generally have the same electrical characteristics.

6. Data such as input/output voltages and currents are specified in manufacturers’ datasheets. Only the maximum or minimum values of these parameters should be used as design information. “Typical” values should be regarded as “information only.”

7. The time required for an input to change as a result of an input change is called propagation delay.

8. Propagation delay is specified as \( t_{pd} \) when an output changes from LOW to HIGH and \( t_{ph} \) when the output goes from HIGH to LOW.

9. Propagation delay in a circuit is the sum of all delays in the slowest input-to-output path. Gates whose outputs do not change are ignored in the calculation.

10. Fanout is the maximum number of device inputs that can be driven by the output of a logic device.

11. The actual value of output current in a driving gate is the sum of all load currents, which are the input currents of the load gates. For \( n \) loads,

\[
I_{OL} = I_{L1} + I_{L2} + \ldots + I_{L_n} = n_L I_L
\]

and

\[
I_{OH} = I_{H1} + I_{H2} + \ldots + I_{Hn} = n_{OH}
\]

12. The fanout of the driving gate in the LOW and HIGH states can be calculated as:

\[
n_L = \frac{I_{OL}}{I_{IL}}
\]

and

\[
n_H = \frac{I_{OH}}{I_{IH}}
\]

13. If the fanout is unequal for LOW and HIGH states, the smaller value must be used.

14. If the fanout of a gate is exceeded, the output voltage of the driving gate will drop if the output is HIGH and rise if the output is LOW. This move away from the nominal value degrades the general performance of the driving gate.

15. Power supply current \( I_{CC} \), and therefore power dissipation \( P_D \), of a TTL device depends on the number of outputs in the device that are HIGH or LOW. \( P_D = V_{CC} I_{CC} = V_{CC} \left( \frac{n_L}{n} I_{CCH} + \frac{n_H}{n} I_{CCL} \right) \) for a device with \( n \) outputs, \( n_H \) of which are HIGH and \( n_L \) of which are LOW.

16. CMOS devices draw most current from the power supply when its outputs are switching and very little when they are static. Power dissipation of a high-speed CMOS device with \( n \) outputs has a static and a dynamic component, given by:

\[
P_D = (C_L + C_{PD})V_{CC}^2 f + \frac{V_{CC} I_{CC}}{n}
\]

At high frequencies (\( \pm 1 \) MHz), the quiescent current can be neglected.

17. Noise margin is a measure of the noise voltage that can be tolerated by a logic device input. In the HIGH state, it is given by \( V_{NH} = V_{OH} - V_{IL} \). In the LOW state, it is given by \( V_{NL} = V_{IH} - V_{IL} \). CMOS devices generally have higher noise margins than TTL.

18. When interfacing two devices from different logic families, the driving gate must satisfy the voltage and current requirements of the load gates.

19. Input current in a CMOS gate is very low, due to its high input impedance. Thus, fanout is generally not a problem with CMOS loads.

20. CMOS devices that have the same values of \( V_{IH} \) and \( V_{IL} \) as TTL are considered to be TTL compatible, since they can be driven directly by TTL drivers.

21. A 74HC or 74HCT device can drive 10 LSTTL loads directly. To calculate fanout, we use the output currents for which the driving gate output voltages are defined.

22. A 74LS device can drive one or more 74HC devices, provided each 74HC input has a pull-up resistor (about 1 k\( \Omega \) to 10 k\( \Omega \)) to supply sufficient voltage in the HIGH state.

23. A 74LS device can drive one or more 74HCT inputs directly.

24. Low-voltage CMOS (e.g., 74LVX or 74LCX) can be driven directly by a TTL device if the CMOS device is operated with a 3.3 V power supply. Noise margins are too small for a low-voltage CMOS driver to drive TTL loads.

25. 74HC or 74HCT gates can be operated at a low value of \( V_{CC} \) (e.g., 3 volts) and interfaced to a higher-voltage driver by an inverting or noninverting buffer, such as the 74HC4049 or 74HC4050. The interface buffer can tolerate relatively high input voltages (up to 15 V) and, if it shares the same supply voltage as the load gate, can provide correct input voltages to the load.

26. A bipolar transistor with a grounded emitter acts as an inverter or a digital switch. A HIGH at the base causes the transistor to conduct, pulling the collector to near-ground potential. If there is a pull-up resistor on the collector, there will be a HIGH state at the collector when the base is LOW.

27. The simplest TTL input is a transistor with its base connected to \( V_{CC} \) through a resistor. It can be treated as two diodes, back-to-back.

28. A TTL LOW input forward-biases the base-emitter junction of the input transistor, supplying a path to ground for input current.

29. A TTL HIGH input reverse-biases the base-emitter junction of the input transistor and forward-biases its base-collector junction. Input current in the HIGH state is restricted to reverse leakage current through the base-emitter junction.

30. An open TTL input is equivalent to a HIGH, as it provides no path to ground.

31. Some types of TTL gates, such as NAND, have multiple-emitter input transistors. Any one input LOW acts as a LOW for the whole circuit.

32. Other TTL gates, such as NOR, have separate transistors for each input. Any HIGH input acts as a HIGH for the whole circuit.

33. An open-collector output has one output transistor that switches on a path to ground (logic LOW) when it is turned on. There is no separate internal circuit for a HIGH output. This must be provided by an external pull-up resistor.

34. Open-collector outputs can be used to parallel outputs (wired-AND), drive high-current loads, or interface to a circuit with a different power supply voltage than the driving gate.

35. A totem pole output has a transistor that switches on for a LOW output and another that switches on for a HIGH output. These output transistors are always in opposite states, except briefly during times when the output is changing states.

36. Totem pole outputs generate noise spikes on the power line of a circuit when they switch between logic states. These
spikes can be amplified by inductance of the power line. Decoupling capacitors placed close to each device help minimize this problem.

37. TTL outputs should never be connected together, as they can be damaged when the outputs are in opposite states. (Too much output current flows.) The logic level under such conditions is not certain.

38. Gates with tristate outputs can generate logic LOW, logic HIGH, or high-impedance states. A high-impedance state is like an open circuit or electrical disconnection of the gate output from the circuit. In this state, both HIGH- and LOW-state output transistors are off.

39. The operation of a tristate output is controlled by the state of a control input. In one control state, the output is either HIGH or LOW. In the opposite control state, the output is in the high-impedance state.

40. CMOS (complementary MOS) devices are based on n-channel and p-channel MOSFETs (metal-oxide-semiconductor field effect transistors).

41. A MOSFET consists of a silicon substrate of a particular type of silicon (e.g., p-type), embedded with wells of the opposite type (e.g., n-type) that form the drain and source regions of the MOSFET. A gate electrode can bias the substrate to create a conduction channel between drain and source.

42. An n-channel enhancement mode MOSFET is biased on when its gate voltage exceeds its source voltage by a given amount called the threshold voltage.

43. A p-channel enhancement mode MOSFET is biased on when its gate voltage is less than its source voltage by a given amount called the threshold voltage.

44. An n-channel and p-channel MOSFET can be connected in such a way that one of the pair of MOSFETs is always on and one is always off. This connection is called a complementary pair and forms the basis for CMOS logic.

45. Logic functions, such as NAND and NOR, can be implemented with a complementary pair of MOSFETs for each input, with the MOSFETs in series or parallel to Vcc or ground, as required.

46. Many TTL families have been designed to incorporate Schottky barrier diodes, which limit the saturation of their transistors, allowing faster internal and output switching speeds.

47. Metal-gate CMOS has been superseded by high-speed (silicon-gate) CMOS, which has a smaller MOSFET size, resulting in faster switching and lower gate capacitance.

48. Speed-power product is a measure of the energy used by a gate. More advanced logic families have smaller values of speed-power product.

**GLOSSARY**

CMOS  Complementary metal-oxide semiconductor. A logic family based on the switching of n- and p-channel metal-oxide-semiconductor field effect transistors (MOSFETs).

Cutoff mode  The operating mode of a transistor when there is no collector or drain current flowing and the path from collector to emitter or drain to source is effectively an open circuit.

Driving gate  A gate whose output supplies current to the inputs of other gates.

ECL  Emitter coupled logic. A high-speed logic family based on bipolar transistors.

Enhancement-mode MOSFET  A MOSFET which creates a conduction path (a channel) between its drain and source terminals when the voltage between gate and source exceeds a specified threshold level.

Fanout  The number of gate inputs that a gate output is capable of driving without possible logic errors.

Floating  An undefined logic state, neither HIGH nor LOW.

High-speed (silicon-gate) CMOS  A CMOS logic family with a smaller device structure and thus higher speed than standard (metal-gate) CMOS.

Icc  Total supply current in a TTL or high-speed CMOS device.

Icch  TTL supply current with all outputs HIGH.

Icl  TTL supply current with all outputs LOW.

Iod  CMOS supply current under static (nonswitching) conditions.

Iih  Current measured at a device input when the input is HIGH.

Iil  Current measured at a device input when the input is LOW.

Ioh  Current measured at a device output when the output is HIGH.

Iol  Current measured at a device output when the output is LOW.

It  When referring to CMOS supply current, the sum of static and dynamic supply currents.

Load gate  A gate whose input current is supplied by the output of another gate.

MOSFET  Metal-oxide-semiconductor field effect transistor. A MOSFET has three terminals—gate, source, and drain—which are analogous to the base, emitter, and collector of a bipolar junction transistor.

n-channel enhancement-mode MOSFET  A MOSFET built on a p-type substrate with n-type drain and source regions. An n-type channel is created in the p-substrate during conduction.

Noise  Unwanted electrical signal, often resulting from electromagnetic radiation.

Noise margin  A measure of the ability of a logic circuit to tolerate noise.

n-type inversion layer  The conducting layer formed between drain and source when an enhancement-mode n-channel MOSFET is biased ON. Also referred to as the channel.

Ohmic region  The MOSFET equivalent of saturation. When a MOSFET is biased ON, it acts like a relatively low resistance, or “ohmically.”

Open-collector output  A TTL output where the collector of the LOW-state output transistor is brought out directly to the output pin. There is no built-in HIGH-state output circuitry which allows two or more open collector outputs to be connected without possible damage.
**p-channel enhancement-mode MOSFET** A MOSFET built on an n-type substrate with p-type drain and source regions. During conduction, a p-type channel is created in the n-substrate.

**Phase splitter** A transistor in a TTL circuit which ensures that the LOW- and HIGH-state output transistors of a totem pole output are always in opposite phase (i.e., one ON, one OFF).

**Power dissipation** The electrical energy used by a logic circuit in a specified period of time. Abbreviation: \( P_D \)

**Propagation delay** The time required for the output of a digital circuit to change states after a change at one or more of its inputs.

**Saturation mode** The operating mode of a bipolar transistor when an increase in base current will not cause a further increase in the collector current and the path from collector to emitter is very nearly (but not quite) a short circuit. This is the ON state of a transistor in a digital circuit.

**Schottky barrier diode** A specialized diode with a forward drop of about 0.4 V.

**Schottky transistor** A bipolar transistor with a Schottky diode across its base-collector junction, which prevents the transistor from going into deep saturation.

**Schottky TTL** A series of unsaturated TTL logic families based on Schottky transistors. Schottky TTL switches faster than standard TTL due to decreased storage time in its transistors.

**Sinking** A terminal on a gate or flip-flop is sinking current when the current flows into the terminal.

**Sourcing** A terminal on a gate or flip-flop is sourcing current when the current flows out of the terminal.

**Speed-power product** A measure of a logic circuit’s efficiency, calculated by multiplying its propagation delay by its power dissipation. Unit: picojoule (pJ)

**Storage time** Time required to transport stored charge away from the base region of a bipolar transistor before it can turn off.

**Substrate** The foundation of n- or p-type silicon on which an integrated circuit is built.

**Threshold voltage, \( V_{GS(Th)} \)** The minimum voltage between gate and source of a MOSFET for the formation of the conducting inversion layer (channel).

**Totem pole output** A type of TTL output with a HIGH and a LOW output transistor, only one of which is active at any time.

\( t_{pHL} \) Propagation delay when the device output is changing from HIGH to LOW.

\( t_{pLH} \) Propagation delay when the device output is changing from LOW to HIGH.

**Tristate output** An output having three possible states: logic HIGH, logic LOW, and a high-impedance state, in which the output acts as an open circuit.

**TTL** Transistor-transistor logic. A logic family based on bipolar transistors.

**TTL Compatible** Able to be driven directly by a TTL output. Usually implies voltage compatibility with TTL.

\( V_{CC} \) Supply voltage for TTL and high-speed CMOS devices.

\( V_{DD} \) Metal-gate CMOS supply voltage.

\( V_{IH} \) Voltage level required to make the input of a logic circuit HIGH.

\( V_{IL} \) Voltage level required to make the input of a logic circuit LOW.

\( V_{OL} \) Voltage level required to make the input of a logic circuit LOW.

\( V_{OH} \) Voltage measured at a device output when the output is HIGH.

\( V_{OL} \) Voltage measured at a device output when the output is LOW.

**Wired-AND** A connection where open-collector outputs of logic gates are wired together. The logical effect is the ANDing of connected functions.

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**Problems**

*Problem numbers set in color indicate more difficult problems: those with underlines indicate most difficult problems.*

**Section 11.1 Electrical Characteristics of Logic Gates**

11.1 Briefly list the advantages and disadvantages of TTL, CMOS, and ECL logic gates.

**Section 11.2 Propagation Delay**

11.2 Explain how propagation delay is measured in TTL devices and CMOS devices. How do these measurements differ?

11.3 Figure 11.65 shows the input and output waveforms of a logic gate. Use the graph to calculate \( t_{pHL} \) and \( t_{pLH} \).

11.4 The inputs of the logic circuit in Figure 11.66 are in state 1 in the following table. The inputs change to state 2, then to state 3.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>State 2</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>State 3</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

a. Draw a timing diagram that uses the above changes of input state to illustrate the effect of propagation delay in the circuit.

b. Calculate the maximum time it takes for the output to change when the inputs change from state 1 to state 2.

c. Calculate the maximum time it takes for the output to change when the inputs change from state 2 to state 3.
11.5 Repeat Problem 11.4, parts b and c, for a 74HC00 NAND and a 74HC02 NOR gate.

11.6 Calculate the maximum number of low-power Schottky TTL loads (74LSNN series) that a 74S86 XOR gate can drive.

11.7 What is the maximum number of 74S32 OR gates that a 74LS00 NAND gate can drive?

11.8 What is the maximum number of 74LS00 NAND gates that a 74S32 OR gate can drive?

11.9 An LSTTL gate is driving seven LSTTL gate inputs, each equivalent to the load presented by a 74LS00 NAND input. Calculate the source and sink currents required from the driving gate.

11.10 Calculate the current values for the circuits shown in Figure 11.67. For each circuit, state the logic level at the output of gate 1.
Section 11.4  Power Dissipation

11.11  The circuit in Figure 11.68 is constructed from the gates of a 74LS08 AND device. Calculate the power dissipation of the circuit for the following input logic levels:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>b.</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>c.</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>d.</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

FIGURE 11.68
Problems 11.11 to 11.13
Logic Circuit

11.12  The gate outputs in Figure 11.68 are switching at an average frequency of 100 kHz, with an average duty cycle of 60%. Calculate the power dissipation if the gates are all 74S08 AND gates.

11.13  The gates in Figure 11.68 are 74HC08A high-speed CMOS gates.

a. Calculate the power dissipation of the circuit if the input state is ABCDE = 010101. (V\textsubscript{CC} = 4.5 V, T\textsubscript{A} = 25°C)

b. Calculate the circuit power dissipation if the outputs are switching at a frequency of 10 kHz, 50% duty cycle.

c. Repeat part b for a frequency of 2 MHz.

11.14  The circuit in Figure 11.69 consists of two 74LS00 NAND gates (gates 4 and 5) and three 74LS02 NOR gates (gates 1, 2, and 3). When this circuit is actually built, there will be two unused NAND gates and one unused NOR gate in the device packages.

FIGURE 11.69
Problem 11.14
Logic Circuit

Calculate the maximum total power dissipation of the circuit when its input state is ABCDE = 01100. Include all unused gates. (Connect unused gate inputs so that they will dissipate the least amount of power.)

11.15  a. Calculate the no-load power dissipation of a single gate at 1 MHz for a 74HC00A quad 2-input NAND gate (V\textsubscript{CC} = 5 V). (Neglect quiescent current.)

b. Calculate the percent change in power dissipation if the gate in part a of this question is operated with a new value of V\textsubscript{CC} = 3.3 V. (f = 1 MHz)

Section 11.5  Noise Margin

11.16  Calculate the maximum noise margins, in both HIGH and LOW states, of:

a. A 74S00 NAND gate
b. A 74LS00 NAND gate
c. A 74AS00 NAND gate
d. A 74ALS00 NAND gate
e. A 74HC00 NAND gate (V\textsubscript{CC} = 5 V)
f. A 74HCT00 NAND gate (V\textsubscript{CC} = 5 V)

Section 11.6  Interfacing TTL and CMOS Gates

11.17  Why can an LSTTL gate drive a 74HC0T gate directly, but not a 74HC? Show calculations.

11.18  Draw a circuit that allows an LSTTL gate to drive a 74HC gate. Explain briefly how it works.

11.19  How many LSTTL loads (e.g., 74LS00) can a 74HC00A NAND gate drive? Use data sheet parameters to support your answer. Assume V\textsubscript{CC} = 4.5 V. Show all calculations.

Section 11.7  Internal Circuitry of TTL Gates

11.20  In what logic state is an open TTL input? Why?

11.21  Briefly describe the operation of the TTL open-collector inverter shown in Figure 11.20. What is the purpose of the diode?

11.22  Briefly explain the operation of a multiple-emitter input transistor used in a TTL NAND gate. Describe how the transistor responds to various combinations of HIGH and LOW inputs.

11.23  Draw a wired-AND circuit consisting of three open-collector NAND gates and an output pull-up resistor. The gate inputs are as follows:

Gate 1: Inputs A, B
Gate 2: Inputs C, D
Gate 3: Inputs E, F

Write the Boolean function of the circuit output.

11.24  Calculate the minimum value of the pull-up resistor if the circuit drawn in Problem 11.23 is to drive a logic gate having input current I\textsubscript{IL} = 0.8 mA and the NAND gates can sink 12 mA in the LOW output state. (Assume that V\textsubscript{OL} = 0.4 V.)
11.25 Draw a circuit consisting only of open-collector gates whose Boolean expression is the product-of-sums expression

\]

11.26 Is an open-collector TTL output likely to be damaged if shorted to ground? Why or why not?

11.27 Is an open-collector TTL output likely to be damaged if shorted to \(V_{CC}\)? Why or why not?

11.28 Draw the totem pole output of a standard TTL gate.

11.29 Refer to the TTL NAND gate in Figure 11.34.
   a. Why are \(Q_3\) and \(Q_4\) never on at the same time (ideally)?
   b. How does switching noise originate in a totem pole output? How can the problem be controlled?

11.30 Explain briefly why two totem pole outputs should not be connected together.

11.31 Two LED driver circuits are shown in Figure 11.70. For each circuit, calculate the current flowing when the LED is ON. Calculate the ratio between the LED ON current and \(I_{OL}\) or \(I_{OH}\) of the inverter, whichever is appropriate for each circuit. State which is the best connection for LED driving and explain why.

11.32 Calculate the current flowing when the lamp in Figure 11.71 is illuminated. Choose one of the following devices as a suitable driver: 74LS04, 74LS05, 74LS06, 74LS16. Explain your choice. (Data sheets for these devices are found in Appendix C.)

Section 11.8 Internal Circuitry of CMOS Gates

11.33 State several precautions that should be taken to prevent electrostatic damage to MOSFET circuits.

11.34 a. Draw the circuit symbols for an \(n\)-channel and a \(p\)-channel enhancement-mode MOSFET.
   b. Describe the required bias conditions for each type of MOSFET in the cutoff and ohmic regions.
   c. State the approximate channel resistance for a MOSFET in the cutoff and ohmic regions.

11.35 Draw the circuit diagram of a CMOS AND gate. Derive the truth table of the gate by analyzing the operation of all the transistors under all possible input conditions.

11.36 Repeat Problem 11.35 for a CMOS OR gate.

11.37 Figure 11.72 shows a circuit that can switch two analog signals to an automotive speedometer/tachometer. Each sensor produces an analog voltage proportional to its measured quantity. Briefly explain how these analog signals are switched to the display output circuitry.

Section 11.9 TTL and CMOS Variations

11.38 Briefly explain how a Schottky barrier diode can improve the performance of a transistor in a TTL circuit.

11.39 Is the speed-power product of a TTL gate affected by the switching frequency of its output? Explain.

11.40 Use data sheets to calculate the speed-power products of the following gates:
   a. 74LS00
   b. 74S00
   c. 74ALS00
   d. 74AS00
   e. 74HC00A (quiescent and 10 MHz)
   f. 74HCT00A (quiescent and 10 MHz)
   g. 74F00
**ANSWERS TO SECTION REVIEW PROBLEMS**

**Section 11.1**

11.1 \( V_{OH} = 2.7 \text{ V min.} \) (We cannot expect typical values for \( V_{OH} \).) \( I_{OH} = -0.4 \text{ mA} \) (The negative sign indicates that the current is leaving the gate. See Figure 11.2.)

**Section 11.2**

11.2 \( t_{pHL1} + t_{pHL2} = 20 \text{ ns} + 22 \text{ ns} = 42 \text{ ns}; t_{pLH2} = 22 \text{ ns} \)

**Section 11.3**

11.3 Source currents: \( I_{OH}, I_{L} \); sink currents: \( I_{OL}, I_{IH} \)

**Section 11.4**

11.4 CMOS draws very little current when its outputs are not switching. Since the majority of current is drawn when the outputs switch, the more often the outputs switch, the more current is drawn from the supply. This is the same as saying that power dissipation increases with frequency.

**Section 11.5**

11.5 \( V_{NH} = 1.98 \text{ V}, V_{SL} = 0.66 \text{ V} \)

**Section 11.6**

11.6 2.5 V. The interface buffer and load should have the same supply voltage so that the output voltage of the buffer and input voltage of the load are compatible.

**Section 11.7a**

11.7 a. Provision of logic HIGH when output transistor is OFF

b. Limitation of \( I_{OL} \) when output transistor is ON

**Section 11.7b**

11.8 \( R_{ext} = 592 \Omega \). Minimum standard value: 680 \( \Omega \)

**Section 11.7c**

11.9 When the output is HIGH, current flows to ground through a low-impedance path, causing \( I_{OH} \) to exceed its rating.
Section 11.7d

11.10 The diode allows the base of $Q_4$ to be pulled LOW through $G$, but will not allow a HIGH at $G$ to turn it on. This keeps both output transistors OFF in the high-impedance state and allows them to be in opposite states when the output is enabled.

Section 11.7e

11.11 Noninverting gates are actually double-inverting gates. They require an extra transistor stage to cancel the inversion introduced by NAND or NOR transistor logic.

Section 11.8a

11.12 The thin oxide layer in the gate region can be damaged by overvoltage, such as that caused by electrostatic discharge. If the oxide layer is damaged, it may no longer insulate the gate terminal from the MOSFET substrate, which causes the transistor to malfunction.

Section 11.8b

11.13 It allows complementary operation with an $n$-channel MOSFET. Specifically, a gate voltage of 0 V turns OFF an $n$-channel device having a grounded source. The same voltage turns ON the $p$-channel device whose source is tied to $V_{CC}$. It does so by making the $p$-channel gate-source voltage more negative than the required threshold.

Section 11.9

Interfacing Analog and Digital Circuits

OUTLINE

12.1 Analog and Digital Signals
12.2 Digital-to-Analog Conversion
12.3 Analog-to-Digital Conversion
12.4 Data Acquisition

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

• Define the terms “analog” and “digital” and give examples of each.
• Explain the sampling of an analog signal and the effects of sampling frequency and quantization on the quality of the converted digital signal.
• Draw the block diagram of a generic digital-to-analog converter (DAC) and circuits of a weighted resistor DAC and an R-2R ladder DAC.
• Calculate analog output voltages of a DAC, given a reference voltage and a digital input code.
• Configure an MC1408 integrated circuit DAC for unipolar and bipolar output, and calculate output voltage from known component values, reference voltage, and digital inputs.
• Describe important performance specifications of a digital-to-analog converter.
• Draw the circuit for a flash analog-to-digital converter (ADC) and briefly explain its operation.
• Define “quantization error” and describe its effect on the output of an ADC.
• Explain the basis of the successive approximation ADC, draw its block diagram, and briefly describe its operation.
• Describe the operation of an integrator with constant input voltage.
• Draw the block diagram of a dual slope (integrating) ADC and briefly explain its operation.
• Explain the necessity of a sample and hold circuit in an ADC and its operation.
• State the Nyquist sampling theorem and do simple calculations of maximum analog frequencies that can be accurately sampled by an ADC system.
• Describe the phenomenon of aliasing and explain how it arises and how it can be remedied.
• Interface an ADC0808 analog-to-digital converter to a CPLD-based state machine.
• Design a 4-channel data acquisition system, including an ADC0808 analog-to-digital converter and a CPLD-based state machine.
Electronic circuits and signals can be divided into two main categories: analog and digital. Analog signals can vary continuously throughout a defined range. Digital signals take on specific values only, each usually described by a binary number.

Many phenomena in the world around us are analog in nature. Sound, light, heat, position, velocity, acceleration, time, weight, and volume are all analog quantities. Each of these can be represented by a voltage or current in an electronic circuit. This voltage or current is a copy, or analog, of the sound, velocity, or whatever.

We can also represent these physical properties digitally, that is, as a series of numbers, each describing an aspect of the property, such as its magnitude at a particular time. To translate between the physical world and a digital circuit, we must be able to convert analog signals to digital and vice versa.

We will begin by examining some of the factors involved in the conversion between analog and digital signals, including sampling rate, resolution, range, and quantization. We will then examine circuits for converting digital signals to analog, since these have a fairly standard form. Analog-to-digital conversion has no standard method. We will study several of the most popular: simultaneous (flash) conversion, successive approximation, and dual slope (integrating) conversion.

12.1 Analog and Digital Signals

**KEY TERMS**

**Continuous** Smoothly connected. An unbroken series of consecutive values with no instantaneous changes.

**Discrete** Separated into distinct segments or pieces. A series of discontinuous values.

**Analog** A way of representing some physical quantity, such as temperature or velocity, by a proportional continuous voltage or current. An analog voltage or current can have any value within a defined range.

**Digital** A way of representing a physical quantity by a series of binary numbers. A digital representation can have only specific discrete values.

**Analog-to-digital converter** A circuit that converts an analog signal at its input to a digital code. (Also called an A-to-D converter, A/D converter, or ADC.)

**Digital-to-analog converter** A circuit that converts a digital code at its input to an analog voltage or current. (Also called a D-to-A converter, D/A converter, or DAC.)

Electronic circuits are tools to measure and change our environment. Measurement instruments tell us about the physical properties of objects around us. They answer questions such as “How hot is this water?”, “How fast is this car going?”, and “How many electrons are flowing past this point per second?” These data can correspond to voltages and currents in electronic instruments.

If the internal voltage of an instrument is directly proportional to the quantity being measured, with no breaks in the proportional function, we say that it is an analog voltage. Like the property being measured, the voltage can vary continuously throughout a defined range.

For example, sound waves are continuous movements in the air. We can plot these movements mathematically as a sum of sine waves of various frequencies. The patterns of magnetic domains on an audio tape are analogous to the sound waves that produce them and electromagnetically represent the same mathematical functions. When the tape is played, the playback head produces a voltage that is also proportional to the original sound waves. This analog audio voltage can be any value between the maximum and minimum voltages of the audio system amplifier.
If an instrument represents a measured quantity as a series of binary numbers, the representation is **digital**. Since the binary numbers in a circuit necessarily have a fixed number of bits, the instrument can represent the measured quantities only as having specific **discrete** values.

A compact disc stores a record of sound waves as a series of binary numbers. Each number represents the amplitude of the sound at a particular time. These numbers are decoded and translated into analog sound waves upon playback. The values of the stored numbers (the encoded sound information) are limited by the number of bits in each stored digital “word.”

The main advantage of a digital representation is that it is not subject to the same distortions as an analog signal. Nonideal properties of analog circuits, such as stray inductance and capacitance, amplification limits, and unwanted phase shifts, all degrade an analog signal. Storage techniques, such as magnetic tape, can also introduce distortion due to the nonlinearity of the recording medium.

Digital signals, on the other hand, do not depend on the shape of a waveform to preserve the encoded information. All that is required is to maintain the integrity of the logic HIGHs and LOWs of the digital signal. Digital information can be easily moved around in a circuit and stored in a latch or on some magnetic or optical medium. When the information is required in analog form, the analog quantity is reproduced as a new copy every time it is needed. Each copy is as good as any previous one. Distortions are not introduced between copy generations, as is the case with analog copying techniques, unless the constituent bits themselves are changed.

Digital circuits give us a good way of measuring and evaluating the physical world, with many advantages over analog methods. However, most properties of the physical world are analog. How do we bridge the gap?

We can make these translations with two classes of circuits. An **analog-to-digital converter** accepts an analog voltage or current at its input and produces a corresponding digital code. A **digital-to-analog converter** generates a unique analog voltage or current for every combination of bits at its inputs.

### Sampling an Analog Voltage

#### Key Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample</td>
<td>An instantaneous measurement of an analog voltage, taken at regular intervals.</td>
</tr>
<tr>
<td><strong>Sampling frequency</strong></td>
<td>The number of samples taken per unit time of an analog signal.</td>
</tr>
<tr>
<td>Quantization</td>
<td>The number of bits used to represent an analog voltage as a digital number.</td>
</tr>
<tr>
<td>Resolution</td>
<td>The difference in analog voltage corresponding to two adjacent digital codes. Analog step size.</td>
</tr>
</tbody>
</table>

Before we examine actual D/A and A/D converter circuits, we need to look at some of the theoretical issues behind the conversion process. We will look at the concept of **sampling** an analog signal and discover how the **sampling frequency** affects the accuracy of the digital representation. We will also examine **quantization**, or the number of bits in the digital representation of the analog sample, and its effect on the quality of a digital signal.

Figure 12.1 shows a circuit that converts an analog signal (a sine pulse) to a series of 4-bit digital codes, then back to an analog output. The analog input and output voltages are shown on the two graphs.

There are two main reasons why the output is not a very good copy of the input. First, the number of bits in the digital representation is too low. Second, the input signal is not
sampled frequently enough. To help us understand the effect of each of these factors, let us examine the conversion process in more detail.

The analog input signal varies between 0 and 8 volts. This is evenly divided into 16 ranges, each corresponding to a 4-bit digital code (0000 to 1111). We say that the signal is quantized into 4 bits. The **resolution**, or analog step size, for a 4-bit quantization is 8 V/16 steps = 0.5 V/step. Table 12.1 shows the codes for each analog range.

### Table 12.1 4-bit Digital Codes for 0 to 8 V Analog Range

<table>
<thead>
<tr>
<th>Analog Voltage</th>
<th>Digital Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00–0.25</td>
<td>0000</td>
</tr>
<tr>
<td>0.25–0.75</td>
<td>0001</td>
</tr>
<tr>
<td>0.75–1.25</td>
<td>0010</td>
</tr>
<tr>
<td>1.25–1.75</td>
<td>0011</td>
</tr>
<tr>
<td>1.75–2.25</td>
<td>0100</td>
</tr>
<tr>
<td>2.25–2.75</td>
<td>0101</td>
</tr>
<tr>
<td>2.75–3.25</td>
<td>0110</td>
</tr>
<tr>
<td>3.25–3.75</td>
<td>0111</td>
</tr>
<tr>
<td>3.75–4.25</td>
<td>1000</td>
</tr>
<tr>
<td>4.25–4.75</td>
<td>1001</td>
</tr>
<tr>
<td>4.75–5.25</td>
<td>1010</td>
</tr>
<tr>
<td>5.25–5.75</td>
<td>1011</td>
</tr>
<tr>
<td>5.75–6.25</td>
<td>1100</td>
</tr>
<tr>
<td>6.25–6.75</td>
<td>1101</td>
</tr>
<tr>
<td>6.75–7.25</td>
<td>1110</td>
</tr>
<tr>
<td>7.25–8.00</td>
<td>1111</td>
</tr>
</tbody>
</table>
The analog input is sampled and converted at the beginning of each time division on the graph. The 4-bit digital code does not change until the next conversion, 1 ms later. This is the same as saying that the system has a sampling frequency of 1 kHz \( f = 1/T = 1/(1 \text{ ms}) = 1 \text{ kHz} \).

Table 12.2 shows the digital codes for samples taken from \( t = 0 \) to \( t = 18 \) ms. The analog voltages in Table 12.2 are calculated by the formula

\[
V_{\text{analog}} = 8 \text{ V} \sin (t \times (10^3/\text{ms}))
\]

For example at \( t = 2 \) ms, \( V_{\text{analog}} = 8 \text{ V} \sin (2 \text{ ms} \times (10^3/\text{ms})) = 8 \text{ V} \sin (20^\circ) = 2.736 \text{ V} \).

The calculated analog values are compared to the voltage ranges in Table 12.1 and assigned the appropriate code. The value 2.736 V is between 2.25 V and 2.75 V and therefore is assigned the 4-bit value of 0101.

The digital-to-analog converter in Figure 12.1 continuously converts the digital codes to their analog equivalents. Each code produces an analog voltage whose value is the midpoint of the range corresponding to that code.

For this particular analog waveform, the A/D converter introduces the greatest inaccuracy at the peak of the waveform, where the magnitude of the input voltage changes the least per unit time. There is not sufficient difference between the values of successive analog samples to map them into unique codes. As a result, the output waveform flattens out at the top.

This is the consequence of using a 4-bit quantization, which allows only 16 different analog ranges in the signal. By using more bits, we could divide the analog signal into a greater number of smaller ranges, allowing more accurate conversion of a signal having small changes in amplitude. For example, an 8-bit code would give us 256 steps (a resolution of 8 V/256 = 31.25 mV). This would yield the code assignments shown in Table 12.3. Note that for an 8-bit code, there is a unique value for every sampled voltage.

Figure 12.2 shows how different levels of quantization affect the accuracy of a digital representation of an analog signal. The analog input is a sine wave, converted to digital...
The graphs show the analog input and three analog outputs, each of which has been sampled 28 times per cycle, but with different quantizations. The corresponding digital codes range from a maximum negative value of $n$ 0s to a maximum positive value of $n$ 1s for an $n$-bit quantization (e.g., for a 4-bit quantization, maximum negative = 0000, maximum positive = 1111).

The first output signal has an infinite number of bits in its quantization. Even the smallest analog change between samples has a unique code. This ideal case is not attainable, since a digital circuit always has a finite number of bits. We can see from the codes in Table 12.3 that an 8-bit quantization is sufficient to give unique codes for this waveform. An infinite quantization implies that the resolution is small enough that each sampled voltage can be represented, not only by a unique code, but as its exact value rather than a point within a range.

The 4-bit and 3-bit quantizations in the next two graphs show progressively worse representation of the original signal, especially at the peaks. The change in analog voltage is too small for each sample to have a unique code at these low quantizations.

Figure 12.3 shows how the digital representation of a signal can be improved by increasing its sampling frequency. It shows an analog signal and three analog waveforms resulting from an analog-digital-analog conversion. All waveforms have infinite quantization, but different numbers of samples in the analog-to-digital conversion. As the number of samples decreases, the output waveform becomes a poorer copy of the input.

In general, the sampling frequency affects the horizontal resolution of the digitized waveform and the quantization affects the vertical resolution.
SECTION 12.1 REVIEW PROBLEM

12.1 An analog signal has a range of 0 to 24 mV. The range is divided into 32 equal steps for conversion to a series of digital codes. How many bits are in the resultant digital codes? What is the resolution of the A/D converter?

12.2 Digital-to-Analog Conversion

KEY TERM

**Full scale** The maximum analog reference voltage or current of a digital-to-analog converter.

Figure 12.4 shows the block diagram of a generalized digital-to-analog converter. Each digital input switches a proportionally weighted current on or off, with the current for the MSB being the largest. The second MSB produces a current half as large. The current generated by the third MSB is one quarter of the MSB current, and so on.

These currents all sum at the operational amplifier’s (op amp’s) inverting input. The total analog current for an \( n \)-bit circuit is given by:

\[
I_a = \frac{b_{n-1}2^{n-1} + \cdots + b_22^2 + b_12^1 + b_02^0}{2^n} I_{ref}
\]

The bit values \( b_0, b_1, \ldots, b_n \) can be only 0 or 1. The function of each bit is to include or exclude a term from the general expression.
The op amp acts as a current-to-voltage converter. The analysis, illustrated in Figure 12.4b, is the same as for an inverting op amp circuit with a constant input current.

The input impedance of the op amp is the impedance between its inverting (−) and noninverting (+) terminals. This value is very large, on the order of 2 MΩ. If this is large compared to other circuit resistances, we can neglect the op amp input current, $I_{in}$.

This implies that the voltage drop across the input terminals is very small; the inverting and noninverting terminals are at approximately the same voltage. Since the noninverting input is grounded, we can say that the inverting input is “virtually grounded.”

Current $I_F$ flows in the feedback loop, through resistor $R_F$. Since $I_a - I_m - I_F = 0$ and $I_m = 0$, then $I_F = I_a$. By Ohm’s law, the voltage across $R_F$ is given by $V_F = I_a R_F$. The feedback resistor is connected to the output at one end and to virtual ground at the other. The op amp output voltage is measured with respect to ground. The two voltages are effectively in parallel. Thus, the output voltage is the same as the voltage across the feedback resistor, with a polarity opposite to $V_F$, calculated above.

$$V_a = -V_F = -I_a R_F = \frac{-b_{n-1}2^{n-1} + \cdots + b_22^2 + b_12^1 + b_02^0}{2^n} I_{ref} R_F$$

The range of analog output voltage is set by choosing the appropriate value of $R_F$.

**EXAMPLE 12.1**

Write the expression for analog current, $I_a$, of a 4-bit D/A converter. Calculate values of $I_a$ for input codes $b_3b_2b_1b_0 = 0000, 0001, 1000, 1010$, and $1111$, if $I_{ref} = 1$ mA.

**Solution** The analog current of a 4-bit converter is:

$$I_a = \frac{b_32^3 + b_22^2 + b_12^1 + b_02^0}{2^4 I_{ref}}$$
1.2 \cdot \text{Digital-to-Analog Conversion} \quad \text{573}

\begin{align*}
\frac{8b_3 + 4b_2 + 2b_1 + b_0}{16} (1 \text{ mA})
\end{align*}

\begin{align*}
b_3b_2b_1b_0 &= 0000, \quad I_a = \frac{(0 + 0 + 0 + 0)(1 \text{ mA})}{16} = 0 \\
b_3b_2b_1b_0 &= 0001, \quad I_a = \frac{(0 + 0 + 0 + 1)(1 \text{ mA})}{16} = \frac{1 \text{ mA}}{16} = 62.5 \mu\text{A} \\
b_3b_2b_1b_0 &= 1000, \quad I_a = \frac{(8 + 0 + 0 + 0)(1 \text{ mA})}{16} = \frac{8 \text{ mA}}{16} = 0.5 \text{ mA} \\
b_3b_2b_1b_0 &= 1010, \quad I_a = \frac{(8 + 0 + 2 + 0)(1 \text{ mA})}{16} = \frac{10 \text{ mA}}{16} = 0.625 \text{ mA} \\
b_3b_2b_1b_0 &= 1111, \quad I_a = \frac{(8 + 4 + 2 + 1)(1 \text{ mA})}{16} = \frac{15 \text{ mA}}{16} = 0.9375 \text{ mA}
\end{align*}

Example 12.1 suggests an easy way to calculate D/A analog current. \(I_a\) is a fraction of the reference current \(I_{\text{ref}}\). The denominator of the fraction is \(2^n\) for an \(n\)-bit converter. The numerator is the decimal equivalent of the binary input. For example, for input \(b_3b_2b_1b_0 = 0111\), \(I_a = (7/16)(I_{\text{ref}})\).

Note that when \(b_3b_2b_1b_0 = 1111\), the analog current is not the full value of \(I_{\text{ref}}\) but 15/16 of it. This is one least significant bit less than full scale.

This is true for any D/A converter, regardless of the number of bits. The maximum analog current for a 5-bit converter is 31/32 of full scale. In an 8-bit converter, \(I_a\) cannot exceed 255/256 of full scale. This is because the analog value 0 has its own code. An \(n\)-bit converter has \(2^n\) input codes, ranging from 0 to \(2^n - 1\).

The difference between the full scale (FS) of a digital-to-analog converter and its maximum output is the resolution of the converter. Since the resolution is the smallest change in output, equivalent to a change in the least significant bit, we can define the maximum output as \(FS - 1\) LSB. (As an example, in the case of an 8-bit converter \(FS - 1\) LSB = 255/256 \(I_{\text{ref}}\).)

\section*{SECTION 12.2A REVIEW PROBLEM}

12.2 Calculate the range of analog voltage of a 4-bit D/A converter having values of \(I_{\text{ref}} = 1 \text{ mA}\) and \(R_F = 10 \text{ k}\Omega\). Repeat the calculation for an 8-bit D/A converter.

\textbf{Weighted Resistor D/A Converter}

Figure 12.5 shows the circuit of a 4-bit weighted resistor D/A converter. The heart of this circuit is a parallel network of binary-weighted resistors. The MSB has a resistor value of \(R\). Successive branches have resistor values that double with each bit: \(2R\), \(4R\), and \(8R\). The branch currents decrease by halves with each descending bit value.

\textbf{FIGURE 12.5} Weighted Resistor D-to-A Converter
The bit inputs, \( b_3, b_2, b_1, \) and \( b_0, \) are either 0 V or \( V_{\text{ref}}. \) When the corresponding bits are HIGH, the branch currents are:

\[
\begin{align*}
I_3 &= V_{\text{ref}} / R \\
I_2 &= V_{\text{ref}} / 2R \\
I_1 &= V_{\text{ref}} / 4R \\
I_0 &= V_{\text{ref}} / 8R
\end{align*}
\]

The sum of branch currents gives us the analog current \( I_a. \)

\[
I_a = \frac{b_3 V_{\text{ref}}}{R} + \frac{b_2 V_{\text{ref}}}{2R} + \frac{b_1 V_{\text{ref}}}{4R} + \frac{b_0 V_{\text{ref}}}{8R}
\]

\[
= \left[ \frac{b_3}{1} + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8} \right] \frac{V_{\text{ref}}}{R}
\]

We can calculate the analog voltage by Ohm’s law:

\[
V_a = -I_a R_F = -I_a (R/2)
\]

\[
= -\left[ \frac{b_3}{1} + \frac{b_2}{2} + \frac{b_1}{4} + \frac{b_0}{8} \right] \frac{V_{\text{ref}}}{R} \times \frac{R}{2}
\]

\[
= -\left[ \frac{b_3}{2} + \frac{b_2}{4} + \frac{b_1}{8} + \frac{b_0}{16} \right] \frac{V_{\text{ref}}}{R}
\]

The choice of \( R_F = R/2 \) makes the analog output a binary fraction of \( V_{\text{ref}}. \)

### EXAMPLE 12.2

Calculate the analog voltage of a weighted resistor D/A converter when the binary inputs have the following values: \( b_3 b_2 b_1 b_0 = 0000, 1000, 1111. \) \( V_{\text{ref}} = 5 \) V.

**Solution**

\[
\begin{align*}
&b_3 b_2 b_1 b_0 = 0000 \\
&V_a = -\left[ \frac{0}{2} + \frac{0}{4} + \frac{0}{8} + \frac{0}{16} \right] \frac{V_{\text{ref}}}{R} = 0
\end{align*}
\]

\[
\begin{align*}
&b_3 b_2 b_1 b_0 = 1000 \\
&V_a = -\left[ \frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{0}{16} \right] \frac{V_{\text{ref}}}{R} = -\frac{1}{2} \ (5 \text{ V}) = -2.5 \text{ V}
\end{align*}
\]

\[
\begin{align*}
&b_3 b_2 b_1 b_0 = 1111 \\
&V_a = -\left[ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \right] \frac{V_{\text{ref}}}{R} = -\frac{15}{16} \ (5 \text{ V}) = -4.69 \text{ V}
\end{align*}
\]

The weighted resistor DAC is seldom used in practice. One reason is the wide range of resistor values required for a large number of bits. Another reason is the difficulty in obtaining resistors whose values are sufficiently precise.

A 4-bit converter needs a range of resistors from \( R \) to \( 8R. \) If \( R = 1 \) k\( \Omega, \) then \( 8R = 8 \) k\( \Omega. \) An 8-bit DAC must have a range from \( 1 \) k\( \Omega \) to \( 128 \) k\( \Omega. \) Standard value resistors are specified to two significant figures; there is no standard 128-k\( \Omega \) resistor. We would need to use relatively expensive precision resistors for any value having more than two significant figures.
Another DAC circuit, the R-2R ladder, is more commonly used. It requires only two values of resistance for any number of bits.

SECTION 12.2B REVIEW PROBLEM

12.3 The resistor for the MSB of a 12-bit weighted resistor D/A converter is 1 kΩ. What is the resistor value for the LSB?

R-2R Ladder D/A Converter

Figure 12.6 shows the circuit of an R-2R ladder D/A converter. Like the weighted resistor DAC, this circuit produces an analog current that is the sum of binary-weighted currents. An operational amplifier converts the current to a proportional voltage.

The circuit requires an operational amplifier with a high slew rate. Slew rate is the rate at which the output changes after a step change at the input. If a standard op amp (e.g., 741C) is used, the circuit will not accurately reproduce changes introduced by large changes in the digital input.

The method of generating the analog current for an R-2R ladder DAC is a little less obvious than for the weighted resistor DAC. As the name implies, the resistor network is a ladder that has two values of resistance, one of which is twice the other. This circuit is expandable to any number of bits simply by adding one resistor of each value for each bit.

The analog output is a function of the digital input and the value of the op amp feedback resistor. If logic HIGH = $V_{ref}$, logic LOW = 0 V, and $R_F = R$, the analog output is given by:

$$V_a = -\left[\frac{b_3}{2} + \frac{b_2}{4} + \frac{b_1}{8} + \frac{b_0}{16}\right] V_{ref}$$

One way to analyze this circuit is to replace the R-2R ladder with its Thévenin equivalent circuit and treat the circuit as an inverting amplifier. Figure 12.7 shows the equivalent circuit for the input code $b_3b_2b_1b_0 = 1000$.

Figure 12.8a shows the equivalent circuit of the R-2R ladder when $b_3b_2b_1b_0 = 1000$. All LOW bits are grounded, and the HIGH bit connects to $V_{ref}$. We can reduce the network to two resistors by using series and parallel combinations.

The two resistors at the far left of the ladder are in parallel: $2R \parallel 2R = R$. This equivalent resistance is in series with another: $R + R = 2R$. The new resistance is in parallel with yet another: $2R \parallel 2R = R$. We continue this process until we get the simplified circuit shown in Figure 12.8b.
Next, we find the Thévenin equivalent of the simplified circuit. To find $E_{Th}$, calculate the terminal voltage of the circuit, using voltage division.

$$E_{Th} = \frac{2R}{2R + 2R} V_{ref} = \frac{V_{ref}}{2}$$

$R_{Th}$ is the resistance of the circuit, as measured from the terminals, with the voltage source short-circuited. Its value is that of the two resistors in parallel: $R_{Th} = 2R \parallel 2R = R$. 

FIGURE 12.7
Equivalent Circuit for $b_3b_2b_1b_0 = 1000$

FIGURE 12.8
R-2R Circuit Analysis for $b_3b_2b_1b_0 = 1000$
The value of the Thévenin resistance of the R-2R ladder will always be $R$, regardless of the digital input code. This is because we short-circuit any voltage sources when we make this calculation, which grounds the corresponding bit resistors. The other resistors are already grounded by logic LOWs. We reduce the circuit to a single resistor, $R$, by parallel and series combinations of $R$ and $2R$. Figure 12.9 shows the equivalent circuit.

$$R_{Th} = R$$

**FIGURE 12.9**
Equivalent Circuit for Calculating $R_{Th}$

On the other hand, the value of $E_{Th}$ will be different for each different binary input. It will be the sum of binary fractions of the full-scale output voltage, as previously calculated for the generic DAC.

Similar analysis of the R-2R ladder shows that when $b_3b_2b_1b_0 = 0100$, $V_o = -V_{ref}/4$, when $b_3b_2b_1b_0 = 0010$, $V_o = -V_{ref}/8$, and when $b_3b_2b_1b_0 = 0001$, $V_o = -V_{ref}/16$.

If two or more bits in the R-2R ladder are active, each bit acts as a separate voltage source. Analysis becomes much more complicated if we try to solve the network as we did for one active bit.

There is no need to go through a tedious circuit analysis to find the corresponding analog voltage. We can simplify the process greatly by applying the Superposition theorem. This theorem states that the effect of two or more sources in a network can be determined by calculating the effect of each source separately and adding the results.

The Superposition theorem suggests a generalized equivalent circuit of the R-2R ladder DAC. This is shown in Figure 12.10. A Thévenin equivalent source and resistance corresponds to each bit. The source and resistance are switched in and out of the circuit, depending on whether or not the corresponding bit is active.

**FIGURE 12.10**
Equivalent Circuit of R-2R DAC
This model is easily expanded. The source for the most significant bit always has the value $V_{\text{ref}}/2$. Each source is half the value of the preceding bit. Thus, for a 5-bit circuit, the source for the least significant bit has a value of $V_{\text{ref}}/32$. An 8-bit circuit has an LSB equivalent source of $V_{\text{ref}}/256$.

**EXAMPLE 12.3**

A 4-bit DAC based on an R-2R ladder has a reference voltage of 10 volts. Calculate the analog output voltage, $V_a$, for the following input codes:

a. 0000
b. 1000
c. 0100
d. 1100

**Solution**

a. $V_a = -(0/16) V_{\text{ref}} = 0 \text{ V}$  
b. $V_a = -(8/16) V_{\text{ref}} = -(1/2) V_{\text{ref}} = -5 \text{ V}$

c. $V_a = -(4/16) V_{\text{ref}} = -(1/4) V_{\text{ref}} = -2.5 \text{ V}$

d. $V_a = -(12/16) V_{\text{ref}} = -(3/4) V_{\text{ref}} = -7.5 \text{ V}$

**EXAMPLE 12.4**

Calculate the output voltage of an 8-bit DAC based on an R-2R ladder for the following input codes. What general conclusion can be drawn about each code when compared to the solutions in Example 12.3?

a. 00000000  
b. 10000000  
c. 01000000  
d. 11000000

**Solution**

a. $V_a = -(0/256) V_{\text{ref}} = 0 \text{ V}$  
b. $V_a = -(128/256) V_{\text{ref}} = -(1/2) V_{\text{ref}} = -5 \text{ V}$

c. $V_a = -(64/256) V_{\text{ref}} = -(1/4) V_{\text{ref}} = -2.5 \text{ V}$

d. $V_a = -(192/256) V_{\text{ref}} = -(3/4) V_{\text{ref}} = -7.5 \text{ V}$

In general, a DAC input code consisting of 1 followed by all 0s generates an output value of $\frac{1}{2}$ full scale. A code of 01 followed by all 0s yields an output of $\frac{1}{4}$ full scale. An output of 11 followed by all 0s generates an output of $\frac{3}{4}$ full scale.

**SECTION 12.2C REVIEW PROBLEM**

12.4 Calculate $V_a$ for an 8-bit R-2R ladder DAC when the input code is 10100001. Assume that $V_{\text{ref}}$ is 10 V.

**MC1408 Integrated Circuit D/A Converter**

**KEY TERM**

Multiplying DAC  
A DAC whose output changes linearly with a change in DAC reference voltage.
A common and inexpensive DAC is the MC1408 8-bit multiplying digital-to-analog converter. This device also goes by the designation DAC0808. A logic symbol for this DAC is shown in Figure 12.11.

**Figure 12.11**
MC1408 DAC

The output current, $I_o$, flows into pin 4. $I_o$ is a binary fraction of the current flowing into pin 14, as specified by the states of the digital inputs. Other inputs select the range of output voltage and allow for phase compensation.

Figure 12.12 shows the MC1408 in a simple D/A configuration. $R_{14}$ and $R_{15}$ are approximately equal. Pin 14 is approximately at ground potential. This implies:

1. That the DAC reference current can be calculated using only $V_{ref}$ and $R_{14}$ ($I_{ref} = V_{ref} / R_{14}$)
2. That $R_{15}$ is not strictly necessary in the circuit. (It is used primarily to stabilize the circuit against temperature drift.)

The reference voltage must be set up so that current flows into pin 14 and out of pin 15. Thus, $V_{ref}$ must be positive with respect to $V_{ref}$. (It is permissible to ground pin 14 if pin 15 is at a negative voltage.)

$I_o$ is given by:

$$I_o = \left[ \frac{b_7}{2} + \frac{b_6}{4} + \frac{b_5}{8} + \frac{b_4}{16} + \frac{b_3}{32} + \frac{b_2}{64} + \frac{b_1}{128} + \frac{b_0}{256} \right] \frac{V_{ref}}{R_{14}}$$

Since the output is proportional to $V_{ref}$, we refer to the MC1408 as a multiplying DAC.

$I_o$ should not exceed 2 mA. We calculate the output voltage by Ohm’s law: $V_o = -I_o R_L$. The output voltage is negative because current flows from ground into pin 4.

The open pin on the Range input allows the output voltage dropped across $R_L$ to range from $0.4$ V to $5.0$ V without damaging the output circuit of the DAC. If the Range input is grounded, the output can range from $+0.4$ to $-0.55$ V. The lower voltage range allows the output to switch about four times faster than it can in the higher range.
EXAMPLE 12.5

The DAC circuit in Figure 12.12 has the following component values: \( R_{14} = R_{15} = 5.6 \ \text{k} \Omega \), \( R_L = 3.3 \ \text{k} \Omega \), \( V_{\text{ref}} (+) \) is +8 V, and \( V_{\text{ref}} (-) \) is grounded.

Calculate the value of \( V_o \) for each of the following input codes: \( b_7b_6b_5b_4b_3b_2b_1b_0 = 00000000, 00000001, 10000000, 10100000, 11111111 \).

What is the resolution of this DAC?

Solution  

First, calculate the value of \( I_{\text{ref}} \).

\[
I_{\text{ref}} = \frac{V_{\text{ref}} (+)}{R_{14}} = \frac{8 \text{ V}}{5.6 \ \text{k} \Omega} = 1.43 \text{ mA}
\]

Calculate the output current by using the binary fraction for each code. Multiply \(-I_o\) by \( R_L \) to get the output voltage.

\[
b_7b_6b_5b_4b_3b_2b_1b_0 = 00000000 \\
I_o = 0, \ V_o = 0 \\
b_7b_6b_5b_4b_3b_2b_1b_0 = 00000001 \\
I_o = (1/256) (1.43 \text{ mA}) = 5.58 \ \mu\text{A} \\
V_o = -(5.58 \ \mu\text{A})(3.3 \ \text{k} \Omega) = -18.4 \text{ mV} \\
b_7b_6b_5b_4b_3b_2b_1b_0 = 10000000 \\
I_o = (1/2) (1.43 \text{ mA}) = 714 \ \mu\text{A} \\
V_o = -(714 \ \mu\text{A})(3.3 \ \text{k} \Omega) = -2.36 \text{ V}
\]
\( b_7b_6b_5b_4b_3b_2b_1b_0 = 10100000 \)
\( I_o = (1/2 + 1/8)(1.43 \text{ mA}) = (5/8)(1.43 \text{ mA}) = 893 \text{ } \mu\text{A} \)
\( V_o = -(893 \text{ } \mu\text{A})(3.3 \text{ k}\Omega) = -2.95 \text{ V} \)
\( b_7b_6b_5b_4b_3b_2b_1b_0 = 11111111 \)
\( I_o = (255/256)(1.43 \text{ mA}) = 1.42 \text{ mA} \)
\( V_o = -(1.42 \text{ mA})(3.3 \text{ k}\Omega) = -4.70 \text{ V} \)

Resolution is the same as the output resulting from the LSB: 18.4 mV/step

### SECTION 12.2D REVIEW PROBLEM

12.5 The output voltage range of an MC1408 DAC can be limited by grounding the Range pin. Why would we choose to do this?

**Op Amp Buffering of MC1408**

The MC1408 DAC will not drive much of a load on its own, particularly when the Range input is grounded. We can use an operational amplifier to increase the output voltage and current. This allows us to select the lower voltage range for faster switching while retaining the ability to drive a reasonable load. The output voltage is limited only by the op amp supply voltages. We use a 34071 high slew rate op amp for fast switching.

Figure 12.13 shows such a circuit. The 0.1-\( \mu \)F capacitor decouples the +5-V supply. (The manufacturer actually recommends that the +5-V logic supply not be used as a reference voltage. It doesn’t matter for a demonstration circuit, but may introduce noise that is unacceptable in a commercial design.) The 75-pF capacitor is for phase compensation.

**FIGURE 12.13**
DAC With Op Amp Buffering
$V_a$ is positive because the voltage drop across $R_F$ is positive with respect to the virtual ground at the op amp (−) input. This feedback voltage is in parallel with (i.e., the same as) the output voltage, since both are measured from output to ground.

We can develop the formula for the analog voltage, $V_a$, in three stages:

1. Calculate the reference current:
   \[ I_{\text{ref}} = V_{\text{ref}}(+) / R_{14} \]

2. Determine the binary-weighted fraction of reference current to get DAC output current:
   \[ I_a = \left[ \frac{b_7 + b_6 + b_5 + b_4 + b_3 + b_2 + b_1 + b_0}{2^7} \right] I_{\text{ref}} \]
   \[ = \left[ \frac{b_7}{2} + \frac{b_6}{4} + \frac{b_5}{8} + \frac{b_4}{16} + \frac{b_3}{32} + \frac{b_2}{64} + \frac{b_1}{128} + \frac{b_0}{256} \right] I_{\text{ref}} \]
   \[ = \left( \frac{\text{digital code}}{256} \right) \left( \frac{V_{\text{ref}}}{R_{14}} \right) \]

3. Use Ohm’s law to calculate the op amp output voltage:
   \[ V_a = I_a R_F = \left( \frac{\text{digital code}}{256} \right) \left( \frac{R_F}{R_{14}} \right) V_{\text{ref}} \]

The resistor values in the above formulae are the total resistances for the corresponding part of the circuit. That is, $R_{14} = R_{14A} + R_{14B}$ and $R_F = R_{FB} + R_{FB}$. These both consist of a fixed and a variable resistor, which has two advantages: (a) The reference current and output voltage can be independently adjusted within a specified range by the variable resistors. (b) The resistances defining the reference and feedback currents cannot go below a specified minimum value, determined by the fixed resistance, ensuring that excessive current does not flow into the reference input or the DAC output terminal.

$V_a$ can, in theory, be any positive value less than the op amp positive supply (+12 V in this case). Any attempt to exceed this voltage makes the op amp saturate. The actual maximum value, if not the same as the op amp’s saturation voltage, depends on the values of $R_F$ and $R_{14}$.

### Example 12.6

Describe a step-by-step procedure that calibrates the DAC circuit in Figure 12.13 so that it has a reference current of 1 mA and a full-scale analog output voltage of 10 V, using only a series of measurements of the analog output voltage. When the procedure is complete, what are the resistance values in the circuit? What is the range of the DAC?

**Solution** Since the maximum output of the DAC is 1 LSB less than full scale, we must indirectly measure the full scale value. We can do so by setting the digital input code to 10000000, which exactly represents the half-scale value of output current, and making appropriate adjustments.

Set the variable feedback resistor to zero so that the output voltage is due only to the fixed feedback resistor and the feedback current. Measure the output voltage of the circuit and adjust $R_{14B}$ so that $V_a = 2.35$ volts. Ohm’s law tells us that this sets the feedback current to $I_F = 2.35 \text{ V} / 4.7 \text{ k}\Omega = 0.5 \text{ mA}$. Since the digital code is set for half scale, $I_{\text{ref}} = 2 I_F = 1 \text{ mA}$.

Adjust $R_{FB}$ so that the half-scale output voltage is 5.00 V.

After adjustment, $R_{14} = 2.7 \text{ k}\Omega + 2.3 \text{ k}\Omega = 5 \text{ k}\Omega$ and $R_F = 4.7 \text{ k}\Omega + 4.3 \text{ k}\Omega = 10 \text{ k}\Omega$. In both cases the variable resistors were selected so that their final values are about halfway through their respective ranges.

The range of the DAC is 0 V to 9.961 V.

\[(FS − 1 \text{ LSB} = 10 \text{ V} − (10 \text{ V}/256) = 9.961 \text{ V}\]
EXAMPLE 12.7

Figure 12.14 shows the circuit of an analog ramp (sawtooth) generator built from an MC1408 DAC, an op amp, and an 8-bit synchronous counter. (A ramp generator has numerous analog applications, such as sweep generation in an oscilloscope and frequency sweep in a spectrum analyzer.)

![Circuit Diagram](image)

**FIGURE 12.14**
Example 16.5
DAC Ramp Generator

Briefly explain the operation of the circuit and sketch the output waveform. Calculate the step size between analog outputs resulting from adjacent codes. Assume that the DAC is set for +6-V output when the input code is 10000000.

Calculate the output sawtooth frequency when the clock is running at 1 MHz.

**Solution**

The 8-bit counter cycles from 00000000 to 11111111 and repeats continuously. This is a total of 256 states.

The DAC output is 0 V for an input code of 00000000 and (12 V - 1 LSB) for a code of 11111111. We know this because a code of 10000000 always gives an output voltage of half the full-scale value (6 V = 12 V/2), and the maximum code gives an output that is one step less than the full-scale voltage. The step size is 12 V/256 steps = 46.9 mV/step. The DAC output advances linearly from 0 to (12 V - 1 LSB) in 256 clock cycles.

Figure 12.15 shows the analog output plotted against the number of input clock cycles. The ramp looks smooth at the scale shown. A section enlarged 32 times shows the analog steps resulting from eight clock pulses.

One complete cycle of the sawtooth waveform requires 256 clock pulses. Thus, if \( f_{CLK} = 1 \text{ MHz}, f_o = 1 \text{ MHz}/256 = 3.9 \text{ kHz}. \) (Note that if we do not use a high slew rate op amp, the sawtooth waveform will not have vertical sides.)
Bipolar Operation of MC1408

Many analog signals are bipolar, that is, they have both positive and negative values. We can configure the MC1408 to produce a bipolar output voltage. Such a circuit is shown in Figure 12.16.

We can model the bipolar DAC as shown in Figure 12.16b. The amplitude of the constant-current sink, \( I_o \), is set by \( V_{ref} (+), R_{14} \), and the binary value of the digital inputs.

\( I_s \) is determined by Ohm’s law: \( I_s = V_{ref} (+)/R_4 \).

The output voltage is set by the value of \( I_F \):

\[
V_a = I_F R_F = I_F (R_{FA} + R_{FB})
\]

By Kirchhoff’s current law:

\[
I_s + I_F - I_o = 0
\]

or

\[
I_F = I_o - I_s
\]

Thus, output voltage is given by:

\[
V_a = (I_o - I_s)R_F = I_o R_F - I_s R_F
\]

\[
= \left[ \frac{b_7}{2} + \frac{b_6}{4} + \frac{b_5}{8} + \frac{b_3}{16} + \frac{b_2}{32} + \frac{b_1}{64} + \frac{b_0}{128} + \frac{b_0}{256} \right] \frac{R_F}{R_{14}} V_{ref} - \frac{R_F}{R_4} V_{ref}
\]

\[
= \left( \frac{\text{digital code}}{256} \right) \frac{R_F}{R_{14}} V_{ref} - \frac{R_F}{R_4} V_{ref}
\]

How do we understand the circuit operation from this mathematical analysis?
The current sink, $I_o$, is a variable element. The voltage source, $V_{ref}$ (+), remains constant. To satisfy Kirchhoff’s current law, the feedback current, $I_F$, must vary to the same degree as $I_o$. Depending on the value of $I_o$ with respect to $I_r$, $I_F$ can be positive or negative.

We can get some intuitive understanding of the circuit operation by examining several cases of the equation $V_a$.
Case 1: $I_o = 0$. This corresponds to the digital input $b_7b_6b_5b_4b_3b_2b_1b_0 = 00000000$. The output voltage is:

$$V_a = (I_o - I_s)R_F = -I_s \frac{R_F}{R_4} V_{ref}$$

This is the maximum negative output voltage.

Case 2: $0 < I_o < I_s$. The term $(I_o - I_s)$ is negative, so output voltage is also a negative value.

Case 3: $I_o = I_s$. The output is given by:

$$V_a = (I_o - I_s)R_F = 0$$

The digital code for this case could be any value, depending on the setting of $R_{14}$. To set the zero-crossing to half-scale, set the digital input to 10000000 and adjust $R_{14}$ for 0 V.

Case 4: $I_o > I_s$. Since the term $(I_o - I_s)$ is positive, output voltage is positive. The largest value of $I_o$ (and thus the maximum positive output voltage) corresponds to the input code $b_7b_6b_5b_4b_3b_2b_1b_0 = 11111111$.

The magnitude of the maximum positive output voltage of this particular circuit is 2 LSB less than the magnitude of the maximum negative voltage. Specifically, $V_a = (127/128)(R_F/R_4)(V_{ref})$ if $R_4 = 2R_{14}$.

To summarize:

<table>
<thead>
<tr>
<th>Input Code</th>
<th>Output Voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000000</td>
<td>Maximum negative*</td>
</tr>
<tr>
<td>10000000</td>
<td>0 V**</td>
</tr>
<tr>
<td>11111111</td>
<td>Maximum positive</td>
</tr>
</tbody>
</table>

*As adjusted by $R_{FB}$

**As adjusted by $R_{14B}$

Negative Range:

00000000 to 01111111 (128 codes)

Positive Range:

10000001 to 11111111 (127 codes)

Zero:

00000000 (1 code)

256 codes

EXAMPLE 12.8

Calculate the values to which $R_{14}$ and $R_F$ must be set to make the output of the bipolar DAC in Figure 12.16 range from $-12 \text{ V}$ to $(+12 \text{ V} - 2 \text{ LSB})$. Describe the procedure you would use to set the circuit output as specified.

Confirm that the calculated resistor settings generate the correct values of maximum and minimum output.

Solution

Set $R_{14}$ so that the DAC circuit has an output of 0 V when input code is $b_7b_6b_5b_4b_3b_2b_1b_0 = 10000000$. We can calculate the value of $R_{14}$ as follows:

$$\frac{R_F}{2R_{14}} V_{ref} - \frac{R_F}{R_4} V_{ref} = 0$$
The first term is set by the value of the input code. Solving for \( R_{14} \), we get:

\[
\left[ \frac{1}{2R_{14}} - \frac{1}{R_4} \right] R_F \ V_{\text{ref}} = 0
\]

\[
\frac{1}{2R_{14}} - \frac{1}{R_4} = 0
\]

\[
\frac{1}{2R_{14}} = \frac{1}{R_4}
\]

\[
2R_{14} = R_4
\]

\[
R_{14} = \frac{R_4}{2} = 10 \, \text{k}\Omega/2 = 5 \, \text{k}\Omega
\]

To set the maximum negative value, set the input code to 00000000 and adjust \( R_{FB} \) for -12 V. \( R_{FB} = R_F - R_{PA} \). Solve the following equation for \( R_F \):

\[
- \frac{R_F}{R_4} V_{\text{ref}} = -12 \, \text{V}
\]

\[
- \frac{R_F}{10 \, \text{k}\Omega} (5 \, \text{V}) = -12 \, \text{V}
\]

\[
R_F = (12 \, \text{V})(10 \, \text{k}\Omega)/5 \, \text{V} = 24 \, \text{k}\Omega
\]

\[
R_{FB} = 24 \, \text{k}\Omega - 18 \, \text{k}\Omega = 6 \, \text{k}\Omega
\]

**Settings**

\( R_{14} = \frac{R_4}{2} = 5 \, \text{k}\Omega \) for zero-crossing at half-scale.

\( R_F = 24 \, \text{k}\Omega \) for output of ±12 V.

**Check Output Range**

For \( b_7b_6b_5b_4b_3b_2b_1b_0 = 00000000 \):

\[
V_a = \left[ \frac{0}{R_{14}} - \frac{1}{R_4} \right] R_F \ V_{\text{ref}} = -\frac{(24 \, \text{k}\Omega)(5 \, \text{V})}{10 \, \text{k}\Omega} = -12 \, \text{V}
\]

For \( b_7b_6b_5b_4b_3b_2b_1b_0 = 11111111 \):

\[
V_a = \left[ \frac{255}{256 R_{14}} - \frac{1}{R_4} \right] R_F \ V_{\text{ref}}
\]

\[
\left[ \frac{255}{(256)(5 \, \text{k}\Omega)} - \frac{1}{10 \, \text{k}\Omega} \right] (24 \, \text{k}\Omega)(5 \, \text{V}) = 11.906 \, \text{V}
\]

(Note: 12 V − 2 LSB = 12 V − (12 V/128) = 12 V − 94 mV = 11.906 V.)

**SECTION 12.2E REVIEW PROBLEM**

12.6 Why is the actual maximum value of an 8-bit DAC less than its reference (i.e., its apparent maximum) voltage?

**DAC Performance Specifications**

A number of factors affect the performance of a digital-to-analog converter. The major factors are briefly described below.
Monotonicity. The output of a DAC is monotonic if the magnitude of the output voltage increases every time the input code increases. Figure 12.17 shows the output of a DAC that increases monotonically and the output of a DAC that does not.

We show the output response of a DAC as a series of data points joined by a straight-line approximation. One input code produces one voltage, so there is no value that corresponds to anything in between codes, but the straight-line approximation allows us to see a trend over the whole range of input codes.

Absolute accuracy. This is a measure of DAC output voltage with respect to its expected value.

Relative accuracy. Relative accuracy is a more frequently used measurement than absolute accuracy. It measures the deviation of the actual from the ideal output voltage as a fraction of the full-scale voltage. The MC1408 DAC has a relative accuracy of $\pm \frac{1}{2}$ LSB = $\pm 0.195\%$ of full scale.

Settling time. The time required for the outputs to switch and settle to within $\pm \frac{1}{2}$ LSB when the input code switches from all 0s to all 1s. The MC1408 has a settling time of 300 ns for 8-bit accuracy, limiting its output switching frequency to $1/300$ ns = 3.33 MHz. Depending on the value of $R_a$, the output resistor, the settling time of the MC1408 may increase to as much as 1.2 $\mu$s when the Range input is open.

Gain error. Gain error primarily affects the high end of the output voltage range. If the gain of a DAC is too high, the output saturates before reaching the maximum output code. Figure 12.18 shows the effect of gain error in a 3-bit DAC. In the high gain response, the last two input codes (110 and 111) produce the same output voltage.
Linearity error. This error is present when the analog output does not follow a straight-line increase with increasing digital input codes. Figure 12.19 shows this error. A linearity error of more than $\pm \frac{1}{2}$ LSB can result in a nonmonotonic output. For example, in Figure 12.17b, the transition from 010 to 011 should result in an output change of $+1$ LSB. Instead, it results in a change of $-\frac{1}{2}$ LSB. This is an error of $-\frac{1}{2}$ LSB, resulting in a non-monotonic output.

In Figure 12.19, the code for 011 has a linearity error of $+\frac{1}{2}$ LSB and the adjacent code (100) has a linearity error of $-\frac{1}{2}$ LSB, yielding a flat output for the two codes. This makes it impossible to distinguish the value of input code for that analog output value.
Differential nonlinearity. This specification measures the difference between actual and expected step size of a DAC when the input code is changed by 1 LSB. An actual step that is smaller than the expected step can result in a nonmonotonic output.

Offset error. This error occurs when the analog output of a positive-value DAC is not 0 V when the input code is all 0s. Figure 12.20 shows the effect of offset error.

EXAMPLE 12.9
An 8-bit DAC has an output range of 0 to (+8 volts − 1 LSB). The hexadecimal value of the input is symbolized by \( x \).

a. What is the value of 1 LSB?

b. Assuming an ideal DAC, what would the output be for a binary input \( x = \text{C}0\text{H} \)?

c. If the DAC has an input of \( x = \text{00H} \) and the output voltage is 0.008 V, calculate the offset error (OE) of the DAC in LSB and as a percentage of the full scale (FS). Assume no other errors.

d. If the DAC has an input of \( x = \text{FFH} \) and the output voltage is 7.98 V, calculate the gain error (GE) of the DAC in LSB and as a percentage of the full scale (FS). Assume no other errors.

e. If the DAC output is 4 V for an input \( x = \text{80H} \) and the output is 0.015 V for an input of \( x = \text{00H} \), calculate the linearity error (LE) and offset error (OE) of the DAC in LSB and as a percentage of the full scale (FS).

Solution

a. 1 LSB = \( FS/2^n = 8 \text{ V}/2^8 = 8 \text{ V}/256 = 31.25 \text{ mV} \)

b. \( \text{C0H} = 192_{10} \)

\[ V_a = (\text{code}/256) \times 8 \text{ V} = (192/256) \times 8 \text{ V} = 6 \text{ V} \]

alternatively: \( \text{C0H} = 11000000 \), which corresponds to \( \frac{3}{4} \text{ FS} = 6 \text{ V} \)

c. When \( x = \text{00H} \), \( V_a \) should be 0 V. Therefore, \( OE = 8 \text{ mV} \).

\[ OE[\text{LSB}] = 8 \text{ mV}/31.25 \text{ mV} = 0.256 \text{ LSB} \]

\[ OE[\%\text{FS}] = (8 \text{ mV}/8 \text{ V}) \times 100\% \text{ FS} = 0.1\% \text{ FS} \]
d. When \( x = \text{FFH} \), \( V_a \) should be \((255/256) \times 8\,\text{V} = 7.969\,\text{V}\). \( GE = 7.98\,\text{V} - 7.9675\,\text{V} = 11.25\,\text{mV} \).
\[ GE[\text{LSB}] = 11.25\,\text{mV}/31.25\,\text{mV} = 0.36\,\text{LSB} \]
\[ GE[\%FS] = (11.25\,\text{mV}/8\,\text{V}) \times 100\%\,FS = 0.14\%\,FS \]
e. Without accounting for other possible errors, the output value for an input of \( 80\text{H} \) appears to be correct. However, we find an offset error of 0.015 V that must be subtracted out of all measured values in the DAC output.

Adjusted value at \( 80\text{H} = 4\,\text{V} - 0.015\,\text{V} = 3.985\,\text{V} \). This error is exactly balanced by the offset error, so both have the same value.

\[ LE[\text{LSB}] = OE[\text{LSB}] = 15\,\text{mV}/31.25\,\text{mV} = 0.48\,\text{LSB} \]
\[ LE[\%FS] = OE[\%FS] = (15\,\text{mV}/8\,\text{V}) \times 100\%\,FS = 0.188\%\,FS \]

### 12.3 Analog-to-Digital Conversion

We saw in an earlier section of this chapter that all digital-to-analog converters can be described by a generic form. This is not true of analog-to-digital converters. There are many circuits for converting analog signals to digital codes, each with its own advantages. We will look at several of the most popular.

#### Flash A/D Converter

**Flash converter (or simultaneous converter)** An analog-to-digital converter that uses comparators and a priority encoder to produce a digital code.

**Priority encoder** An encoder that will produce a binary output corresponding to the subscript of the highest-priority active input. This is usually defined as the input with the largest subscript.

Figure 12.21 shows the circuit for a 3-bit **flash analog-to-digital converter**. The circuit consists of a resistive voltage divider, seven analog comparators, a **priority encoder**, and an output latch array.

The voltage divider has a total resistance of \( 8R \). The resistors are selected to produce seven equally spaced reference voltages (\( V_{\text{ref}/16}, 3V_{\text{ref}/16}, 5V_{\text{ref}/16}, \ldots, 15V_{\text{ref}/16} \); each is separated by \( V_{\text{ref}/8} \)). Each reference voltage is fed to the inverting input of a comparator.

A comparator output goes HIGH if the voltage at its noninverting (+) input is higher than the voltage at its inverting (−) input. If the (−) input voltage is greater than the (+) input voltage, the comparator output is LOW.

The analog voltage, \( V_a \), is applied to the noninverting inputs of all comparators simultaneously. Thus, if the analog voltage exceeds the reference voltage of a particular comparator, that comparator switches its output to the HIGH state.

For most analog input values, more than one comparator will have a HIGH output. For example, the reference voltage of comparator 3 is \( (5V_{\text{ref}/16}) \). Comparator 4 has a reference voltage of \( (7V_{\text{ref}/16}) \). If the analog voltage is in the range \( (5V_{\text{ref}/16}) \leq V_a < (7V_{\text{ref}/16}) \), comparators 3, 2, and 1 all have HIGH outputs and comparators 4, 5, 6, and 7 all have LOW outputs.

The priority encoder recognizes that input \( D_3 \) is the highest-priority active input and produces the digital code 011 at its outputs. The output latches store this value when the \( CLK \) input is pulsed.
We can regularly sample an analog signal by applying a pulse waveform to the \( CLK \) input of the latch circuit. The sampling frequency is the same as the clock frequency.

The \( D_0 \) input of the priority encoder is grounded, rather than connected to a comparator output. No comparator is needed for this input; if \( V_a \leq (V_{ref}/16) \), all comparator outputs are LOW and the resulting digital code is 000.

Figure 12.22 shows the transfer characteristic of the flash ADC with a reference voltage of 8 V. The digital steps are centered on the analog voltages that are whole-number fractions (1/8, 1/4, 3/8, . . . , 7/8) of the reference voltage. The transitions are midway between these points. This is why the resistor for the least significant bit is \( R/2 \), rather than \( R \).

**FIGURE 12.21**
Flash Converter (ADC)

**FIGURE 12.22**
Transfer Characteristic of Flash ADC
The general form of this circuit has $2^n - 1$ comparators for an $n$-bit output. For example, an 8-bit flash converter has $2^8 - 1 = 255$ comparators. For any large number of bits, the circuit becomes overly complex.

The main advantage of this circuit is its speed. Since the analog input is compared to the threshold values of all possible input codes at one time, conversion occurs in one clock cycle.

### Successive Approximation A/D Converter

**Key Terms**

| **Successive approximation register** | A state machine used to generate a sequence of closer and closer binary approximations to an analog signal. |
| **Quantization error** | Inaccuracy introduced into a digital signal by the inability of a fixed number of bits to represent the exact value of an analog signal. |

Probably the most widely used type of analog-to-digital converter is the successive approximation ADC. The idea behind this type of converter is a technique a computer programmer would call “binary search.”

The analog voltage to be converted is a number within a defined range. The search technique works by narrowing down progressively smaller binary fractions of the known range of numbers.

Suppose we know that the analog voltage is a number between 0 and 255, inclusive. We can find the binary value of any randomly chosen number in this range in no more than eight guesses, or approximations, since $2^8 = 256$. Each approximation adds one more bit to the estimated digital value.

The first approximation determines which half of the range the number is in. The second test finds which quarter of the range, the third test which eighth, the fourth test which sixteenth, and so on until we run out of bits.

---

### Example 12.10

Use a binary search technique to find the value of a number in the range 0 to 255. (The number is 44.)

**Solution**

1. The number must be in the upper or lower half of the range. Cut the range in half:
   - 0–127, 128–125.
   - Is $x \geq 128$? No, $0 \leq x < 128$.
2. Cut the remaining range in half: 0–63, 64–127.
   - Is $x \geq 64$? No, $0 \leq x < 64$.
3. Cut the remaining range in half: 0–31, 32–63.
   - Is $x \geq 32$? Yes, $32 \leq x < 64$.
   - Is $x \geq 48$? No, $32 \leq x < 48$.
5. Cut the remaining range in half: 32–39, 40–47.
   - Is $x \geq 40$? Yes, $40 \leq x < 48$.
6. Cut the remaining range in half: 40–43, 44–47.
   - Is $x \geq 44$? Yes, $44 \leq x < 48$.
7. Cut the remaining range in half: 44–45, 46–47.
   - Is $x \geq 46$? No, $44 \leq x < 46$.
8. Cut the remaining range in half: 44–45.
   - Is $x \geq 45$? No, $x = 44$. 

---

The analog-to-digital conversion process and the successive approximation technique are fundamental in digital signal processing, enabling the conversion of analog signals into digital form for processing and storage.
The test criteria for each step in Example 12.10 are phrased so that the answer is always yes or no. (For example, $x \equiv 64$? can only be answered yes or no.) Assume that a 1 means yes and a 0 means no. The tests in Example 12.10 give the following sequence of results: 00101100. The decimal equivalent of this binary number is 44, our original value.

A successive approximation ADC such as the one shown in Figure 12.23 applies a similar technique. The circuit has three main components: an analog comparator, a digital-to-analog converter, and a state machine called a successive approximation register (SAR). The SAR is an 8-bit register whose bits can be set and cleared individually, according to a specific control sequence and the logic value at the output of the analog comparator.

When a pulse activates the Start Conversion input, bit $Q_7$ of the SAR is set. This makes the SAR output 10000000. The DAC converts the SAR output to an analog equivalent. When only the MSB is set, this is one half the reference voltage of the DAC.

The DAC output voltage is compared to an analog input voltage. (In effect, the SAR asks, “Is this approximation greater or less than the actual analog voltage?”)

If $V_{\text{analog}} > V_{\text{DAC}}$, the comparator output is HIGH and the MSB remains set. Otherwise, the comparator output is LOW and the MSB is cleared. The process is repeated for all bits.

After all bits have been set or cleared, the End of Conversion (EOC) output changes state. This can be used to load the final digital value into an 8-bit latch.

**EXAMPLE 12.11**

An 8-bit successive approximation ADC has an analog input voltage of 9.5 V. Describe the steps the circuit performs to generate an 8-bit digital equivalent value if the DAC in the circuit has a reference voltage of 12 V.

**Solution** Figure 12.24 shows the steps the converter performs to generate the 8-bit digital equivalent of 9.5 V. The conversion process is also summarized in Table 12.4.
**Table 12.4** 8-Bit Successive Approximation Conversion

<table>
<thead>
<tr>
<th>Bit</th>
<th>New Digital Value</th>
<th>Analog Equivalent</th>
<th>$V_{\text{analog}} \geq V_{\text{DAC}}$?</th>
<th>Comparator Output</th>
<th>Accumulated Digital Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_7$</td>
<td>110000000</td>
<td>6 V</td>
<td>Yes</td>
<td>1</td>
<td>100000000</td>
</tr>
<tr>
<td>$Q_6$</td>
<td>111000000</td>
<td>9 V</td>
<td>Yes</td>
<td>1</td>
<td>110000000</td>
</tr>
<tr>
<td>$Q_5$</td>
<td>111000000</td>
<td>10.5 V</td>
<td>No</td>
<td>0</td>
<td>110000000</td>
</tr>
<tr>
<td>$Q_4$</td>
<td>111000000</td>
<td>9.75 V</td>
<td>No</td>
<td>0</td>
<td>110000000</td>
</tr>
<tr>
<td>$Q_3$</td>
<td>110010000</td>
<td>9.375 V</td>
<td>Yes</td>
<td>1</td>
<td>110010000</td>
</tr>
<tr>
<td>$Q_2$</td>
<td>110011000</td>
<td>9.5625 V</td>
<td>Yes</td>
<td>1</td>
<td>110010000</td>
</tr>
<tr>
<td>$Q_1$</td>
<td>110010100</td>
<td>9.46875 V</td>
<td>Yes</td>
<td>1</td>
<td>110010010</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>110010110</td>
<td>9.515625 V</td>
<td>No</td>
<td>0</td>
<td>110010010</td>
</tr>
</tbody>
</table>
The following steps occur for each bit:

1. The bit is set.
2. The digital output is converted to an analog voltage and compared to the actual analog input.
3. If the analog voltage is greater than the DAC output voltage, the bit remains set. Otherwise it is cleared.

There is no exact 8-bit binary value for the analog voltage specified in Example 12.11 (9.5 V). The final answer is within 13 mV, out of 12 V, which is pretty close but not exact. This difference is called quantization error. The maximum value of quantization error is $\pm \frac{1}{2}$ LSB for any ADC, except on the lowest step, where the error is $+\frac{1}{2}, -0$ LSB, and on the highest step, where the error is $+1, -\frac{1}{2}$ LSB.

As more bits are added to the accumulated digital value, the analog equivalent of the approximation acquires more decimal places of accuracy. Note that once the analog value extends beyond the decimal point, the last decimal digit is always 5.

An advantage of a successive approximation ADC is that the conversion time is always the same, regardless of the analog input voltage. This is not true with all types of analog-to-digital converters. The constant conversion time allows the output to be synchronized so that it can be read at known intervals.

The conversion time can be as few as $(n + 1)$ clock pulses for an $n$-bit device, if a bit is set by a clock edge and cleared asynchronously or by the opposite clock edge. Some SARs require four or more clock pulses per bit.

### Dual Slope A/D Converter

**Key Terms**

**Integrator** A circuit whose output is the accumulated sum of all previous input values. The integrator’s output changes linearly with time when the input voltage is constant.

**Dual slope ADC** Also called an integrating ADC. An analog-to-digital converter based on an integrator. The name derives from the fact that during the conversion process the integrator output changes linearly over time, with two different slopes.

A **dual slope analog-to-digital converter** is based on an integrator circuit, such as the one shown in Figure 12.25. The circuit output is proportional to the integral of the input voltage.

![Integrator](image)

**FIGURE 12.25**

Integrator
voltage as a function of time. Integration with respect to time is the summing of instantaneous values of a function over a specified period of time. In other words, the output of an integrator is the accumulated total of all previous values of input voltage.

We can analyze the circuit without calculus under special conditions, such as when the input voltage is constant. An integrator is similar to an inverting amplifier and can be analyzed using similar techniques. Since the input impedance of the op amp is large, there is very little current flowing into its (−) terminal. Ohm’s law thus implies that there is very little voltage difference between the (+) and (−) terminals. Since they are at almost the same potential and the (+) terminal is grounded, we can say that the (−) terminal is “virtually grounded.”

If the input voltage is constant, a DC current, \( I \), flows in \( R \). Since \( R \) is connected to the positive terminal of the input voltage source at one end and virtual ground at the other, the entire source voltage drops across the resistor. By Ohm’s law,

\[
I = \frac{V_{in}}{R}
\]

Since the op amp input impedance is large, most current flows into the capacitor, causing it to charge over time. The current direction defines a polarity for \( V_c \), the capacitor voltage.

The op amp output voltage is measured with respect to ground. The capacitor is connected from the op amp output to virtual ground. Therefore, the output voltage, \( V_o \), is dropped across the capacitor. Notice that the polarities defined for \( V_o \) and \( V_c \) are opposite:

\[
V_o = -V_c
\]

The capacitor voltage is determined by the stored charge, \( Q \), and the value of capacitance, \( C \):

\[
V_c = \frac{Q}{C}
\]

The current \( I \) is the amount of charge flowing past a given point in a fixed time:

\[
I = \frac{Q}{t}
\]

Thus,

\[
V_c = \frac{It}{C}
\]

and

\[
V_o = -\frac{It}{C}
\]

Substitute the expression for \( I \) into this equation to get

\[
V_o = -\left(\frac{t}{RC}\right)V_{in}
\]

The output of an integrator with a constant input changes linearly with time, with a slope equal to \(-\frac{V_{in}}{RC}\).

This equation describes the change in output voltage due to a constant input. When the input goes to 0 V, the capacitor holds its charge (ideally forever; in practice until it leaks away through circuit impedances) and maintains the output voltage at its final value. If a new input voltage is applied, we can use the integrator equation to calculate the change in output, which must then be added to the previous value.

**EXAMPLE 12.12**

The integrator circuit of Figure 12.25 has the following component values:

\[
C = 0.025 \, \mu F, \quad R = 10 \, k\Omega
\]

Sketch the graph of the output voltage if the waveform shown in the graph of Figure 12.26a is applied to the integrator input. The integrator output is originally at 0 V.
Solution  We must examine the graph in two sections:
1. From 0 to 3 ms
2. From 3 to 9 ms

A different constant input voltage is applied for each section of the graph.

0 to 3 ms:
The output at 3 ms is given by:
\[
\text{slope} = \frac{V_{\text{in}}}{RC} = \frac{1\text{V}}{(0.025\mu\text{F})(10\text{ k}\Omega)} = -4\text{ V/ms}
\]
\[
V_o(3\text{ ms}) = v_o(0) + (t/RC) V_{\text{in}}
\]
\[
= 0\text{V} + [(3\text{ ms})(-4\text{ V/ms})] = -12\text{ V}
\]
The output changes at a rate of $-4\text{ V/ms}$ for 3 ms.

3 to 9 ms:
The output at 9 ms is given by:
\[
\text{slope} = \frac{V_{\text{in}}}{RC} = \frac{-0.5\text{V}}{(0.025\mu\text{F})(10\text{ k}\Omega)} = +2\text{ V/ms}
\]
\[ V_o (9 \text{ ms}) = v_o(3 \text{ ms}) - (t/RC) V_{in} \]
\[ = -12 \text{ V} + [(6 \text{ ms})(+2 \text{ V/ms})] \]
\[ = -12 \text{ V} + (+12 \text{ V}) \]
\[ = 0 \text{ V} \]

The output changes at a rate of +2 V/ms for 6 ms. This cancels the effect of the original input.

Figure 12.27 shows the block diagram of an 8-bit dual slope analog-to-digital converter. Integrator output voltages for several input values are shown in Figure 12.28. Assume that the integrator has the same \( R \) and \( C \) values as in Figure 12.25.
Before conversion starts, an auto-zero circuit sets the comparator output to 0 V by applying a compensating voltage to the comparator.

The input analog voltage causes the integrator output to increase in magnitude, as shown in the left half of Figure 12.28. As soon as this integrator voltage is nonzero, the comparator enables a counter via the control logic.

When the counter overflows (i.e., recycles to 00000000), the integrator input is switched from the analog input to $-V_{\text{ref}}$.

The reference voltage causes the integrator output to move toward 0 V at a known rate, as shown in the right half of Figure 12.28. During this rezeroing time, the counter continues to clock. When the integrator output voltage reaches 0 V, the comparator disables the counter. The digital equivalent of the analog voltage is now contained in the counter.

The reason this works is that in the initial integrating phase, the integrator output operates for a known time, producing a final output proportional to the input voltage. In the second phase, the output moves toward zero at a known rate, reaching zero in a time proportional to the final voltage of the first phase.

For example, assume that the components of the integrator and the clock rate of the counter are such that a 1-V input corresponds to the full-scale digital output ($FS$). The integrator output reaches a value of $-12$ V in 3 ms. The time required to rezero the integrator is the same as the initial integrating phase, 3 ms. The counter completes one cycle in the integrating phase and another cycle in the rezeroing phase, so that its final value is 00000000. (Note that this is the result obtained when 1 LSB is added to 11111111.)

If the input voltage is 0.25 volts, the integrator output is $-3$ V after 3 ms (one counter cycle). Since the integrator always rezeros at the same rate (4 V/ms), the rezeroing time is 0.75 ms, or one fourth of a counter cycle (since $12$ V/4 = 3 V). The counter has time to reach state 01000000 or $\frac{1}{4} FS$.

If we attempt to measure a voltage beyond that corresponding to full scale, the integrator output cannot rezero within the second counter cycle. Usually, an output pin on the ADC activates to show this condition. Some digital multimeters that use dual slope ADCs show an overvoltage or out-of-range condition by blanking the display, except for a leading digit 1.

One advantage of a dual slope ADC is its accuracy. One particular dual slope ADC is accurate to within $\pm 0.05\% \pm 1$ count. This accuracy is balanced against a relatively slow conversion time, in the milliseconds, compared to microseconds for a successive approximation ADC and nanoseconds for a flash converter.

Another advantage is the ability of the integrator to reject noise. If we assume that noise voltage is random, then it will be positive about half the time and negative about half the time. Over time it should average out to zero.

As was alluded to above, a common application of this device is as a voltmeter circuit, where speed is less important than accuracy.

SECTION 12.3 REVIEW PROBLEMS

12.7 Suppose that the dual slope ADC described above (same component values) has an input voltage of 0.375 V (3/8 full scale).

a. What is the slope of the integrator voltage during the integrating phase?

b. What is its slope during the rezeroing phase?

c. How much time elapses during the rezeroing phase?

d. What digital code is contained in the output latch after the conversion is complete?
Analog-to-Digital Conversion

Sample and Hold Circuit

**KEY TERM**

Sample and hold circuit  A circuit that samples an analog signal at periodic intervals and holds the sampled value long enough for an ADC to convert it to a digital code.

For the sake of analysis, we have been assuming that the analog input voltage of any analog-to-digital converter is constant. This is an actual requirement. Most of these circuits will not produce a correct digital code if the analog voltage at the input changes during conversion time.

Unfortunately, most analog signals are not constant. Usually, we want to sample these signals at periodic intervals and generate a series of digital codes that tells us something about the way the input signal is changing over time. A circuit called a sample and hold circuit must be used to bridge the gap between a changing analog signal and a requirement for a constant ADC input voltage.

Figure 12.29 shows a basic sample and hold circuit. The voltage followers act as buffers with high input and low output impedances. The transmission gate is enabled during the sampling period, during which it charges the hold capacitor to the current value of the analog signal. During the hold period, the capacitor retains its charge, thus preserving the sampled analog voltage. The high input impedance of the second voltage follower prevents the capacitor from discharging significantly during the hold period.

**FIGURE 12.29**
Sample and Hold Circuit

Figure 12.30 shows how a sample and hold circuit produces a steady series of constant analog voltages for an ADC input. Since these sampled values have yet to be converted to digital codes, they can take on any value within the analog range; they are not yet limited by the number of bits in the quantization.

**FIGURE 12.30**
Sample and Hold Output
Ideally, a sample and hold circuit should charge quickly in sample mode and discharge slowly in hold mode. These characteristics are facilitated by the low output impedance and high input impedance of the voltage follower circuits.

\[ Z_{o} \text{ and } Z_{i} \]

**Figure 12.31**
Equivalent Circuits for Sample-and-Hold Circuit

Figure 12.31 shows the equivalent circuits of the sample and hold modes of the circuit in Figure 12.29. In sample mode, the capacitor charges through the output impedance, \( Z_{o} \), of the first voltage follower. Since this is a very small value (about \( 75 \times 10^{-5} \) \( \Omega \)), the capacitor will charge quickly. In the hold mode, the capacitor discharges slowly through the very high input impedance of the second voltage follower (about \( 2 \times 10^{11} \) \( \Omega \)).

NOTE

The input and output impedances of the voltage follower are significantly different from the open-loop op amp values. This is because, in the voltage follower configuration, the input impedance is divided by the open loop gain (about \( 75 \Omega /100,000 \)) and the output impedance is multiplied by the open loop gain (about \( 2 \text{M} \Omega \times 100,000 \)).

A variation of the sample and hold circuit is the track and hold circuit. The difference is not so much in the circuit as in the way it is operated. A sample and hold circuit is restricted by the charging speed of its hold capacitor. If there is a large change in signal level between samples, the hold capacitor may not be able to keep up with the change. A track and hold circuit samples the analog signal continuously, minimizing charging delays of the hold capacitor. When the analog signal needs to be converted, the track and hold circuit reverts to hold mode by closing the analog transmission gate. Many high-speed ADCs have a track and hold circuit as an integral part of the device.

**Sampling Frequency and Aliasing**

**KEY TERMS**

**Nyquist sampling theorem**  A theorem from information theory that states that, in order to preserve all information in a signal, it must be sampled at a rate of twice the highest-frequency component of the signal. \( (f_s \geq 2f_{\text{max}}) \)

**Aliasing**  A phenomenon that produces an unwanted low-frequency component in a sampled analog signal due to a sampling frequency that is too slow relative to the sampled analog signal.

**Anti-aliasing filter**  A low-pass filter with a corner frequency of twice the maximum frequency of a sampled signal, used to prevent aliasing in an ADC.
In the first section of this chapter, we saw that the sampling frequency of an ADC has a great effect on the quality of the digital representation of an analog signal. We may ask, what is the minimum value of the sampling frequency for any particular analog signal and what happens if this criterion is not met?

A theorem in information theory, called the **Nyquist sampling theorem**, states that a periodic signal must be sampled at least twice a cycle to preserve all its information. In practice, this means that the sampling frequency of a particular system must be twice the maximum frequency of any signal to be sampled by the system. (These frequencies might also include harmonics of a signal that add to the basic signal to give it its characteristic shape.) This can be expressed mathematically as \( f_s \geq 2f_{\text{max}} \) for a sampling frequency \( f_s \) and a maximum-frequency component of \( f_{\text{max}} \).

For example, the sampling frequency for compact disc audio is 44.1 kHz, which allows signals of up to 22.05 kHz to be sampled accurately. This fits in nicely with the statistical range of human hearing: 20 Hz–20 kHz. (People who have listened to any amount of rock music in their youth can probably only get up to 12 kHz.) Telephone-quality signals are sampled at 8 kHz, yielding a maximum frequency of 4 kHz, which is a bit more than the classical telephone-line bandwidth of 300 Hz–3300 Hz.

A sampling frequency of an ADC system that does not meet the criterion required by the Nyquist sampling theorem results in **aliasing**, a phenomenon that generates a false low-frequency component of the digital sample.

To get an idea of how aliasing works, let us examine a sine wave with a period of 12 μs \( (f = 83.3 \text{ kHz}) \), shown in Figure 12.32. If we sampled the signal every 1 μs, we would capture the values listed in Table 12.5.

<table>
<thead>
<tr>
<th>Actual signal</th>
<th>Implied signal (alias)</th>
<th>Sampling pulses</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![FIGURE 12.32](image)

**FIGURE 12.32**
Effect of Sampling Too Slowly

The points in Table 12.5 can be used to accurately reconstruct the original sine wave. (The reconstructed output would need to be filtered to eliminate introduced high-frequency components, but the fundamental frequency would be correct.)

Suppose now that we sample the same sine wave at less than twice a cycle. Table 12.6 shows the samples captured by a series of sampling pulses that are spaced by 13 μs. The first four samples in the table are shown by vertical lines in Figure 12.32.

The samples in Table 12.6 have exactly the same amplitude as those taken in Table 12.5. However, the samples are spaced at 13 μs intervals, rather than 1 μs. For example, the sample at 13 μs measures the sine wave amplitude at 390°, which is the same as 30° of
the second cycle. If these samples were used to reconstruct a sine wave, it would have a period of 156 \( \mu \text{s} \), rather than 12 \( \mu \text{s} \). This false low-frequency component is shown by the broken line connecting the first four samples in Figure 12.32, which represent measurements, not in a single cycle, but in four cycles of the original analog signal. Figure 12.33 shows one complete cycle of the alias frequency as a broken line and thirteen cycles of the sampled signal as a solid line.

Aliasing can be prevented by filtering the analog input to an ADC with an anti-aliasing filter, as shown in Figure 12.34. The anti-aliasing filter is a low-pass filter with the corner frequency set to 2 \( f_{\text{max}} \). Frequencies less than 2 \( f_{\text{max}} \) are allowed to pass to the analog input of the ADC. Frequencies greater than 2 \( f_{\text{max}} \) are attenuated. In this way, the ADC never converts any signal with a frequency greater than 2 \( f_{\text{max}} \) and thus an alias frequency cannot develop.
12.4 Data Acquisition

**KEY TERM**

Data acquisition network  A circuit that gathers and digitizes data from several analog sources.

CPLD Interface for an ADC

Figure 12.35 shows the symbol for an ADC0808 analog-to-digital converter. This successive approximation ADC can form the basis of a data acquisition network, a system that can convert analog information from up to eight channels and store the converted values in a series of output latches.

The ADC0808 has a built-in 8-channel analog multiplexer with inputs IN0 through IN7, which are selected by the states of three address inputs, ADD C, ADD B, and ADD A, where ADD C is the most significant bit. Before an analog input can be converted, its address must be stored in an internal address latch by a high-going pulse on ALE (Address Latch Enable).
The conversion process starts with a high-going pulse on the *START* input. (START and ALE can be tied together.) End-of-conversion is indicated by the *EOC* output. The conversion process is driven by the *CLOCK* input. After conversion is complete, the digital output can be read by making *OE* (Output Enable) HIGH. When *OE* is not active, the digital outputs are in the high-impedance state.

**FIGURE 12.36**
Timing Diagram for an ADC0808

Figure 12.36 shows a timing diagram relating the various control signals of the ADC0808. Figure 12.37 is an excerpt from the ADC0808 data sheet that shows relevant timing information. The ADC is reset on the rising edge of *START*. After *START/ALE* goes LOW, the ADC makes *EOC* go LOW within 8 clock cycles + 2 μs. *EOC* stays LOW until conversion is complete. The simplest way to operate the ADC on a stand-alone basis is to tie the *EOC* line to the *START/ALE* line so that the ADC starts as soon as *EOC* goes HIGH and the ADC continuously updates the value of the digital output.

We can design a state machine that controls the ADC and stores output values in an octal latch automatically. Figure 12.38 shows such a circuit. The controller is a state machine that will accept a LOW pulse from a pushbutton switch labeled *go*, perform one analog-to-digital conversion from one of eight analog channels and store the resulting 8-bit digital value in an octal latch. The analog channel is manually selected by DIP switches at the address select inputs. The entire state machine and latch portion of the circuit is contained in one CPLD, such as the Altera EPM7128SLC84-7 on the Altera UP-1 circuit board.

**FIGURE 12.39**
State Diagram of the controller

Figure 12.39 shows the state diagram of the controller, with two synchronous inputs called *go* and *eoc*. The asynchronous reset, which sets the machine to the *idle* state, is not shown on the state diagram. Outputs are *sc* (start conversion), *oe* (output enable), and *en* (latch enable). The states are as follows:

- **idle**—Wait for *go* = 0 (switch pressed). All outputs are LOW.
- **start**—Wait for *go* = 1 (switch released). Transition to wait1 makes *sc* = 1 (START/ALE pulse). Other outputs are LOW.
- **wait1**—Wait for *eoc* = 0. (Wait for conversion to start. EOC is LOW during, but not before, conversion. Do not test for *eoc* = 1 until after conversion.) All outputs LOW.
- **wait2**—Wait for *eoc* = 1. (Conversion complete.) When complete, transition to read. At that time, *oe* = 1, *en* = 1.
- **read**—Enable ADC output (*oe* = 1) and make latch transparent (*en* = 1).
- **store**—Keep ADC output enabled, put latch in store mode (*en* = 0). ADC digital output is now stored in the output latch.

Figure 12.40 shows a simulation of the controller. Both the latch and controller can be implemented as VHDL design entities and instantiated as components in the top level of a VHDL hierarchy. This and later VHDL examples will be saved as exercises for the lab manual accompanying this book. The VHDL files are available to instructors in the Online Companion to this book.

We should note that if the controller/latch circuit is to be implemented on the Altera
### Electrical Characteristics (Continued)

#### Digital Levels and DC Specifications:

ADC0808CCN, ADC0808CCV, ADC0809CCN and ADC0809CCV, 4.75\text{V}<\text{V}_{\text{CC}}<5.25\text{V},

\(-40\text{C}<\text{T}_{\text{A}}<85\text{C}\) unless otherwise noted.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Conditions</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_{\text{OUT}(1)})</td>
<td>Logical (\text{Hi})-\text{&quot;1&quot;} Output Voltage</td>
<td>(\text{V}<em>{\text{CC}}=4.75\text{V}), (\text{I}</em>{\text{OUT}}=\pm360\mu\text{A})</td>
<td>2.4</td>
<td>4.5</td>
<td>V(min)</td>
<td>V(min)</td>
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<tr>
<td>(V_{\text{OUT}(0)})</td>
<td>Logical (\text{Hi})-\text{&quot;0&quot;} Output Voltage</td>
<td>(\text{I}_{\text{O}}=1.6\text{mA})</td>
<td>0.45</td>
<td>V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_{\text{OUT}(0)})</td>
<td>Logical (\text{Hi})-\text{&quot;0&quot;} Output Voltage EOC</td>
<td>(\text{I}_{\text{O}}=1.2\text{mA})</td>
<td>0.45</td>
<td>V</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(I_{\text{OUT}})</td>
<td>TRI-STATE Output Current</td>
<td>(\text{V}<em>{\text{O}}=\pm5\text{V}), (\text{V}</em>{\text{O}}=0)</td>
<td>3</td>
<td>(\mu\text{A})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Timing Specifications:

\(\text{V}_{\text{CC}}=\text{V}_{\text{REF}(+)}=5\text{V}, \text{V}_{\text{REF}(\text{a})}=\text{GND}\), \(\text{t}_{\text{R}}=\text{t}_{\text{F}}=20\text{ns}\) and \(\text{T}_{\text{A}}=25\text{C}\) unless otherwise noted.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Conditions</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Units</th>
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</thead>
<tbody>
<tr>
<td>(t_{\text{WS}})</td>
<td>Minimum Start Pulse Width</td>
<td>(Figure 5)</td>
<td>100</td>
<td>200</td>
<td>ns</td>
<td></td>
</tr>
<tr>
<td>(t_{\text{WALE}})</td>
<td>Minimum ALE Pulse Width</td>
<td>(Figure 5)</td>
<td>100</td>
<td>200</td>
<td>ns</td>
<td></td>
</tr>
<tr>
<td>(t_{\text{A}})</td>
<td>Minimum Address Set-Up Time</td>
<td>(Figure 5)</td>
<td>2</td>
<td>25</td>
<td>50</td>
<td>ns</td>
</tr>
<tr>
<td>(t_{\text{H}})</td>
<td>Minimum Address Hold Time</td>
<td>(Figure 5)</td>
<td>2</td>
<td>25</td>
<td>50</td>
<td>ns</td>
</tr>
<tr>
<td>(t_{\text{D}})</td>
<td>Analog MUX Delay Time From ALE</td>
<td>(R_{\text{L},=0\Omega}) (Figure 5)</td>
<td>1</td>
<td>2.5</td>
<td>(\mu\text{s})</td>
<td></td>
</tr>
<tr>
<td>(t_{\text{R}},t_{\text{H}})</td>
<td>OE Control to Q Logic State</td>
<td>(C_{\text{L}}=50\text{pF}, R_{\text{L},=10k}) (Figure 8)</td>
<td>125</td>
<td>250</td>
<td>ns</td>
<td></td>
</tr>
<tr>
<td>(t_{\text{HR}},t_{\text{OH}})</td>
<td>OE Control to Hi-Z</td>
<td>(C_{\text{L}}=10\text{pF}, R_{\text{L},=10k}) (Figure 8)</td>
<td>125</td>
<td>250</td>
<td>ns</td>
<td></td>
</tr>
<tr>
<td>(t_{\text{C}})</td>
<td>Conversion Time (f_{\text{C}}=640\text{kHz}), (Figure 5) (Note 7)</td>
<td>90</td>
<td>100</td>
<td>116</td>
<td>(\mu\text{s})</td>
<td></td>
</tr>
<tr>
<td>(t_{\text{C}})</td>
<td>Clock Frequency</td>
<td>(f_{\text{C}}=640\text{kHz}) (Figure 5)</td>
<td>10</td>
<td>640</td>
<td>1280</td>
<td>kHz</td>
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<tr>
<td>(t_{\text{EOC}})</td>
<td>EOC Delay Time</td>
<td>(Figure 5)</td>
<td>0</td>
<td>8\pm2\mu\text{s}</td>
<td>Clock Periods</td>
<td></td>
</tr>
<tr>
<td>(C_{\text{IN}})</td>
<td>Input Capacitance</td>
<td>At Control Inputs</td>
<td>10</td>
<td>15</td>
<td>pF</td>
<td></td>
</tr>
<tr>
<td>(C_{\text{OUT}})</td>
<td>TRI-STATE Output Capacitance</td>
<td>At TRI-STATE Outputs</td>
<td>10</td>
<td>15</td>
<td>pF</td>
<td></td>
</tr>
</tbody>
</table>

**Note 1:** Absolute Maximum Ratings indicate limits beyond which damage to the device may occur. DC and AC electrical specifications do not apply when operating the device beyond its specified operating conditions.

**Note 2:** All voltages are measured with respect to GND, unless otherwise specified.

**Note 3:** A zener diode exists, internally, from \(\text{V}_{\text{CC}}\) to GND and has a typical breakdown voltage of 7\text{V}.

**Note 4:** Two on-chip diodes are tied to each analog input which will forward conduct for analog input voltages one diode drop below ground or one diode drop greater than the \(\text{V}_{\text{CC}}\) supply. The spec allows 100 mV forward bias of either diode. This means that as long as the analog \(\text{V}_{\text{AI}}\) does not exceed the supply voltage by more than 100 mV, the output code will be correct. To achieve an absolute \(\text{V}_{\text{CC}}\) to 5\text{V} input range will therefore require a minimum supply voltage of 4.900\text{V} over temperature variations, initial tolerance and loading.

**Note 5:** Total unadjusted error includes offset, full-scale, linearity, and multiplexer errors. See Figure 3. None of these A/Ds requires a zero or full-scale adjust. However, if an all zero code is desired for an analog input other than 0.0\text{V}, or if a narrow full-scale span exists (for example: 0.5\text{V} to 4.5\text{V} full-scale) the reference voltages can be adjusted to achieve this. See Figure 13.

**Note 6:** Comparator input current is a bias current into or out of the chopper stabilized comparator. The bias current varies directly with clock frequency and has little temperature dependence (Figure 6). See paragraph 4.0.

**Note 7:** The outputs of the data register are updated one clock cycle before the rising edge of EOC.

**Note 8:** Human body model, 100 pF discharged through a 1.5 k\text{Ω} resistor.

---

**FIGURE 12.37**

Extract from ADC0808 Datasheet (Reprinted with permission of National Semiconductor)
UP-1 board, we must also include a clock divider circuit. The Altera UP-1 board has an onboard oscillator that runs at 25.175 MHz. The clock rate, as defined by the ADC0808 data sheet, must be in the range $10\text{kHz} \leq f_c \leq 1280 \text{kHz}$.

A 5-bit counter can serve as a divide-by-32 circuit ($2^5 = 32$), as shown in Figure 12.41. If the UP-1 oscillator is applied to the counter clock, the $Q_4$ output frequency is given by $25.175 \text{MHz}/32 = 786.7 \text{kHz}$, which is within the required range for the ADC clock. $Q_4$ should then be used to clock the state machine, as well as any other synchronous circuitry used in conjunction with the ADC0808.

We can make a few minor changes to the ADC interface in Figure 12.38 to make it run as a continuous-conversion circuit. First, we eliminate the \textit{go} input and associated pushbutton. Second, we change the state diagram to eliminate the \textit{idle} state, as shown in Figure 12.42. With no pushbutton to press and then release, we eliminate two wait transitions (previously associated with the \textit{idle} and \textit{start} states) from the state diagram. Otherwise, the circuit and controller remain the same. A simulation of the modified controller is shown in Figure 12.43.
FIGURE 12.39
State Diagram for an ADC Controller

FIGURE 12.40
Simulation of State Machine ADC Controller

FIGURE 12.41
5-Bit Counter as Divide-by-32 Clock Divider
EXAMPLE 12.13
From the ADC0808 data sheet extract in Figure 12.37, determine the number of clock cycles required for the conversion of an analog signal.

Solution
For a clock frequency of 640 kHz, typical conversion time is given as 100 µs.

\[
\frac{640 \times 10^3 \text{ clock cycles}}{\text{second}} \times (100 \times 10^{-6} \text{ seconds}) = 64 \text{ clock cycles}
\]

EXAMPLE 12.14
Calculate the highest-frequency analog input that can be accurately converted by an ADC0808 controlled by a state machine represented by the state diagram of Figure 12.42 if the system clock frequency is 787 kHz.

Solution
One conversion cycle, \(T_s\), requires 64 clock cycles for the ADC and an overhead of 13 clocks +2 µs for the state machine for a total of 77 clock cycles +2 µs. (Start to wait1 requires one clock cycle. An additional 8 cycles +2 µs are needed before eoc goes LOW. According to Note 7 in Figure 12.37, the ADC conversion is complete one clock cycle before EOC goes HIGH. From this point back to start is 4 clocks.)
According to the Nyquist sampling theorem, the maximum-frequency component of the sampled analog signal is \( f_{\text{max}} = \frac{f_s}{2} \), where \( f_s = \frac{1}{T_s} \). Given \( T_s = \frac{1}{787 \times 10^3} \) clock cycles/second, we have:

\[
T_s = \frac{1}{787 \times 10^3} \text{ clock cycles/second} \times 77 \text{ clock cycles} + 2 \mu s = 99.8 \times 10^{-6} \text{ seconds}
\]

\[
f_s = \frac{1}{T_s} = \frac{1}{99.8 \times 10^{-6}} = 10.02 \text{ kHz}
\]

According to the Nyquist sampling theorem, the maximum-frequency component of the sampled analog signal is \( f_{\text{max}} = f_s/2 = 10.02 \text{ kHz}/2 = 5.01 \text{ kHz} \). This is of the same order of magnitude as a telephone-quality audio signal.

### CPLD-Based Data Acquisition Network

Figure 12.44 shows a data acquisition system that continuously converts and stores data from four analog channels. All the circuitry within the broken line is contained within a single CPLD, such as the Altera EPM7128SLC84. The operation is similar to the system in...
Figure 12.38, except that with multiple latches in the circuit, a counter and decoder are required to keep track of the selected channel.

The controller, whose state diagram is shown in Figure 12.45, generates the same control signals for the ADC as the system in Figure 12.38. When the conversion is complete and the controller detects a LOW on its eoc input, it reads the ADC output and transfers the contents to the selected 8-bit latch. The latch is selected, via the decoder, by the value of the counter (e.g., \(Q_1Q_0 = 11\) selects analog input channel 3, decoder output \(Y_3\), and latch 3). The selected latch input is enabled (i.e., made transparent) by the controller during the transition from wait2 to read. At all other times all decoder outputs are LOW, disabling all latches, thus placing them in store mode. After the ADC data have been stored, the controller sets cnt_en HIGH, which allows the counter to be incremented on the next clock pulse. The next channel is now ready for a convert-and-store cycle. After all channels have been sampled, converted, and stored, the cycle begins again at channel 0 and continues indefinitely.

![State Diagram for 4-channel Data Acquisition System](image)

**FIGURE 12.45**
State Diagram for 4-channel Data Acquisition System

Figure 12.46 shows a simulation of the controller, counter, and decoder for the data acquisition system of Figure 12.44. During the read-and-store part of the cycle, only one of the latch enables, \(y_0\) to \(y_3\), is active when oe is active. The number of the active latch enable is the same as the counter value on the second last waveform. The last line in the simulation (controller|outputs3.Q) is the cnt_en line from the controller to the counter. The counter is incremented on the first positive edge of the clock after this line goes HIGH. This point is indicated by the cursor line on the transition from channel 1 to channel 2.

The circuit in Figure 12.44 could be expanded to convert all eight analog channels from the ADC, but the chosen CPLD (EPM7128SLC84) does not have enough I/O pins. Eight 8-bit latch outputs require 64 pins; the CPLD only has 60 user I/Os. An 8-channel system could be implemented if it used eight external latches, such as eight 74HC373 octal latches, or internal latches on a different CPLD. Note that the CPLD has enough logic cells to implement the system, just not enough I/O pins. The identical device in a different package (EPM7128SQC100; 100-pin quad flat-pack) can accommodate the entire system.

For an 8-channel system, the counter would need to be expanded to 3 bits and the decoder to a 3-line-to-8-line device.
SECTION 12.4 REVIEW PROBLEM

12.8 Calculate the highest-frequency component of an analog signal that can be accurately converted by the 4-channel data acquisition system in Figure 12.44. Assume the system clock is running at 787 kHz.

SUMMARY

1. An analog system can represent a physical property (e.g., temperature, pressure, or velocity) by a proportional voltage or current. The mathematical function describing the analog voltage or current is continuous throughout a defined range.

2. A digital system can represent a physical property by a series of binary numbers of a fixed bit size.

3. Digital representations of data are not subject to the same distortions as analog representations. They are also easier to store and reproduce than analog.

4. The quality of a digital representation depends on the sampling frequency and quantization (number of bits) of the system that converts an analog input to a digital output.

5. The resolution of a system is a function of the number of bits in its digital representation. A greater number of bits implies that the sampled analog input can be broken up into more, smaller segments, allowing each segment to more closely approximate the original input value.

6. A digital-to-analog converter (DAC) uses electronic switches to sum binary-weighted currents to a total analog output current. Analog current can be calculated by:

\[ I_a = b_n b_{n-1} 2^{n-1} + b_{n-2} 2^{n-2} + \cdots + b_2 2^2 + b_1 2^1 + b_0 2^0 \]

\[ \text{or, more simply:} \]

\[ I_a = \frac{\text{digital code}}{2^n} I_{\text{ref}} \]

for an \( n \)-bit DAC, where \( b_n b_{n-1} \cdots b_2 b_1 b_0 \) is the digital input code,

\( I_a \) is the analog output current, and

\( I_{\text{ref}} \) is the DAC reference (full scale) current.

7. The maximum output of a DAC is full scale (FS) minus the value represented by a change in the least significant bit of the input (\( FS - 1 \) LSB). For example, for a 4-bit converter (1 LSB = 1/16 FS), the maximum output is \( FS - 1/16 \) FS. For an 8-bit converter (1 LSB = 1/256 FS), the maximum output is \( FS - 1/256 \) FS.
8. A weighted-resistor DAC derives its binary-weighted currents from binary-weighted resistors connected to the reference voltage supply.

9. An R-2R ladder DAC derives its binary weighted currents from a resistor ladder network that consists of resistors of two values only, one of which is twice the other. The R-2R ladder is more common than the weighted resistor DAC.

10. A DAC input code consisting of a 1 followed by all 0s represents an output of \( \frac{1}{2^k} \) FS, regardless of the number of bits in the DAC input. A code of 01 followed by all 0s represents an output of \( \frac{1}{2} \) FS. A code of 11 followed by all 0s is \( \frac{3}{2} \) FS.

11. The MC1408 DAC is an example of a monolithic (single-chip) DAC. Output current at pin 4 is a binary-weighted fraction of the reference current at pin 14:

\[
I_o = \left( \frac{\text{digital code}}{256} \right) \left( \frac{V_{\text{ref}}}{R_{14}} \right)
\]

12. If the output of an MC1408 DAC is buffered by a noninverting op amp with a feedback resistance of \( R_f \), the output voltage is given by:

\[
V_o = I_o R_f = \left( \frac{\text{digital code}}{256} \right) \left( \frac{R_f}{R_{14}} \right) V_{\text{ref}}
\]

13. An 8-bit DAC can be used as a ramp generator by connecting an 8-bit binary counter to the digital inputs.

14. An MC1408 DAC can be configured for bipolar output by connecting a pull-up resistor (\( R_3 \)) from the output (pin 4) to the reference voltage supply. Output is given by:

\[
V_o = I_o R_f - I_o R_F = \left( \frac{\text{digital code}}{256} \right) \left( \frac{R_F}{R_{14}} \right) V_{\text{ref}} - \frac{R_F}{R_3} V_{\text{ref}}
\]

15. A DAC is monotonic if every increase in binary input results in an increase in analog output.

16. DAC errors include: offset error (nonzero output for zero input code), gain error (output falling above or below \( FS - 1 \) LSB for maximum input code due to an incorrect slope), linearity error (deviation from straight-line approximation between codes), and differential nonlinearity (deviation of step sizes from ideal of one step per LSB).

17. DAC linearity error of greater than \( \pm \frac{1}{2} \) LSB can result in a nonmonotonic output.

18. Several popular types of analog-to-digital converters (ADC) are flash or simultaneous, successive approximation, and dual slope or integrating.

19. A flash ADC consists of a voltage divider with the same number of steps as output codes, a set of comparators (one for every output code), and a priority encoder. All comparators whose reference input is less than the analog input will fire, the priority encoder will detect the highest-value active comparator, and generate the corresponding output code. A flash ADC is fast, but requires \( 2^n \) comparators for an \( n \)-bit output code.

20. An ADC transfer characteristic is set up so that all codes are 1 LSB wide, except for the first and last codes. The code for 0 is \( \frac{1}{2} \) LSB wide and the maximum code is \( 1\frac{1}{2} \) LSB wide. This offset places the nominal analog value of the code in the center of the code’s range of analog input values.

21. A successive approximation ADC consists of a state machine called a successive approximation register (SAR) whose bits can be set and cleared individually in a specific sequence, a digital-to-analog converter, and an analog comparator.

22. A successive approximation ADC sets each bit of the SAR in turn as an approximation of the required digital code. For each bit, the approximation is converted back to analog form and compared with the incoming analog value. If the converted value is less than the actual analog value, the bit remains set and the next bit is tried. If the converted value is greater than the actual analog input, the bit is cleared and the next bit is tried.

23. A dual slope ADC consists of an integrator, comparator, counter, and control logic. The integrator output changes with a slope of \( \frac{-V_{\text{ref}}}{R_{F}C} \) for a constant input. This ADC allows the integrator to charge for the time required for the counter to complete one full cycle (known time). At that time, the integrator input is switched to a reference voltage of opposite polarity. The reference voltage discharges the integrator at a known rate. The time required to do this is stored in the counter and represents the fraction of full scale analog voltage applied to the converter.

24. A sample and hold circuit may be required to hold the input value of an ADC constant for the conversion time of the ADC. It samples an analog signal at periodic intervals and holds the sampled value in a capacitor until the next sample is taken. A track and hold circuit performs a similar function, but allows the capacitor to charge and discharge along with the changing analog signal, holding its value only during the conversion time of the ADC.

25. In order to preserve the information in an analog signal, it must be sampled at a frequency of at least twice the maximum-frequency component of the signal (\( f_s \geq f_{\text{max}} \)). This criterion is called the Nyquist sampling theorem.

26. If the Nyquist sampling theorem is violated, an alias frequency, or false low-frequency component, will be added to the digital representation of the analog signal.

27. Aliases frequencies can be eliminated with an anti-aliasing filter, a low-pass filter used to pass only frequencies less than \( 2f_s \) to the input of an ADC. This input frequency range automatically satisfies the Nyquist criterion at the ADC input.

28. An ADC0808 successive approximation ADC contains an 8-channel analog MUX and can be used as the basis for an 8-channel data acquisition system.

29. The conversion sequence for the ADC0808 is as follows:
   a. an analog input channel is selected by setting the appropriate address on lines ADD C, ADD B, and ADD A.
   b. ALE and START are pulsed HIGH.
   c. EOC (end-of-conversion) goes LOW no later than 8 clock cycles +2 \( \mu \)s after START.
   d. EOC goes HIGH when conversion is complete.
   e. OE (output enable) is set HIGH to read converted output.

This sequence can be controlled by a CPLD-based state machine.

30. A data acquisition system based on an ADC0808 requires an octal latch for each analog channel, a state-machine controller, and a counter/decoder circuit to select the active analog channel and latch.
GLOSSARY

Aliasing  A phenomenon that produces an unwanted low-frequency component in a sampled analog signal due to a sampling frequency that is too slow relative to the sampled analog signal.

Anti-aliasing filter  An low-pass filter with a corner frequency of twice the maximum frequency of a sampled signal, used to prevent aliasing in an ADC.

Analog  A way of representing some physical quantity, such as temperature or velocity, by a proportional continuous voltage or current. An analog voltage or current can have any value within a defined range.

Analog-to-digital converter  A circuit that converts an analog signal at its input to a digital code. (Also called an A-to-D converter, A/D converter, or ADC.)

Continuous  Smoothly connected. An unbroken series of consecutive values with no instantaneous changes.

Data acquisition network  A circuit that gathers and digitizes data from several analog sources.

Digital  A way of representing a physical quantity by a series of binary numbers. A digital representation can have only specific discrete values.

Digital-to-analog converter  A circuit that converts a digital code at its input to an analog voltage or current. (Also called a D-to-A converter, D/A converter, or DAC.)

Discrete  Separated into distinct segments or pieces. A series of discontinuous values.

Dual slope ADC  Also called an integrating ADC. An analog-to-digital converter based on an integrator. The name derives from the fact that during the conversion process the integrator output changes linearly over time, with two different slopes.

Flash converter (or simultaneous converter)  An analog-to-digital converter that uses comparators and a priority encoder to produce a digital code.

Full scale  The maximum analog reference voltage or current of a digital-to-analog converter.

Integrator  A circuit whose output is the accumulated sum of all previous input values. The integrator’s output changes linearly with time when the input voltage is constant.

Multiplying DAC  A DAC whose output changes linearly with a change in DAC reference voltage.

Nyquist sampling theorem  A theorem from information theory that states that, in order to preserve all information in a signal, it must be sampled at a rate of twice the highest-frequency component of the signal. \( f_s \geq 2f_{\text{max}} \)

Priority encoder  An encoder that will produce a binary output corresponding to the subscript of the highest-priority active input. This is usually defined as the input with the largest subscript.

Quantization  The number of bits used to represent an analog voltage as a digital number.

Quantization error  Inaccuracy introduced into a digital signal by the inability of a fixed number of bits to represent the exact value of an analog signal.

Resolution  The difference in analog voltage corresponding to two adjacent digital codes. Analog step size.

Sample  An instantaneous measurement of an analog voltage, taken at regular intervals.

Sample and hold circuit  A circuit that samples an analog signal at periodic intervals and holds the sampled value long enough for an ADC to convert it to a digital code.

Sampling frequency  The number of samples taken per unit time of an analog signal.

Successive approximation register  A state machine used to generate a sequence of closer and closer binary approximations to an analog signal.

PROBLEMS

Problem numbers set in color indicate more difficult problems: those with underlines indicate most difficult problems.

Section 12.1 Analog and Digital Signals

12.1 An analog signal with a range of 0 to 12 V is converted to a series of 3-bit digital codes. Make a table similar to Table 12.1 showing the analog range for each digital code.

12.2 Sketch the positive half of a sine wave with a peak voltage of 12 V. Assume that this signal will be quantized according to the table constructed in Problem 12.1. Write the digital codes for the points 0, \( T/8 \), \( T/4 \), 3\( T/8 \), . . . , \( T \) where \( T \) is the period of the half sine wave.

12.3 Repeat Problems 12.1 and 12.2 for a 4-bit quantization.

12.4 Write the 3-bit and 4-bit digital codes for the points 0, \( T/16 \), \( T/8 \), 3\( T/16 \), . . . , \( T \) for the half sine wave described in Problem 12.2.

12.5 An analog-to-digital converter divides the range of an analog signal into 64 equal parts. The analog input has a range of 0 to 500 mV. How many bits are there in the resultant digital codes? What is the resolution of the A/D converter?

12.6 Repeat Problem 12.5 if the analog range is divided into 256 equal parts.

12.7 The analog range of a signal is divided into \( m \) equal parts, yielding a digital quantization of \( n \) bits. If the range is divided into \( 2m \) parts, how many bits are in the equivalent digital codes? (That is, how many extra bits do we get for each doubling of the number of codes?)

Section 12.2 Digital-to-Analog Conversion

12.8 a. Calculate the analog output voltage, \( V_o \), for a 4-bit DAC when the input code is 1010.

b. Calculate \( V_o \) for an 8-bit DAC when the input code is 10100000.

c. Compare the results of parts a and b. What can you conclude from this comparison?
12.9  a. Calculate the analog output voltage, $V_a$, for a 4-bit DAC when the input code is 1100.
   b. Calculate $V_a$ for an 8-bit DAC when the input code is 11001000.
   c. Compare the results of parts a and b. What can you conclude from this comparison? How does this differ from the comparison made in Problem 12.8?

12.10 Refer to the generalized D/A converter in Figure 12.4. For $I_{ref} = 500 \mu A$ and $R_F = 22 k\Omega$, calculate the range of analog output voltage, $V_a$, if the DAC is a 4-bit circuit. Repeat the calculation for an 8-bit DAC.

12.11 The resistor for the MSB of a 16-bit weighted resistor D/A converter is 1 kΩ. List the resistor values for all bits. What component problem do we encounter when we try to build this circuit?

12.12 Draw the circuit for an 8-bit R-2R ladder DAC.

12.13 Calculate the value of $V_a$ of an R-2R ladder DAC when digital inputs are as follows. $V_{ref} = 12 V$.

   DCBA
   a. 1111
   b. 1011
   c. 0110
   d. 0011

12.14 An MC1408 DAC is configured as shown in Figure 12.12. $R_{14} = R_{15} = 6.8 k\Omega$, $V_{ref}(+) = +12 V$, $V_{ref}(-) = 0$, and $R_L = 2.2 k\Omega$. Calculate the output voltage, $V_a$, for the following digital input codes: 00000000, 00000001, 10000000, 10101010, 11100010, 11111111.

12.15 Calculate the resolution of the DAC in Problem 12.14.

12.16 Refer to the op amp-buffered DAC in Figure 12.13. Assume the resistor values are changed as follows: $R_{14A} = 270 \Omega$, $R_{14B} = 2 k\Omega$ (max), $R_{FA} = 1.2 k\Omega$, $R_{FB} = 5 k\Omega$ (max). Describe a step-by-step procedure that calibrates the DAC so that it has a reference current of 4 mA and a full scale analog output voltage of 12 volts, using only a series of measurements of the analog output voltage. When the procedure is complete, what are the resistance values in the circuit? What is the range of the DAC?

12.17 The resistor networks shown in the DAC circuit of Figure 12.13 allow us to set our input reference current and output gain to values within a specified range. Using the values shown in Figure 12.13, fill in Table 12.7 for the cases when $V_a$ is at minimum and maximum, and when the potentiometers are at their midpoint values. Assume the DAC input is set to 1111 1111. Show all calculations.

<table>
<thead>
<tr>
<th>Table 12.7 DAC Output Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{14}$ ($\Omega$)</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>Minimum $V_a$</td>
</tr>
</tbody>
</table>

12.18 The waveform in Figure 12.47 is observed at the output of the DAC ramp generator of Figure 12.14. (Compare this to the proper waveform, found in Figure 12.15.) What is likely to be the problem with the circuit? Can it be easily fixed? How?

12.19 The waveform in Figure 12.48 is observed at the output of the DAC ramp generator in Figure 12.14. What is likely to be the problem with the circuit?
12.20 Refer to the bipolar DAC circuit in Figure 12.16. Describe how you would adjust the output for a range of \(-10 \text{ V to } (+10 \text{ V } - 2 \text{ LSB})\). Include values of variable components. Calculate the resolution of this circuit.

12.21 A 3-bit DAC has a reference voltage of 12 V and a transfer characteristic summarized in Table 12.8. Plot the data on a graph similar to those in Figures 12.18 through 12.20. From the data in Table 12.8, determine the offset error, gain error, and linearity error of the DAC, both in % of full scale and as a fraction of an LSB.

Table 12.8 DAC Transfer Characteristic for Problem 12.21

<table>
<thead>
<tr>
<th>Digital Code</th>
<th>Analog Output (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0.5</td>
</tr>
<tr>
<td>001</td>
<td>2.0</td>
</tr>
<tr>
<td>010</td>
<td>3.5</td>
</tr>
<tr>
<td>011</td>
<td>5.0</td>
</tr>
<tr>
<td>100</td>
<td>6.5</td>
</tr>
<tr>
<td>101</td>
<td>8.0</td>
</tr>
<tr>
<td>110</td>
<td>9.5</td>
</tr>
<tr>
<td>111</td>
<td>11.0</td>
</tr>
</tbody>
</table>

12.22 A 3-bit DAC has a reference voltage of 8 V and a transfer characteristic summarized in Table 12.9. Plot the data on a graph. From the data in Table 12.9, determine the offset error, gain error, linearity error, and differential nonlinearity of the DAC, both in % of full scale and as a fraction of an LSB.

Table 12.9 DAC Transfer Characteristic for Problem 12.22

<table>
<thead>
<tr>
<th>Digital Code</th>
<th>Analog Output (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0.000</td>
</tr>
<tr>
<td>001</td>
<td>1.036</td>
</tr>
<tr>
<td>010</td>
<td>2.071</td>
</tr>
<tr>
<td>011</td>
<td>3.107</td>
</tr>
<tr>
<td>100</td>
<td>4.143</td>
</tr>
<tr>
<td>101</td>
<td>5.179</td>
</tr>
<tr>
<td>110</td>
<td>6.214</td>
</tr>
<tr>
<td>111</td>
<td>7.250</td>
</tr>
</tbody>
</table>

12.23 A 3-bit DAC has a reference voltage of 4 V and a transfer characteristic summarized in Table 12.10. Plot the data on a graph. From the data in the Table 12.10, determine the offset error, gain error, and linearity error of the DAC, both in % of full scale and as a fraction of an LSB.

Table 12.10 DAC Transfer Characteristic for Problem 12.23

<table>
<thead>
<tr>
<th>Digital Code</th>
<th>Analog Output (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>0.000</td>
</tr>
<tr>
<td>001</td>
<td>0.500</td>
</tr>
<tr>
<td>010</td>
<td>1.025</td>
</tr>
<tr>
<td>011</td>
<td>1.525</td>
</tr>
<tr>
<td>100</td>
<td>1.985</td>
</tr>
<tr>
<td>101</td>
<td>2.675</td>
</tr>
<tr>
<td>110</td>
<td>3.000</td>
</tr>
<tr>
<td>111</td>
<td>3.500</td>
</tr>
</tbody>
</table>

Section 12.3 Analog-to-Digital Conversion

12.24 How many comparators are needed to construct an 8-bit flash converter? Sketch the circuit of this converter. (It is only necessary to show a few of the comparators and indicate how many there are.)

12.25 Briefly explain the operation of a flash ADC. What is the purpose of the priority encoder? Explain how the latch can be used to synchronize the output to a particular sampling frequency.

12.26 Why do we choose a value of \( R/2 \) for the LSB resistor of a flash ADC?

12.27 An 8-bit successive approximation ADC has a reference voltage of +16 V. Describe the conversion sequence for the case where the analog input is 4.75 V. Summarize the steps in Table 12.11. (Refer to Example 12.11.)

12.28 What is displayed on the seven-segment display in Figure 12.49 when \( v_{\text{analog}} = 5.25 \text{ V} \)? Assume that the reference voltage is 12 V and that the display can show hex digits.

12.29 Describe the operation of each part of the successive approximation ADC shown in Figure 12.49 when the analog input changes from 5.25 V to 8.0 V. What is the new number displayed on the seven-segment display?

Table 12.11 Table for Problem 16.23

<table>
<thead>
<tr>
<th>Bit</th>
<th>New Digital Value</th>
<th>Analog Equivalent ( v_{\text{analog}} )</th>
<th>Comparator Output</th>
<th>Accumulated Digital Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_7 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_6 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_4 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_3 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_1 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_0 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12.30 a. An 8-bit successive approximation ADC has a reference voltage of 12 V. Calculate the resolution of this ADC.

b. The analog input voltage to the ADC in part a is 8 V. Can this input voltage be represented exactly? What digital code represents the closest value to 8 V? What exact analog value does this represent? Calculate the percent error of this conversion.

12.31 What is the maximum quantization error of an ADC, relative to a fraction of 1 LSB?

12.32 An 8-bit dual slope analog-to-digital converter has a reference voltage of 16 V. The integrator component values are: \( R = 80 \, \text{k} \Omega \), \( C = 0.1 \, \mu \text{F} \). The analog input voltage is 14 V.

Calculate the slope of the integrator voltage during:

a. the integrating phase, and

b. the rezeroing phase.

c. How much time elapses during the rezeroing phase?

(Assume that (1) the integrating and rezeroing time are equal if the integrator output is at full scale, and (2) the reference voltage will rezero the integrator from full scale in exactly one counter cycle.)

d. Sketch the integrator output waveform.

e. What digital code is contained in the output latch after the conversion is complete?

12.33 Repeat Problem 12.32 if the analog input voltage is 3 V.

12.34 Repeat Problem 12.32 if the analog input voltage is 18 V.

12.35 make a sketch of a basic sample and hold circuit and briefly explain its operation.

12.36 Explain why a sample and hold circuit may be needed at the input of an analog-to-digital converter.

12.37 What is the highest-frequency component of an analog signal that can be accurately represented digitally if it is sampled at a rate of 100 kHz?

12.38 Calculate the minimum sampling frequency required to preserve all information when sampling a sine wave with a frequency of 130 kHz.

12.39 Suppose a sine wave with a period of 4.8 ms is sampled every 5.2 ms. What alias frequency will result? (Hint: see Figure 12.33.)

12.40 Calculate the corner frequency of an anti-aliasing filter for an ADC with a sampling frequency of 8 kHz. What type of filter (low-pass, high-pass, bandpass, etc.) is required?

Section 12.4 Data Acquisition

12.41 Refer to the data acquisition system in Figure 12.38. Write a VHDL file to implement the continuous-convert version of the ADC controller, as represented in the state diagram of Figure 12.42. Create a simulation in MAX+PLUS II to verify the operation of the controller.

12.42 Use the state machine controller from Problem 12.41 and an octal latch as components in a VHDL hierarchy that represents the ADC interface of Figure 12.38. Create a simulation in MAX+PLUS II to verify the operation of the design.

12.43 The data acquisition system in Figure 12.38 is designed with the controller from Problem 12.41. (The controller state diagram is shown in Figure 12.42.) Assume the controller and latch are interfaced with a different ADC that has a conversion time of 16 \( \mu \text{s} \), which is equivalent to 64 clock cycles. Calculate the highest-frequency component that can be accurately converted with this system for a clock rate of 787 kHz.

12.44 Repeat Problem 12.43 for a 4-channel data acquisition system, assuming the same conversion rate for the ADC and the controller state diagram of Figure 12.45.
ANSWERS TO SECTION REVIEW PROBLEMS

Section 12.1

12.1 5 bits (2^5 = 32). Resolution = 24 mV/32 steps = 0.75 mV/step.

Section 12.2a

12.2 4-bit: \( I_a = 0 \) to \( (15/16)(1 \text{ mA}) = 0 \) to 0.9375 mA; \( V_a = -I_a R_F \) = 0 to \(-9.375 \text{ V}\) 8-bit: \( I_a = 0 \) to \( (255/256)(1 \text{ mA}) = 0 \) to 0.9961 mA; \( V_a = 0 \) to \(-9.961 \text{ V}\)

Section 12.2b

12.3 2.048 M\(\Omega\).

Section 12.2c

12.4 \( V_o = (10 \text{ V}/2) + (10 \text{ V}/8) + (10 \text{ V}/256) = 6.29 \text{ V} \) or \( V_o = (161/256)10 \text{ V} = 6.29 \text{ V} \)

Section 12.2d

12.5 The maximum switching speed is higher if we choose the lower range of output voltage.

Section 12.2e

12.6 The output 0 V requires its own code. This leaves 255, not 256, codes for the remaining output values. The maximum value of a positive-only output is 255/256 of the reference voltage. A bipolar DAC ranges from \(-128/128\) to \(+127/128\) of the reference voltage.

Section 12.3

12.7 a. \(-1.5 \text{ V/ms}\);  b. \(+4 \text{ V/ms}\); c. 1.125 ms; d. 01100000.

Section 12.4

12.8 1.26 kHz
CHAPTER 13

Memory Devices and Systems

OUTLINE

13.1 Basic Memory Concepts
13.2 Random Access Read/Write Memory (RAM)
13.3 Read Only Memory (ROM)
13.4 Sequential Memory: FIFO and LIFO
13.5 Dynamic RAM Modules
13.6 Memory Systems

CHAPTER OBJECTIVES

Upon successful completion of this chapter, you will be able to:

• Describe basic memory concepts of address and data.
• Understand how latches and flip-flops act as simple memory devices and sketch simple memory systems based on these devices.
• Distinguish between random access read/write memory (RAM) and read only memory (ROM).
• Describe the uses of tristate logic in data bussing.
• Sketch the circuits of static and dynamic RAM cells.
• Sketch a block diagram of a static or dynamic RAM chip.
• Describe various types of ROM cells and arrays: mask-programmed, UV erasable, and electrically erasable.
• Use various types of ROM in simple applications, such as digital function generation.
• Describe the basic configuration of flash memory.
• Describe the basic configuration and operation of two types of sequential memory: first-in-first-out (FIFO) and last-in-first-out (LIFO).
• Describe how dynamic RAM is configured into high capacity memory modules.
• Sketch a basic memory system, consisting of several memory devices, an address and a data bus, and address decoding circuitry.
• Represent the location of various memory device addresses on a system memory map.
• Recognize and eliminate conditions leading to bus contention in a memory system.
• Expand memory capacity by parallel bussing and CPLD-based decoding.
In recent years, memory has become one of the most important topics in digital electronics. This is tied closely to the increasing prominence of cheap and readily available microprocessor chips. The simplest memory is a device we are already familiar with: the D flip-flop. This device stores a single bit of information as long as necessary. This simple concept is at the heart of all memory devices.

The other basic concept of memory is the organization of stored data. Bits are stored in locations specified by an “address,” a unique number which tells a digital system how to find data that have been previously stored. (By analogy, think of your street address: a unique way to find you and anyone you live with.)

Some memory can be written to and read from in random order; this is called random access read/write memory (RAM). Other memory can be read only: read only memory (ROM). Yet another type of memory, sequential memory, can be read or written only in a specific sequence. There are several variations on all these basic classes.

Memory devices are usually part of a larger system, including a microprocessor, peripheral devices, and a system of tristate busses. If dynamic RAM is used in such a system, it is often in a memory module of some type. The capacity of a single memory chip is usually less than the memory capacity of the microprocessor system in which it is used. In order to use the full system capacity, it is necessary to use a method of memory address decoding to select a particular RAM device for a specified portion of system memory.

### 13.1 Basic Memory Concepts

**KEY TERMS**

- **Memory** A device for storing digital data in such a way that they can be recalled for later use in a digital system.
- **Data** Binary digits (0s and 1s) that contain some kind of information. The digital contents of a memory device.
- **Address** A number, represented by the binary states of a group of inputs or outputs, uniquely defining the location of data stored in a memory device.
- **Write** Store data in a memory device.
- **Read** Retrieve data from a memory device.
- **Byte** A group of 8 bits.
- **Nibble** Half a byte; 4 bits.

**Address and Data**

A memory is a digital device or circuit that can store one or more bits of data. The simplest memory device, a D-type latch, shown in Figure 13.1, can store 1 bit. A 0 or 1 is stored in the latch and remains there until changed.

A simple extension of the single D-type latch is an array of latches, shown in Figure 13.2, that can store 8 bits (1 byte) of data. Figure 13.3 shows this octal latch used as a component in a MAX+PLUS II graphic file and configured as an 8-bit memory.

When the WRITE line goes LOW, then HIGH, data at the DATA_IN are stored in the eight latches. Data are available at the DATA_OUT pins when READ is HIGH. Note that although the READ and WRITE inputs are separate in this design, their functions would often be implemented as opposite logic levels of the same pin.

Figure 13.4 shows a simulation of the 8-bit memory. The LOW pulses on WRITE write the data, shown as two hexadecimal digits on the DATA_IN line, into the latches. To read the values stored in the eight latches, we set READ HIGH. In between read states, all DATA_OUT lines are in the high-impedance state, indicated by the notation ZZ.
FIGURE 13.1
D-Type Latch

FIGURE 13.2
Octal Latch
Figure 13.5 shows an expanded version of the octal latch memory circuit. Four octal latches are configured to make a 4 × 8-bit memory that can store and recall four separate 8-bit words. The octal latches are based on 8-bit latches instantiated in VHDL from the Altera Library of Parameterized Modules (LPM). The remaining components of Figure 13.5 are behaviorally-designed VHDL components.

The 8-bit input data are applied to the inputs of all four octal latches simultaneously. Data are written to a particular latch when a 2-bit address and a LOW on WRITEn cause an output of a 2-line-to-4-line decoder to enable the selected latch. For example, when ADDR[1..0] = 01 AND WRITEn = 0, decoder output Y1 goes HIGH, activating the ENABLE input on latch 1. The values at DATA_IN[7..0] are transferred to latch 1 and stored there when WRITEn goes HIGH.

The latch outputs are applied to the data inputs of an octal 4-to-1 multiplexer. Recall that this circuit will direct one of four 8-bit inputs to an 8-bit output. The selected set of inputs correspond to the binary value at the MUX select inputs, which is the same as the address applied to the decoder in the write phase. The MUX output is directed to the DATA_OUT lines by an octal tristate bus driver, which is enabled by the READ line. To read the contents of latch 1, we set the address to 01, as before, and make the READ line HIGH. If READ is LOW, the DATA_OUT lines are in the high-impedance state.

Figure 13.6 shows a simulation of the 4 × 8-bit memory. The address inputs change in a continuous binary sequence. For each address, a write pulse loads 8-bit data into the selected latch. After all four latches have been loaded, the latches are read in a rotating sequence. To read any new data from the memory, we would first have to write the new data into one or more of the latch locations.
FIGURE 13.5
4 × 8-bit Memory from Octal Latches
RAM and ROM

**Random access memory (RAM)** A type of memory device where data can be accessed in any order, that is, randomly. The term usually refers to random access read/write memory.

**Read only memory (ROM)** A type of memory where data are permanently stored and can only be read, not written.

The memory circuit in Figure 13.5 is one type of random access memory, or RAM. Data can be stored in or retrieved from any address at any time. The data can be accessed randomly, without the need to follow a sequence of addresses, as would be necessary in a sequential storage device such as magnetic tape.

RAM has come to mean random access read/write memory, memory that can have its data changed by a write operation, as well as have its data read. The data in another type of memory, called read only memory, or ROM, can also be accessed randomly, although it cannot be changed, or at least not changed as easily as RAM; there is no write function; hence the name “read only.” Even though both types of memory are random access, we generally do not include ROM in this category.

Memory Capacity

**b** Bit.

**B** Byte.

**K** \(1024 = 2^{10}\). Analogous to the metric prefix “k” (kilo-).

**M** \(1,048,576 = 2^{20}\). Analogous to the metric prefix “M” (mega-).

The capacity of a memory device is specified by the address and data sizes. The circuit shown in Figure 13.5 has a capacity of \(4 \times 8\) bits (“four-by-eight”). This tells us that the memory can store 32 bits, organized in groups of 8 bits at 4 different locations.

For large memories, with capacities of thousands or millions of bits, we use the short-hand designations **K** or **M** as prefixes for large binary numbers. The prefix **K** is analogous to, but not the same as, the metric prefix k (kilo). The metric kilo (lowercase k) indicates a multiplier of \(10^3 = 1000\); the binary prefix **K** (uppercase) indicates a multiplier of \(2^{10} = 1024\). Thus, one kilobit (Kb) is 1024 bits.
Similarly, the binary prefix M is analogous to the metric prefix M (mega). Both, unfortunately, are represented by uppercase M. The metric prefix represents a multiplier of $10^6 = 1,000,000$; the binary prefix M represents a value of $2^{20} = 1,048,576$. One megabit (Mb) is 1,048,576 bits. The next extension of this system is the multiplier G ($= 2^{30}$), which is analogous to the metric prefix G (giga; $10^9$).

There is a move afoot to untangle all the inconsistencies in this notation and develop separate units for binary and metric applications, but to date, such new notation is not very widely used.

---

**EXAMPLE 13.1**

A small microcontroller system (i.e., a stand-alone microcomputer system designed for a particular control application) has a memory with a capacity of 64 Kb, organized as $8K \times 8$. What is the total memory capacity of the system in bits? What is the memory capacity in bytes?

**Solution**  The total number of bits in the system memory is:

$$8K \times 8 = 8 \times 8 \times 1K = 64 \text{ Kb} = 64 \times 1024 \text{ bits} = 65,536 \text{ bits}$$

The number of bytes in system memory is:

$$\frac{64 \text{ Kb}}{8\text{B/byte}} = 8 \text{ KB}$$

Usually, the range of numbers spanning 1K is expressed as the 1024 numbers from $0_{10}$ to $1023_{10}$ (0000000000 to 11111111112). This is the full range of numbers that can be expressed by 10 bits. In hexadecimal, the range of numbers spanning 1K is from 00H to 3FFH. The range of numbers in 1M is given as the full hexadecimal range of 20-bit numbers: 00000H to FFFFFH.

The range of numbers spanning 8K can be written in 13 bits ($8 \times 1K = 2^3 \times 2^{10} = 2^{13}$). The addresses in an $8K \times 8$ memory range from 000000000000 to 1111111111111111, or 0000 to 1FFF in hexadecimal. Thus, a memory device that is organized as $8K \times 8$ has 13 address lines and 8 data lines.

Figure 13.7 shows the address and data lines of an $8K \times 8$ memory and a map of its contents. The addresses progress in binary order, but the contents of any location are the

![FIGURE 13.7](image-url)
last data stored there. Since there is no way to predict what those data are, they are essentially random. For example, in Figure 13.7, the byte at address 0000000000100_2 (0004H) is 01110111_2 (77H). (One can readily see the advantage of using hexadecimal notation.)

EXAMPLE 13.2

How many address lines are needed to access all addressable locations in a memory that is organized as 64K \times 4? How many data lines are required?

**Solution**

Address lines: \(2^n = 64\mathrm{K}\)

\[
64\mathrm{K} = 64 \times 1\mathrm{K} = 2^6 \times 2^{10} = 2^{16}
\]

\(n = 16\) address lines

Data lines: There are 4 data bits for each addressable location. Thus, the memory requires 4 data lines.

Control Signals

Two memory devices are shown in Figure 13.8. The device in Figure 13.8a is a 1K \times 4 random access read/write memory (RAM). Figure 13.8b shows 8K \times 8 erasable programmable read only memory (EPROM). The address lines are designated by \(A\) and the data lines by \(DQ\). The dual notation \(DQ\) indicates that these lines are used for both input \((D)\) and output \((Q)\) data, using the conventional designations of D-type latches. The input and output data are prevented from interfering with one another by a pair of opposite-direction tristate buffers on each input/output pin. One buffer goes to a memory cell input; the other comes from the memory cell output. The tristate outputs on the devices in Figure 13.8 allow the outputs to be electrically isolated from a system data bus that would connect several such devices to a microprocessor.

**FIGURE 13.8**

Address, Data, and Control Signals
In addition to the address and data lines, most memory devices, including those in Figure 13.8, have one or more of the following control signal inputs. (Different manufacturers use different notation, so several alternate designations for each function are listed.)

**E (or CE or CS).** Enable (or Chip Enable or Chip Select). The memory is enabled when this line is pulled LOW. If this line is HIGH, the memory cannot be written to or read from.

**W (or WE or R/W).** Write (or Write Enable or Read/Write). This input is used to select the read or write function when data input and output are on the same lines. When HIGH, this line selects the read (output) function if the chip is selected. When LOW, the write (input) function is selected.

**G (or OE).** Gate (or Output Enable). Some memory chips have a separate control to enable their tristate output buffers. When this line is LOW, the output buffers are enabled and the memory can be read. If this line is HIGH, the output buffers are in the high-impedance state. The chip select performs this function in devices without output enable pins.

The electrical functions of these control signals are illustrated in Figure 13.9.
13.2 Random Access Read/Write Memory (RAM)

**KEY TERMS**

- **Volatile**  A memory is volatile if its stored data are lost when electrical power is lost.
- **Static RAM**  A random access memory that can retain data indefinitely as long as electrical power is available to the chip.
- **Dynamic RAM**  A random access memory that cannot retain data for more than a few milliseconds without being “refreshed.”
- **RAM cell**  The smallest storage unit of a RAM, capable of storing 1 bit.

Random access read/write memory (RAM) is used for temporary storage of large blocks of data. An important characteristic of RAM is that it is volatile. It can retain its stored data only as long as power is applied to the memory. When power is lost, so are the data. There are two main RAM configurations: static (SRAM) and dynamic (DRAM).

**Static RAM (SRAM)** consists of arrays of memory cells that are essentially flip-flops. Data can be stored in a static RAM cell and left there indefinitely, as long as power is available to the RAM.

A **dynamic RAM cell** stores a bit as the charged or discharged state of a small capacitor. Since the capacitor can hold its charge for only a few milliseconds, the charge must be restored (“refreshed”) regularly. This makes a dynamic RAM (DRAM) system more complicated than SRAM, as it introduces a requirement for memory refresh circuitry.

DRAMs have the advantage of large memory capacity over SRAMs. At the time of this writing, the largest SRAMs have a capacity of about 4 Mb, whereas the largest DRAMs have a capacity of 256 Mb. DRAM modules, that is, groups of DRAM chips on a small circuit board, have capacities of up to 1 GB. These figures are constantly increasing and are never up to date for very long. (The most famous estimate of the growth rate of semiconductor memory capacity, Moore’s law, estimates that it doubles every 18 months. My casual observation is that this is accurate to within an order of magnitude.)

**Static RAM Cells**

The typical static RAM cell consists of at least two transistors that are cross-coupled in a flip-flop arrangement. Other parts of the cell include pull-up circuitry that can be active (transistor switches) or passive (resistors) and some decoding/switching logic. Figure 13.10 shows an SRAM cell in three technologies: bipolar, NMOS, and CMOS.

Each of these cells can store 1 bit of data, a 0 or a 1, as the state of one of the transistors in the cell. The data are available in true or complement form, as the \( \text{BIT} \) and \( \overline{\text{BIT}} \) outputs of the flip-flop.

All types of SRAM cells operate in more or less the same way. We will analyze the operation of the NMOS cell (Figure 13.10b) and then compare it to the other types.

Transistors \( Q_1 \) and \( Q_2 \) are permanently biased ON, making them into pull-up resistors. Channel width and length are chosen to give a resistance of about 1 k\( \Omega \). These NMOS load transistors are considered passive pull-ups, as they do not switch on and off.

A bit is stored as \( V_{DS3} \), the drain voltage of \( Q_3 \) with respect to its source. If this voltage is HIGH, the gate of \( Q_4 \) is HIGH with respect to its source and \( Q_4 \) is biased ON. This completes a conduction path from the drain of \( Q_4 \) to its source, making \( V_{DSS} \) logic LOW. This LOW is fed back to the gate of \( Q_3 \), turning it OFF. There is no conduction path between the drain and source of \( Q_3 \), so \( V_{DS3} = V_{DD} \) or logic HIGH. The cell is storing a 1.

This bit can be read by making the **ROW SELECT** line HIGH. This turns \( Q_5 \) and \( Q_6 \) ON, which puts the data onto the **BIT** and \( \overline{\text{BIT}} \) lines where it can be read by other circuitry inside the RAM chip.

To change the cell contents to a 0, we make the **BIT** line LOW and the **ROW SELECT** line HIGH. The **ROW SELECT** line gives access to the cell by turning on \( Q_5 \) and \( Q_6 \), com-
13.2 • Random Access Read/Write Memory (RAM)

Completing the conduction path between the BIT lines and the flip-flop inputs. The LOW on the BIT line pulls the gate of $Q_4$ LOW, turning it OFF. This breaks the conduction path from $Q_4$ drain to source and makes $V_{DS4} = V_{DD}$, a logic HIGH. This HIGH is applied to the gate of $Q_3$, turning it ON. A conduction path is established between $Q_3$ drain and source, pulling the drain of $Q_3$ LOW. The cell now stores a logic 0.

**NOTE**

The contents of an SRAM cell must be changed by introducing a LOW on the BIT or the BIT line. The data cannot be changed by pulling an input HIGH without pulling the opposite input LOW. If a MOSFET gate is at the LOW state, a HIGH applied to that gate will be pulled down by the LOW level already existing there and will not cause the cell to change state.
The CMOS cell (Figure 13.10c) functions in the same way, except for the actions of \( Q_1 \) and \( Q_2 \). \( Q_1 \) and \( Q_3 \) are a complementary pair, as are transistors \( Q_2 \) and \( Q_4 \). For each of these pairs, when the\( p \)-channel transistor is ON, the\( n \)-channel is OFF, and vice versa. This arrangement is more energy efficient than the NMOS cell, since there is not the constant current drain associated with the load transistors. Power is consumed primarily during switching between states.

The main design goal of new memory technology is to increase speed and capacity while reducing power consumption and chip area. The NMOS cell has the advantage of being constructed from only one type of component. This makes it possible to manufacture more cells in the same chip area than can be done in either the CMOS or bipolar technologies. NMOS chips, however, are slower than bipolar. New advances in high-speed CMOS technologies have made possible CMOS memories that are as dense or denser than NMOS and faster. Because of this, NMOS will probably decline in importance over time.

Bipolar SRAMs can be either TTL, as shown in Figure 13.10a, or ECL, which is not shown. Of the two bipolar technologies, ECL is the faster. Historically, all bipolar SRAMs have had the advantage of speed over NMOS and CMOS chips. New CMOS devices however, have exceeded the speeds of TTL.

The bipolar SRAM cell is the least suitable for high-density memory. Both bipolar transistors and resistors are large components compared to a MOSFET. Thus, the bipolar cell is inherently larger than the CMOS or NMOS cell. Bipolar memories historically have been used when a small amount of high-speed memory is required.

The operation of the bipolar SRAM cell is similar to that of the MOSFET cells. In the quiescent state, the \textit{ROW SELECT} line is LOW. In either the Read or the Write mode, the \textit{ROW SELECT} line is HIGH. To change the data in the cell, pull one of the emitters LOW. When the emitter of \( Q_1 \) goes LOW, the cell contents become 0. When the emitter of \( Q_2 \) is pulled LOW, the cell contents are 1.

### Static RAM Cell Arrays

**KEY TERMS**

- **Word-organized** A memory is word-organized if one address accesses one word of data.
- **Word** Data accessed at one addressable location.
- **Word length** Number of bits in a word.

Static RAM cell arrays are arranged in a square or rectangular format, accessible by groups in rows and columns. Each column corresponds to a complementary pair of \textit{BIT} lines and each row to a \textit{ROW SELECT} line, as shown in Figure 13.11.

The column lines have MOSFETs configured as pull-up resistors at one end and a circuit called a sense amplifier at the other. The sense amp is a large RAM cell that amplifies the charge of an active storage cell on the same \textit{BIT} line. Having a larger RAM cell as a sense amp allows the storage cells to be smaller, since each individual cell need not carry the charge required for a logic level output.

Figure 13.12 shows the block diagram of a 4 megabit (Mb) SRAM array, including blocks for address decoding and output circuitry. The RAM cells are arrayed in a pattern of 512 rows and 8192 columns for efficient packaging. When a particular address is applied to address lines \( A_{18} \ldots A_0 \), the row and column decoders select an SRAM cell in the memory array for a read or write by activating the associated sense amps for the column and the row select line for the cell.

The columns are further subdivided into groups of eight, so that one column address selects eight bits (one byte) for a read or write operation. Thus, there are 512 separate row addresses (9 bits) and 1024 separate column addresses (10 bits) for every unique group of 8 data bits, requiring a total of 19 address lines and 8 data lines. The capacity of the SRAM can be written as \( 512 \times 1024 \times 8 \).
13.2 • Random Access Read/Write Memory (RAM)

**FIGURE 13.11**
SRAM Cell Array

**FIGURE 13.12**
Block Diagram of a 4Mb (512 KB) SRAM
Since one address reads or writes 8 cells, we say that the SRAM in Figure 13.12 is word-organized and that the word length of the SRAM is 8 bits. Other popular word lengths for various memory arrays are 4, 16, 32, and 64 bits.

SECTION 13.2A REVIEW PROBLEM

13.1 If an SRAM array is organized as $512 \times 512 \times 16$, how many address and data lines are required? How does the bit capacity of this SRAM compare to that of Figure 13.12?

Dynamic RAM Cells

**KEY TERM**

Refresh cycle  The process that periodically recharges the storage capacitors in a dynamic RAM.

A dynamic RAM (DRAM) cell consists of a capacitor and a pass transistor, as shown in Figure 13.13. A bit is stored in the cell as the charged or discharged state of the capacitor. The bit location is read from or written to by activating the cell MOSFET via the Word Select line, thus connecting the capacitor to the BIT line.

![FIGURE 13.13](Dynamic RAM Cell)

The major disadvantage of dynamic RAM is that the capacitor will eventually discharge by internal leakage current and must be recharged periodically to maintain integrity of the stored data. The recharging of the DRAM cell capacitors, known as refreshing the memory, must be done every 8 to 64 ms, depending on the device.

The refresh cycle adds an extra level of complication to the DRAM hardware and also to the timing of the read and write cycles, since the memory might have to be refreshed between read and write tasks. DRAM timing cycles are much more complicated than the equivalent SRAM cycles.

This inconvenience is offset by the high bit densities of DRAM, which are possible due to the simplicity of the DRAM cell. Up to 256 megabits of data can be stored on a single chip.
13.2 • Random Access Read/Write Memory (RAM) 635

DRAM Cell Arrays

**KEY TERMS**

- **Bit-organized**  A memory is bit-organized if one address accesses one bit of data.
- **Address multiplexing**  A technique of addressing storage cells in a dynamic RAM that sequentially uses the same inputs for the row address and column address of the cell.
- **RAS**  Row address strobe. A signal used to latch the row address into the decoding circuitry of a dynamic RAM with multiplexed addressing.
- **CAS**  Column address strobe. A signal used to latch the column address into the decoding circuitry of a dynamic RAM with multiplexed addressing.

Dynamic RAM is sometimes **bit-organized** rather than word-organized. That is, one address will access one bit rather than one word of data. A bit-organized DRAM with a large capacity requires more address lines than a static RAM (e.g., 4 Mb × 1 DRAM requires 22 address lines \(2^{22} = 4,194,304 = 4M\) and 1 data line to access all cells).

In order to save pins on the IC package, a system of **address multiplexing** is used to specify the address of each cell. Each cell has a row address and a column address, which use the same input pins. Two negative-edge signals called **row address strobe** (RAS) and **column address strobe** (CAS) latch the row and column addresses into the DRAM’s decoding circuitry. Figure 13.14 shows a simplified block diagram of the row and column addressing circuitry of a 1 Mb × 1 dynamic RAM.

Figure 13.15 shows the relative timing of the address inputs of a dynamic RAM. The first part of the address is applied to the address pins and latched into the row address buffers.

---

**FIGURE 13.14**  
Row and Column Decoding in a 1M × 1 Dynamic RAM

**FIGURE 13.15**  
DRAM Address Latch Signals
when RAS goes LOW. The second part of the address is then applied to the address pins and latched into the column address buffers by the CAS signal. This allows a 20-bit address to be implemented with 12 pins: 10 address and 2 control lines. Adding another address line effectively adds 2 bits to the address, allowing access to 4 times the number of cells.

The memory cell array in Figure 13.14 is rectangular, not square. One of the Row Address lines is connected internally to the Column Address decoder, resulting in a 512-row-by-2048-column memory array.

One advantage to the rectangular format shown is that it cuts the memory refresh time in half, since all the cells are refreshed by accessing the rows in sequence. Fewer rows means a faster refresh cycle. All cells in a row are also refreshed by normal read and write operations.

SECTION 13.2B REVIEW PROBLEMS

13.2 How many address and data lines are required for the following sizes of dynamic RAM, assuming that each memory cell array is organized in a square format, with common Row and Column Address pins?
   a. 1M × 1
   b. 1M × 4
   c. 4M × 1

13.3 Read Only Memory (ROM)

The main advantage of read only memory (ROM) over random access read/write memory (RAM) is that ROM is nonvolatile. It will retain data even when electrical power is lost to the ROM chip. The disadvantage of this is that stored data are difficult or impossible to change.

ROM is used for storing data required for tasks that never or rarely change, such as software instructions for a bootstrap loader in a personal computer or microcontroller. (The bootstrap loader—a term derived from the whimsical idea of pulling oneself up by one’s bootstraps, that is, starting from nothing—is the software that gives the personal computer its minimum startup information. Generally, it contains the instructions needed to read a magnetic disk containing further operating instructions. This task is always the same for any given machine and is needed every time the machine is turned on, thus making it the ideal candidate for ROM storage.)

Software instructions stored in ROM are called firmware.

Mask-Programmed ROM

Mask-programmed ROM A type of read only memory (ROM) where the stored data are permanently encoded into the memory device during the manufacturing process.
The most permanent form of read only memory is the **mask-programmed ROM**, where the stored data are manufactured into the memory chip. Due to the inflexibility of this type of ROM and the relatively high cost of development, it is used only for well-developed high-volume applications. However, even though development cost of a mask-programmed ROM is high, volume production is cheaper than for some other types of ROM.

Examples of applications suitable to mask-programmed ROM include:

- Bootstrap loaders and BIOS (basic input/output system) for PCs.
- Character generators (decoders that convert ASCII codes into alphanumeric characters on a CRT display)
- Function lookup tables (tables corresponding to binary values of trigonometric, exponential, or other functions)
- Special software instructions that must be permanently stored and never changed (firmware)

Figure 13.16 shows a ROM based on a matrix of MOSFETs. Each cell is manufactured with a MOSFET and its gate and source connections. LOWs are programmed by making a connection between the drain of the cell’s MOSFET and the corresponding Bit

---

**FIGURE 13.16**

Mask-Programmed ROM
line. When the appropriate Row Select goes HIGH, the MOSFET turns ON, providing a path to ground from the selected Bit line. Cells programmed HIGH have no connection between the MOSFET drain and the Bit line, which thus cannot be pulled LOW when the cell is selected.

These connections can be made by a custom overlay of connections (a mask) on top of the standard-cell layer. The standard-cell-plus-custom-overlay format is cheaper to manufacture than custom cells for each bit, even if many of the MOSFETs are never used.

**EPROM**

**KEY TERMS**

**EPROM** Erasable programmable read only memory. A type of ROM that can be programmed (“burned”) by the user and erased later, if necessary, by exposing the chip to ultraviolet radiation.

**FAMOS FET** Floating-gate avalanche MOSFET. A MOSFET with a second, “floating” gate in which charge can be trapped to change the MOSFET’s gate-source threshold voltage.

Mask-programmed ROM is useful because of its nonvolatility, but it is hard to program and impossible to erase. **Erasable programmable read only memory (EPROM)** combines the nonvolatility of ROM with the ability to change the internal data if necessary.

This erasability is particularly useful in the development of a ROM-based system. Anyone who has built a complex circuit or written a computer program knows that there is no such thing as getting it right the first time. Modifications can be made easily and cheaply to data stored in an EPROM. Later, when the design is complete, a mask ROM version can be prepared for mass production. Alternatively, if the design will be produced in small numbers, the ROM data can be stored in EPROMs, saving the cost of preparing a mask-programmed ROM.

The basis of the EPROM memory cell is the **FAMOS FET**, whose circuit symbol is shown in Figure 13.17. FAMOS stands for floating-gate avalanche metal-oxide-semiconductor. (“Avalanche” refers to electron behavior in a semiconductor under certain bias conditions.) This is a MOSFET with a second, or floating, gate that is insulated from the first by a thin oxide layer.

The floating gate has no electrical contact with either the first gate or the source and drain terminals. As is the case in a standard MOSFET, conduction between drain and source terminals is effected by the voltage of the gate terminal with respect to the source. If this voltage is above a certain threshold level, the transistor will turn ON, allowing current to flow between drain and source.

In the unprogrammed state the FAMOS transistor’s threshold voltage is low enough for the transistor to be turned ON by a 5-V read signal on the Row Select line. During the
programming operation, a relatively high voltage pulse (about 12 V to 25 V, depending on the device) on the Row Select line drives high-energy electrons into the floating gate and traps them there. This raises the threshold voltage of the programmed cell to a level where the cell won’t turn ON when selected by a 5-V read.

The EPROM cells are configured so that an unprogrammed location contains a logic HIGH and the programming signal forces it LOW.

To erase an EPROM, the die (i.e., the silicon chip itself) must be exposed for about 20 to 45 minutes to high-intensity ultraviolet light of a specified wavelength (2537 angstroms) at a distance of 2.5 cm (1 inch). The high-energy photons that make up the UV radiation release the electrons trapped in the floating gate and restore the cell threshold voltages to their unprogrammed levels.

EPROMS are manufactured with a quartz window over the die to allow the UV radiation in. Since both sunlight and fluorescent light contain UV light of the right wavelength to erase the EPROM over time (several days to several years, depending on the intensity of the source), the quartz window should be covered by an opaque label after the EPROM has been programmed.

**EPROM Application: Digital Function Generator**

An EPROM can be used as the central component of a digital function generator. Other components in the system include a clock generator, a counter, a digital-to-analog converter, and an output op amp buffer. The portion of the circuit including the last three of these components is shown in Figure 13.18.

The generator can produce the usual analog waveforms—sine, square, triangle, sawtooth—and any other waveforms that you wish to store in the EPROM. A single cycle of

![FIGURE 13.18](digital_function_generator.png)  
Digital Function Generator
each waveform is stored as 256 consecutive 8-bit numbers. For example, the data for one cycle of the sine waveform are stored at addresses 0000H to FFFFH, as shown in hex form in Table 13.1. (FF is maximum positive, 80 is zero, and 00 is maximum negative.) The square wave data are stored at addresses 0100 to 01FF, also shown in Table 13.1. The data for other functions, stored in subsequent 256-byte blocks, are not shown. A full list of the function data and an ANSI C program to generate an EPROM record file (Intel format) are included in Appendix E.

The counter and EPROM can also be implemented in a CPLD. Alternatively, a VHDL-designed state machine can replace the counter and EPROM, except for the sine function.

### Table 13.1 EPROM Sine and Square Wave Data

#### SINE

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</table>
These configurations are designed and built as exercises in the lab manual that accompanies this book.

The most significant bits of the EPROM address select the waveform function by selecting a block of 256 address. The 8 least significant bits of the EPROM address are connected to an 8-bit (mod-256) counter, which continuously cycles through the 256 selected addresses. A 27C64 EPROM (8K × 8) has 13 address lines. After the eight lower lines are accounted for, the remaining five lines can be used to select up to 32 digital functions. With the two binary Function Select switches, we can potentially select 4 functions.

For example, to select the Sine function, inputs A9 and A8, which comprise the most significant digit of the EPROM address, are set to 00. Thus, the 8-bit counter cycles through addresses 0000–00FF, the location of the sine data. The Square Wave function is selected by setting A9 and A8 to 01, thus selecting the address block 0100–01FF. Other functions can be similarly selected.

The data at each address are sent to the D/A converter (MC1408), which, in combination with the op amp, is configured to produce a bipolar (both positive and negative) output. (We use a high slew rate op amp so that the generated square waves will have vertical sides.) The circuit generates a continuous waveform by retracing the data points in one 256-byte section of the EPROM over and over.

The DAC/op amp combination produces a maximum negative voltage for a hex input of 00, a 0 V output for an input of 80, and a maximum positive voltage for an input of FF. (You might wish to refer to the section Bipolar Operation of MC1408 in Chapter 12 for details of the DAC operation.)

You can see from the Sine function data in Table 13.1 that 8 bits are not sufficient to represent each of the 256 steps of a digital sine function as a unique number. The peaks of the waveform are changing too slowly to be represented accurately by an 8-bit quantization, and as a result, the top of the sine wave is flat for several clock pulses. (Mathematically, a sine function is tangential to a horizontal line at its peak. However, since tangential means touching at one point, the flat top is a distortion.) A unique number for each of 256 steps of a sine function needs at least 13 bits, but this requires additional bits on the D/A converter input, and therefore a different DAC and an expanded memory word length.

The output frequency of the function generator is 1/256 of the clock rate. Given that the settling time of the MC1408 DAC is about 300 ns, the maximum clock rate of the circuit is 1/300 ns = 3.33 MHz. At this rate, the output frequency is 3.33 MHz/256 = 13 kHz.

**EEPROM**

<table>
<thead>
<tr>
<th>Key Term</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EEPROM (or E²PROM)</strong> Electrically erasable programmable read only memory. A type of read only memory that can be field-programmed and selectively erased while still in a circuit.</td>
</tr>
</tbody>
</table>

As was discussed in the previous section, EPROMs have the useful property of being erasable. The problem is that they must be removed from the circuit for erasure, and bits

---

1. Bits required:
   - \(360°/256\) steps = 1.40625°/step.
   - Sine function changes most slowly at peak, so calculate \(A \sin(90° ± 1.40625°)\) to find smallest amplitude change.
   - The smallest power-of-2 amplitude, \(A\), for which \(A \sin(90°) = A \sin(90° ± 1.40625°) \geq 1\) is 4096.
   - The amplitude range \(-4096 \leq A \leq 4095\) can be represented by a 13-bit number.
cannot be selectively erased; the whole memory cell array is erased as a unit.

**Electrically erasable programmable read only memory** (EEPROM or E²PROM) provides the advantages of EPROM along with the additional benefit of allowing erasure of selected bits while the chip is in the circuit; it combines the read/write properties of RAM with the nonvolatility of ROM. EEPROM is useful for storage of data that need to be changed occasionally, but that must be retained when power is lost to the EEPROM chip. One example is the memory circuit in an electronically tuned car radio that stores the channel numbers of local stations.

Like the UV-erasable EPROM, the memory cell of the EEPROM is based on the FAMOS transistor. Unlike the EPROM, the FAMOS FET is coupled with a standard MOS-FET, as shown in Figure 13.19.

The FAMOS FET is programmed in the same way as UV-erasable EPROM: a programming voltage pulse ($V_{pp}$) drives high-energy electrons into the floating gate of the FAMOS transistor, where they remain trapped and change the threshold voltage of the transistor. The cell is read by keeping the programming line at 5 V and making the cell’s Row Select line HIGH. The FAMOS transistor will or will not turn on, depending on its programmed state.

The FAMOS transistors used in EPROM and EEPROM differ in one important respect. The EEPROM transistor is manufactured with a very thin oxide layer between the drain and the upper (nonfloating) gate. This construction allows trapped electrons in the floating gate to be forced out electrically, thus erasing the cell contents.

Given the obvious advantages of EEPROM, why doesn’t it replace all other types of memory? There are several reasons:

1. EEPROM has a much slower access time than RAM and is thus not good for high-speed applications.
2. The currently available EEPROMs have significantly smaller bit capacities than commercially available RAM (especially dynamic RAM) and EPROM.
3. EEPROM has a fixed number of write/erase cycles, typically 100,000. After that, new data cannot be programmed into the device.
Flash Memory

**KEY TERMS**

- **Flash memory** A nonvolatile type of memory that can be programmed and erased in sectors, rather than byte-at-a-time.
- **Sector** A segment of flash memory that forms the smallest amount that can be erased and reprogrammed at one time.
- **Boot block** A sector in a flash memory reserved for primary firmware.
- **Top boot block** A boot block sector in a flash memory placed at the highest address in the memory.
- **Bottom boot block** A boot block sector in flash memory placed at the lowest address in the memory.

A popular variation on EEPROM is flash memory. This type of nonvolatile memory generally has a larger byte capacity (e.g., 8 Mb) than EEPROM devices and thus can be used to store fairly large amounts of firmware, such as the BIOS (basic input/output system) of a PC.

A flash memory is divided into sectors, groups of bytes that are programmed and erased at one time. One sector is designated as the boot block, which is either the sector with the highest (top boot block) or lowest (bottom boot block) address. The primary firmware is usually stored in the boot block, with the idea that the system using the flash memory is configured to look there first for firmware instructions. The boot block can also be protected from unauthorized erasure or modification (e.g., by a virus), thus adding a security feature to the device.

Figure 13.20 shows the arrangement of sectors of a 512K × 8-bit (4 Mb) flash memory with a bottom boot block architecture. The range of addresses are shown alongside the blocks. For example, sector S0 (the boot block) has a 16 KB address range of 00000H to 03FFFH. Sector S1 has an 8 KB address range from 04000H to 05FFFH. The first 64 KB of the memory are divided into one 16 KB, two 8 KB, and one 32 KB sectors. The remainder of the memory is divided into equal 64 KB sectors. Note that even though the boot block is drawn at the top of Figure 13.20, it is a bottom boot block because it is the sector with the lowest address.

A flash memory with a top boot block would have the same proportions given over to its sectors, but mirror-image to the diagram in Figure 13.20. That is, S10 (boot block) would be a 16 KB sector from 7C000H to 7FFFFH. The other sectors would be identical to the bottom boot block architecture, but in reverse order.

As with other EEPROM devices, a flash memory can be erased and reprogrammed while installed in a circuit. The memory cells in a flash device have a limited number of program/erase cycles, like other EEPROMs. The sector architecture of the flash memory makes it faster to erase and program than other EEPROM-based memories which must erase or program bytes one at a time. This same characteristic makes it unsuitable for use as system RAM, which must be able to program single bytes.

### SECTION 13.3 REVIEW PROBLEM

13.3 A flash memory has a capacity of 8 Mb, organized as 1M × 8-bit. List the address range for the 32 KB boot block sector of the memory if the device has a bottom boot block architecture and if it has a top boot block architecture.
### FIGURE 13.20
Sectors in a 512K × 8b Flash Memory (Bottom Boot Block)
13.4 Sequential Memory: FIFO and LIFO

**KEY TERMS**

**Sequential Memory**  Memory in which the stored data cannot be read or written in random order, but must be addressed in a specific sequence.

**FIFO**  First-in first-out. A sequential memory in which the stored data can be read only in the order in which it was written.

**Queue**  A FIFO memory.

**LIFO**  Last-in first-out. A sequential memory in which the last data written are the first data read.

**Stack**  A LIFO memory.

The RAM and ROM devices we have examined up until now have all been random access devices. That is, any data could be read from or written to any sequence of addresses in any order. There is another class of memory in which the data must be accessed in a particular order. Such devices are called **sequential memory**.

There are two main ways of organizing a sequential memory—as a **queue** or as a **stack**. Figure 13.21 shows the arrangement of data in each of these types of memory.

A queue is a **first-in first-out** (FIFO) memory, meaning that the data can be read only in the same order they are written, much as railway cars always come out of a tunnel in the same order they go in.

One common use for FIFO memory is to connect two devices that have different data rates. For instance, a computer can send data to a printer much faster than the printer can use it. To keep the computer from either waiting for the printer to print everything or periodically interrupting the computer’s operation to continue the print task, data can be sent in a burst to a FIFO, where the printer can read them as needed. The only proviso is that there...
must be some logic signal to the computer telling it when the queue is full and not to send more data and another signal to the printer letting it know that there are some data to read from the queue.

The last-in-first-out (LIFO), or stack, memory configuration, also shown in Figure 13.21, is not available as a special chip, but rather is a way of organizing RAM in a memory system.

The term “stack” is analogous to the idea of a spring-loaded stack of plates in a cafeteria line. When you put a bunch of plates on the stack, they settle into the recessed storage area. When a plate is removed, the stack springs back slightly and brings the second plate to the top level. (The other plates, of course, all move up a notch.) The top plate is the only one available for removal from the stack, and plates are always removed in reverse order from that in which they were loaded.

Figure 13.21b shows how data are transferred to and from a LIFO memory. A block of addresses in a RAM is designated as a stack, and one or two bytes of data in the RAM store a number called the stack pointer, which is the current address of the top of the stack.

In Figure 13.21, the value of the stack pointer changes with every change of data in the stack, pointing to the last-in data in every case. When data are removed from the stack, the stack pointer is used to locate the data that must be read first. After the read, the stack pointer is modified to point to the next-out data. Some stack configurations have the stack pointer painting to the next empty location on the stack.

The most common application for LIFO memory is in a computer system. If a program is interrupted during its execution by a demand from the program or some piece of hardware that needs attention, the status of various registers within the computer are stored on a stack and the computer can pay attention to the new demand, which will certainly change its operating state. After the interrupting task is finished, the original operating state of the computer can be taken from the top of the stack and reloaded into the appropriate registers, and the program can resume where it left off.

SECTION 13.4 REVIEW PROBLEM

13.4 State the main difference between a stack and a queue.

13.5 Dynamic RAM Modules

**KEY TERMS**

- **Memory module**: A small circuit board containing several dynamic RAM chips.
- **Single in-line memory module (SIMM)**: A memory module with DRAMs and connector pins on one side of the board only.
- **Dual in-line memory module (DIMM)**: A memory module with DRAMs and connector pins on both sides of the board.

Dynamic RAM chips are often combined on a small circuit board to make a memory module. This is because the data bus widths of systems requiring the DRAMs are not always the same as the DRAMs themselves. For example, Figure 13.22 shows how four 64M×8 DRAMs are combined to make a 64M×32 memory module. The block diagram of the module is shown in Figure 13.22, and the mechanical outline is shown in Figure 13.23. The data input/output lines are separate from one another so that there are 32 data I/Os (DQ). The address lines (ADDR[12..0]) for the module are parallel on all chips. With address multiplexing, this 13-bit address bus yields a 26-bit address, giving a 64M address range. Chip selects (CS) for all devices are connected together so that selecting the module selects all chips on the module.

This particular memory module is configured as a single in-line memory module (SIMM), which has the DRAM chips and pin connections on one side of the board only. A
dual in-line memory module (DIMM) has the DRAMs mounted on both sides of the circuit board and pin connections on both sides of the board as well.

SECTION 13.5 REVIEW PROBLEM

13.5 A SIMM has a capacity of 16M × 32. How many 16M × 8 DRAMs are required to make this SIMM? How many address lines does the SIMM require? How should the DRAMs be connected?
13.6 Memory Systems

**KEY TERMS**

- **Address decoder**: A circuit enabling a particular memory device to be selected by the address bus of a larger memory system.
- **Address space**: A block of addresses in a memory system.
- **Bus contention**: The condition that results when two or more devices try to send data to a bus at the same time. Bus contention can damage the output buffers of the devices involved.
- **Memory map**: A diagram showing the total address space of a memory system and the placement of various memory devices within that space.

In the section on memory modules, we saw how multiple memory devices can be combined to make a system that has the same number of addressable locations as the individual devices making up the system, but with a wider data bus. We can also create memory systems where the data I/O width of the system is the same as the individual chips, but where the system has more addressable locations than any chip within the system.

In such a system, the data I/O and control lines from the individual memory chips are connected in parallel, as are the lower bits of an address bus connecting the chips. However, it is important that only one memory device be enabled at any given time, in order to avoid **bus contention**, the condition that results when more than one output attempts to drive a common bus line. To avoid bus contention, one or more additional address lines must be decoded by an **address decoder** that allows only one chip to be selected at a time.

Figure 13.24 shows two 32K × 8 SRAMs connected to make a 64K × 8 memory system. A single 32K × 8 SRAM, as shown in Figure 13.24a, requires 15 address lines, 8 data lines, a write enable (WE), and chip select (CS) line. To make a 64K × 8 SRAM system, all of these lines are connected in parallel, except the CS lines. In order to enable only one at a time, we use one more address line, A15, and enable the top SRAM when A15 = 0 and the bottom SRAM when A15 = 1.

The address range of one 32K × 8 SRAM is given by the range of states of the address lines A[14..0]:

- Lowest single-chip address: 000 0000 0000 0000 = 0000H
- Highest single-chip address: 111 1111 1111 1111 = 7FFFH

The address range of the whole system must also account for the A15 bit:

- Lowest system address: 0000 0000 0000 0000 = 0000H
- Highest system address: 1111 1111 1111 1111 = FFFFH

Within the context of the system, each individual SRAM chip has a range of addresses, depending on the state of A15. Assume SRAM0 is selected when A15 = 0 and SRAM1 is selected when A15 = 1.

- Lowest SRAM0 address: 0000 0000 0000 0000 = 0000H
- Highest SRAM0 address: 0111 1111 1111 1111 = 7FFFH
- Lowest SRAM1 address: 1000 0000 0000 0000 = 8000H
- Highest SRAM1 address: 1111 1111 1111 1111 = FFFFH

Figure 13.25 shows a memory map of the 64K × 8 SRAM system, indicating the range of addresses for each device in the system. The total range of addresses in the system is called the **address space**.
a. Single 32 K × 8 SRAM

b. Two 32 K × 8 SRAMs connected to make 64 K × 8 SRAM system

FIGURE 13.24
Expanding Memory Space

FIGURE 13.25
Memory Map
EXAMPLE 13.3

Figure 13.26 shows a memory map for a system with an address space of 64K (16 address lines). Two 16K \times 8 blocks of SRAM are located at start addresses of 0000H and 8000H, respectively. Sketch a memory system that implements the memory map of Figure 13.26.

Solution

A 16K address block requires 14 address lines, since

\[ 16K = 16 \times 1024 = 2^4 \times 2^{10} = 2^{14} \]

The entire 64K address space requires 16 address lines, since

\[ 64K = 64 \times 1024 = 2^6 \times 2^{10} = 2^{16} \]

The highest address in a block is the start address plus the block size.

- **SRAM0:**
  - Lowest address: 0000 0000 0000 0000 = 0000H
  - Highest address: 0011 1111 1111 1111 = 3FFFH

- **SRAM2:**
  - Lowest Address: 1000 0000 0000 0000 = 8000H
  - Highest Address: 1011 1111 1111 1111 = BFFFH

**A_{15}\overline{A_{14}} = 00** for the entire range of the SRAM0 block. **A_{15}\overline{A_{14}} = 10** for the entire SRAM2 range. These can be decoded by the gates shown in Figure 13.27.

Address Decoding with $n$-line-to-$m$-line Decoders

Figure 13.28 shows a 64K memory system with four 16K chips: one EPROM at 0000H and three SRAMs at 4000H, 8000H, and C000H, respectively. In this circuit, the address decoding is done by a 2-line-to-4-line decoder, which can be an off-the-shelf MSI decoder, such as a 74HC139 decoder or a PLD-based design.

Table 13.2 shows the address ranges decoded by each decoder output. The first two address bits are the same throughout any given address range. Figure 13.29 shows the memory map for the system.
Table 13.2  Address Decoding for Figure 13.28

<table>
<thead>
<tr>
<th>$A_{15}$</th>
<th>$A_{14}$</th>
<th>Active Decoder Output</th>
<th>Device</th>
<th>Address Range</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>$Y_0$</td>
<td>EPROM</td>
<td>0000 0000 0000 0000 = 0000H</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0011 1111 1111 1111 =</td>
</tr>
<tr>
<td>3FFFH</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$Y_1$</td>
<td>SRAM$_1$</td>
<td>0100 0000 0000 0000 = 4000H</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0111 1111 1111 1111 =</td>
</tr>
<tr>
<td>7FFFH</td>
<td>0</td>
<td>$Y_2$</td>
<td>SRAM$_2$</td>
<td>1000 0000 0000 0000 = 8000H</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1011 1111 1111 1111 =</td>
</tr>
</tbody>
</table>
SECTION 13.6 REVIEW PROBLEM

13.6 Calculate the number of 128K memory blocks will fit into a 1M address space. Write the start addresses for the blocks.

SUMMARY

1. A memory is a device that can accept data and store them for later recall.
2. Data are located in a memory by an address, a binary number at a set of address inputs that uniquely locates the block of data.
3. The operation that stores data in a memory is called the write function. The operation that recalls the stored data is the read function. These functions are controlled by functions such as write enable (WE), chip select (CS), and output enable (OE).
4. RAM is random access memory. RAM can be written to and read from in any order of addresses. RAM is volatile. That is, it loses its data when power is removed from the device.
5. ROM is read only memory. Original ROM devices could not be written to at all, except at the time of manufacture. Modern variations can also be written to, but not as easily as RAM. ROM is nonvolatile; it retains its data when power is removed from the device.
6. Memory capacity is given as $m \times n$ for $m$ addressable locations and an $n$-bit data bus. For example, a 64K x 8 memory has 65,536 addressable locations, each with 8-bit data.
7. Large blocks of memory are designated with the binary prefixes $K$ ($2^{10} = 1024$), $M$ ($2^{20} = 1,048,576$), and $G$ ($2^{30} = 1,073,741,824$).
8. RAM can be divided into two major classes: static RAM (SRAM) and dynamic RAM (DRAM). SRAM retains its data as long as power is applied to the device. DRAM requires its data to be refreshed periodically.
9. Typically DRAM capacity is larger than SRAM because DRAM cells are smaller than SRAM cells. An SRAM cell is essentially a flip-flop consisting of several transistors. A DRAM cell has only one transistor and a capacitor.
10. RAM cells are arranged in rectangular arrays for efficient packaging. Internal circuitry locates each cell at the intersection of a row and column within the array.
11. For packaging efficiency, DRAM addresses are often multiplexed so that the device receives half its address as a row address, latched in to the device by a RAS (row address strobe) signal and the second half as a column address, latched in by a CAS (column address strobe) signal.
12. Read only memory (ROM) is used where it is important to retain data after power is removed.
13. Mask-programmed ROM is programmed at the time of manufacture. Programming is done by making a custom overlay of connections onto a standard cell array. Data cannot be changed. This is suitable for mature designs in high-volume production.
14. Erasable programmable read only memory (EPROM) can be programmed by the user and erased by exposure to ultraviolet light of a specified frequency and intensity. An EPROM must be removed from its circuit for erasing and reprogramming.
15. Electrically erasable read only memory (EEPROM or E²PROM) can be programmed and erased in-circuit. It is nonvolatile, but unsuitable for use as system RAM due to
its long programming/erase times and finite number of program/erase cycles.

16. Flash memory is a type of EEPROM that is organized into sectors that are erased all at once. This is faster than other EEPROM, which must be erased byte-by-byte.

17. Flash memory is often configured with one sector as a boot block, where primary firmware is stored. A bottom boot block architecture has the boot block at the lowest chip address. A top boot block architecture has the boot block at the highest chip address.

18. Sequential memory must have its data accessed in sequence. Two major classes are first-in first-out (FIFO) and last-in first-out (LIFO). FIFO is also called a queue and LIFO is called a stack.

19. Dynamic RAM chips are often configured as memory modules, small circuit boards with multiple DRAMs. The modules usually have the same number of address locations as the individual chips on the module, but a wider data bus.

20. Memory systems can be configured to have the same data width as individual memory devices comprising the system, but with more addressable locations than any chip in the system. The additional addresses require additional system address lines, which are decoded to enable one chip at a time within the system.

## Glossary

**Address** A number, represented by the binary states of a group of inputs or outputs, uniquely defining the location of data stored in a memory device.

**Address decoder** A circuit enabling a particular memory device to be selected by the address bus of a larger memory system.

**Address multiplexing** A technique of addressing storage cells in a dynamic RAM which sequentially uses the same inputs for row address and column address of the cell.

**Address Space** A block of addresses in a memory system.

<table>
<thead>
<tr>
<th>b</th>
<th>Bit.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>Byte.</td>
</tr>
</tbody>
</table>

**Bit-organized** A memory is bit-organized if one address accesses one bit of data.

**Boot block** A sector in a flash memory reserved for primary firmware.

**Bottom boot block** A boot block sector in flash memory paced at the lowest address in the memory.

**Bus** A group of parallel conductors carrying related logic signals, such as multi-bit data or addresses.

**Bus contention** The condition that results when two or more devices try to send data to a bus at the same time. Bus contention can damage the output buffers of the devices involved.

**Byte** A group of 8 bits.

**CAS** Column address strobe. A signal used to latch the column address into the decoding circuitry of a dynamic RAM with multiplexed addressing.

**Data** Binary digits (0s and 1s) which contain some kind of information. In the context of memory, the digital contents of a memory device.

**Dual in-line memory module (DIMM)** A memory module with DRAMs and connector pins on both sides of the board.

**Dynamic RAM** A random access memory which cannot retain data for more than a few (e.g., 64) milliseconds without being "refreshed."

**EEPROM (or E²PROM)** Electrically erasable programmable read only memory. A type of read only memory that can be field-programmed and selectively erased while still in a circuit.

**EPROM** Erasable programmable read only memory. A type of ROM that can be programmed ("burned") by the user and erased later, if necessary, by exposing the chip to ultraviolet radiation.

**FAMOS FET** Floating-gate avalanche. MOSFET. A MOSFET with a second, "floating" gate in which charge can be trapped to change the MOSFET’s gate-source threshold voltage. A FAMOS transistor is the memory element in an EPROM cell.

**FIFO** First-in first-out. A sequential memory in which the stored data can only be read in the order in which it was written.

**Firmware** Software instructions permanently stored in ROM.

**Flash memory** A nonvolatile type of memory that can be programmed and erased in sectors, rather than byte-at-a-time.

**Hardware** The electronic circuit of a digital or computer system.

**I/O** Input/output.

<table>
<thead>
<tr>
<th>K</th>
<th>1024 (=2¹⁰) Analogous to the metric prefix “k” (kilo).</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIFO</td>
<td>Last-In first-out. A sequential memory in which the last data written is the first data read.</td>
</tr>
<tr>
<td>M</td>
<td>1,048,576 (=2²⁰) Analogous to the metric prefix “M” (mega).</td>
</tr>
</tbody>
</table>

**Mask-programmed ROM** A type of read only memory (ROM) where the stored data are permanently encoded into the memory device during the manufacturing process.

**Memory** A device for storing digital data in such a way that it can be recalled for later use in a digital system.

**Memory map** A diagram showing the total address space of a memory system and the placement of various memory devices within that space.

**Memory module** A small circuit board containing several dynamic RAM chips.

**Nibble** Half a byte; 4 bits.

**PROM** Programmable read only memory. A type of ROM whose data need not be manufactured into the chip, but can be programmed by the user.

**Queue** A FIFO memory.

**RAM cell** The smallest storage unit of a RAM, capable of storing one bit.
**Random access memory (RAM)** A type of memory device where data at any address can be accessed in any order, that is, randomly. The term usually refers to random access read/write memory.

**RAS** Row address strobe. A signal used to latch the row address into the decoding circuitry of a dynamic RAM with multiplexed addressing.

**Read** Retrieve data from a memory device.

**Read only memory (ROM)** A type of memory where data is permanently stored and can only be read, not written.

**Refresh cycle** The process which periodically recharges the storage capacitors in a dynamic RAM.

**Sector** A segment of flash memory that forms the smallest amount that can be erased and reprogrammed at one time.

**Sequential memory** Memory in which the stored data cannot be read or written in random order, but must be addressed in a specific sequence.

---

**Problems**

**Section 13.2 Basic Memory Concepts**

13.1 How many address lines are necessary to make an 8 x 8 memory similar to the 4 x 8 memory in Figure 13.5? How many address lines are necessary to make a 16 x 8 memory?

13.2 Briefly explain the difference between RAM and ROM.

13.3 Calculate the number of address lines and data lines needed to access all stored data in each of the following sizes of memory:
   a. 64K x 8
   b. 128K x 16
   c. 128K x 32
   d. 256K x 16
   Calculate the total bit capacity of each memory.

13.4 Explain the difference between the chip enable (E) and the output enable (G) control functions in a RAM.

13.5 Refer to Figure 13.9. Briefly explain the operation of the W, E, and G functions of the RAM shown.

**Section 13.2 Random Access Read/Write Memory (RAM)**

13.6 Draw the circuit for an NMOS static RAM cell. Label one output BIT and the other BIT.

13.7 Refer to the NMOS static RAM cell drawn in Problem 13.6. Assume that BIT = 1. Describe the operation required to change BIT to 0.

13.8 Describe the main difference between a CMOS and an NMOS static RAM cell.

13.9 Explain how a particular RAM cell is selected from a group of many cells.

13.10 How many address lines are required to access all elements in a 1M x 1 dynamic RAM with address multiplexing?

13.11 What is the capacity of an address-multiplexed DRAM with one more address line than the DRAM referred to in Problem 13.10? With two more address lines?

13.12 How many address lines are required to access all elements in a 256M x 16 DRAM with address multiplexing?

**Section 13.3 Read Only Memory (ROM)**

13.13 Briefly list some of the differences between mask-programmed ROM, UV-erasable EPROM, EEPROM, and flash memory.

13.14 Briefly describe the programming and erasing process of a UV-EPROM.

13.15 Briefly explain the difference between flash memory and other EEPROM. What is the advantage of each configuration?

13.16 A flash memory has a capacity of 8 Mb, organized as 512K x 16-bit. List the address range for the 16 KB boot block sector of the memory if the device has a bottom boot block architecture and if it has a top boot block architecture.

13.17 Briefly state why EEPROM is not suitable for use as system RAM.

13.18 Briefly state why flash memory is unsuitable for use as system RAM.

**Section 13.4 Sequential Memory**

13.19 State one possible application for a FIFO and for a LIFO memory.

**Section 13.5 Memory Modules**

13.20 A SIMM has a capacity of 32M x 64. How many 32M x 8 DRAMs are required to make this SIMM? How many address lines does the SIMM require? How should the DRAMs be connected?
Section 13.6  Memory Systems

13.21 A microcontroller system with a 16-bit address bus is connected to a 4K × 8 RAM chip and an 8K × 8 RAM chip. The 8K address begins at 6000H. The 4K address block starts at 2000H.

Calculate the end address for each block and show address blocks for both memory chips on a 64K memory map.

13.22 Draw the memory system of Problem 13.21.

13.23 A microcontroller system with a 16-bit address bus has the following memory assignments:

<table>
<thead>
<tr>
<th>Memory</th>
<th>Size</th>
<th>Start Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAM₀</td>
<td>16K</td>
<td>4000H</td>
</tr>
<tr>
<td>RAM₁</td>
<td>8K</td>
<td>8000H</td>
</tr>
<tr>
<td>RAM₂</td>
<td>8K</td>
<td>A000H</td>
</tr>
</tbody>
</table>

Show the blocks on a 64K memory map.

13.24 Draw the memory system described in Problem 13.23.

13.25 The memory map of a microcontroller system with a 16-bit address bus is shown in Figure 13.30. Make a table of start and end addresses for each of the blocks shown. Indicate the size of each block.

13.26 Sketch the memory system described in Problem 13.25.

13.27 How many 16M × 32 DIMMs are required to make a 256M × 32 memory system? Make a table showing the start and end addresses of each block.

ANSWERS TO SECTION REVIEW PROBLEMS

Section 13.2a

13.1 18 address lines, 16 data lines; capacity = 4Mb, same as Figure 3.12

Section 13.2b

13.2 a. 10 address, 1 data; b. 10 address, 4 data; c. 11 address, 1 data.

Section 13.3

13.3 Bottom boot block: 00000H to 07FFFH; top boot block: F8000H to FFFFFH.

Section 13.4

13.4 A stack is a last-in first-out (LIFO) memory and a queue is a first-in first-out (FIFO) memory.

Section 13.5

13.5 Four DRAMs. 12 address lines. Address and control lines are in parallel with all DRAMs. Data I/O lines are separate.

Section 13.6

APPENDIX A

Altera UP-1 User Guide

Current versions of the Altera UP-1 board are shipped with version 9.23 of MAX+PLUS II software. See the file SE_READ.txt on the accompanying CD for installation instructions.
UP 1 Education Board

The UP 1 Education Board is a stand-alone experiment board based on two of Altera’s leading device families: MAX® 7000 and FLEX® 10K. Its simple design, when used with the MAX+PLUS II software, provides a superior platform for learning digital logic design using high-level development tools and PLDs.

The UP 1 Education Board was designed to meet the needs of the educator and the design laboratory environment. The UP 1 Education Board supports both product-term based and look-up table (LUT)-based architectures and includes two PLDs. The EPM7128S device can be programmed in-system with the ByteBlaster download cable. The EPF10K20 device can be configured in-system with either the ByteBlaster download cable or an EPC1 Configuration EPROM (not included).

EPM7128S Device

The EPM7128S device, a mid-density member of the high-density, high-performance MAX 7000S family, is based on EEPROM elements. The EPM7128S device comes in a socket-mounted 84-pin PLCC package and has 128 macrocells. Each macrocell has a programmable-AND/fixed-OR array as well as a configurable register with independently programmable clock, clock enable, clear, and preset functions. With a capacity of 2,500 gates and a simple architecture, the EPM7128S device is ideal for introductory designs as well as larger combinatorial and sequential logic functions.

For more information on MAX 7000 devices, go to the MAX 7000 Programmable Logic Device Family Data Sheet.

EPF10K20 Device

The EPF10K20 device, a member of Altera’s high-density FLEX 10K family, is based on reconfigurable SRAM elements. The EPF10K20 device comes in a 240-pin QFP package and has 1,152 logic elements (LEs) and 6 embedded array blocks (EABs). Each LE consists of a 4-input look-up table (LUT), a programmable flipflop, and dedicated signal paths for carry and cascade functions. Each EAB provides 2,048 bits of memory, which can be used to create RAM, ROM, or first-in first-out (FIFO) functions. The EABs can also be used to implement logic functions, such as multipliers, microcontrollers, state machines, and digital signal processing (DSP) functions. With a typical gate count of 20,000 gates, the EPF10K20 device is ideal for advanced designs, including computer architecture, communications, and DSP applications.

For more information on FLEX 10K devices, go to the FLEX 10K Embedded Programmable Logic Family Data Sheet.
ByteBlaster Parallel Port Download Cable

Designs can be easily and quickly downloaded into the UP 1 Education Board using the ByteBlaster download cable, which is a hardware interface to a standard parallel port. This cable channels programming or configuration data between the MAX+PLUS II software and the UP 1 Education Board. Because design changes are downloaded directly to the devices on the board, prototyping is easy and multiple design iterations can be accomplished in quick succession.

For more information on the ByteBlaster download cable, go to the ByteBlaster Parallel Port Download Cable Data Sheet.

UP 1 Education Board Description

The UP 1 Education Board contains the elements described in this section. Figure 1 shows a block diagram of the UP 1 Education Board.

Figure 1. UP 1 Education Board Block Diagram
University Program Design Laboratory Package User Guide

DC_IN & RAW Power Input

The DC_IN power input accepts a 2.5 mm × 5.55 mm female connector. The acceptable DC input is 7 to 12 V DC at a minimum of 250 mA. The RAW power input consists of two holes for connecting an unregulated power source. The hole marked with a plus sign (+) is the positive input; the hole marked with a minus sign (−) is board common.

On-Board Voltage Regulator

The on-board voltage regulator, an LM340T, regulates the DC positive input at 5 V. The DC input consist of two holes for connecting a 5-V DC regulated power source. The hole marked with a plus sign (+) is the positive input; the hole marked with a minus sign (−) is board common. A green light-emitting diode (LED) labeled POWER is illuminated when current is flowing from the 5-V DC regulated power source.

Oscillator

The UP 1 Education Board contains a 25.175-MHz crystal oscillator. The output of the oscillator drives the global clock input on the EPM7128S device (pin 83) and the global clock input on the EPF10K20 device (pin 91).

JTAG_IN Header

The 10-pin female plug on the ByteBlaster download cable connects with the JTAG_IN 10-pin male header on the UP 1 Education Board. The UP 1 Education Board provides power and ground to the ByteBlaster download cable. Data is shifted into the devices via the TDI pin and shifted out of the devices via the TDO pin. Table 1 identifies the JTAG_IN pin names when the ByteBlaster is operating in JTAG mode.
Table 1. JTAG_IN 10-Pin Header Pin-Outs

<table>
<thead>
<tr>
<th>Pin</th>
<th>JTAG Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TCK</td>
</tr>
<tr>
<td>2</td>
<td>GND</td>
</tr>
<tr>
<td>3</td>
<td>TDO</td>
</tr>
<tr>
<td>4</td>
<td>VCC</td>
</tr>
<tr>
<td>5</td>
<td>TMS</td>
</tr>
<tr>
<td>6</td>
<td>No Connect</td>
</tr>
<tr>
<td>7</td>
<td>No Connect</td>
</tr>
<tr>
<td>8</td>
<td>No Connect</td>
</tr>
<tr>
<td>9</td>
<td>TDI</td>
</tr>
<tr>
<td>10</td>
<td>GND</td>
</tr>
</tbody>
</table>

Jumpers

The UP 1 Education Board contains four three-pin jumpers (TDI, TDO, DEVICE, and BOARD) that set the JTAG configuration. You can set the JTAG chain for a variety of configurations (i.e., to program only the EPM7128S device, to configure only the EPF10K20 device, to configure/program both devices, or to connect multiple UP 1 Education Boards together). Figure 1 shows the positions of the three connectors (C1, C2, and C3) on each of the four jumpers.

Figure 2. Position of C1, C2 & C3 Connectors

<table>
<thead>
<tr>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE</th>
<th>BOARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>C1</td>
<td>C1</td>
<td>C1</td>
</tr>
<tr>
<td>C2</td>
<td>C2</td>
<td>C2</td>
<td>C2</td>
</tr>
<tr>
<td>C3</td>
<td>C3</td>
<td>C3</td>
<td>C3</td>
</tr>
</tbody>
</table>

Table 2 defines the settings for each configuration.
Table 2. JTAG Jumper Settings

<table>
<thead>
<tr>
<th>Desired Action</th>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE</th>
<th>BOARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Program EPM7128S device only</td>
<td>C1 &amp; C2</td>
<td>C1 &amp; C2</td>
<td>C1 &amp; C2</td>
<td>C1 &amp; C2</td>
</tr>
<tr>
<td>Configure EPF10K20 device only</td>
<td>C2 &amp; C3</td>
<td>C2 &amp; C3</td>
<td>C1 &amp; C2</td>
<td>C1 &amp; C2</td>
</tr>
<tr>
<td>Program/configure both devices, Note (1)</td>
<td>C2 &amp; C3</td>
<td>C1 &amp; C2</td>
<td>C2 &amp; C3</td>
<td>C1 &amp; C2</td>
</tr>
<tr>
<td>Connect multiple boards together, Note (2)</td>
<td>C2 &amp; C3</td>
<td>OPEN</td>
<td>C2 &amp; C3</td>
<td>C2 &amp; C3</td>
</tr>
</tbody>
</table>

Notes:
(1) The first device in the JTAG chain is the EPF10K20, and the second device is the EPM7128S.
(2) The first device in the JTAG chain is the EPF10K20, and the second device is the EPM7128S. The last board in the chain must be set for a single board configuration (i.e., for programming only the EPM7128S device, configuring only the EPF10K20 device, or configuring/programming both devices). The last board cannot be set for connecting multiple boards together.

During configuration, the green CONF_D LED will turn off and the green TCK LED will modulate to indicate that data is transferring. After the device has successfully configured, the CONF_D LED will illuminate.

For information on how to program or configure EPM7128S and EPF10K20 devices, see “Programming or Configuring Devices” on page 18.

EPM7128S Device

The UP 1 Education Board provides the following resources for the EPM7128S device.

- Socket-mounted 84-pin PLCC package
- Signal pins that are accessible via female headers
- JTAG chain connection for the ByteBlaster
- 2 momentary push-button switches
- 2 octal dip switches
- 16 LEDs
- Dual-digit 7-segment display
- On-board oscillator (25.175 MHz)
- Expansion port with 42 I/O pins and the dedicated global CLR, OE1, and OE2/GCLK2 pins
The pins from the EPM7128S device are not pre-assigned to switches and LEDs on the board, but are instead connected to female headers. With direct access to the pins, students can concentrate on design fundamentals and learn about the programmability of I/O pins and PLDs. After successfully compiling and verifying a design with MAX+PLUS II, students can easily connect the assigned I/O pins to the switches and LEDs using common hook-up wire. Students can then download their design into the device and compare their design’s simulation to the actual hardware implementation.

**EPM7128S Prototyping Headers**

The EPM7128S prototyping headers are female headers that surround the device and provide access to the device’s signal pins. The 21 pins on each side of the 84-pin PLCC package connect to one of the 22-pin, dual-row 0.1-inch female headers. The pin numbers for the EPM7128S device are printed on the UP 1 Education Board; an X indicates an unassigned pin. Table 3 lists the pin numbers for the four female headers: P1, P2, P3, and P4. The power, ground, and JTAG signal pins are not accessible through these female headers.

<table>
<thead>
<tr>
<th>Table 3. Pin Numbers for Each Prototyping Header</th>
<th>Note (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>P1</strong></td>
<td><strong>P2</strong></td>
</tr>
<tr>
<td><strong>Outside</strong></td>
<td><strong>Inside</strong></td>
</tr>
<tr>
<td>75</td>
<td>76</td>
</tr>
<tr>
<td>77</td>
<td>78</td>
</tr>
<tr>
<td>79</td>
<td>80</td>
</tr>
<tr>
<td>81</td>
<td>82</td>
</tr>
<tr>
<td>83</td>
<td>84</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>X</td>
</tr>
</tbody>
</table>

**Note:**
(1) Inside refers to the row of female headers closest to the device; outside refers to the row of female headers furthest from the device.
MAX_PB1 & MAX_PB2 Push-Buttons

MAX_PB1 and MAX_PB2 are two push-buttons that provide active-low signals and are pulled-up through 10-KΩ resistors. Connections to these signals are easily made by inserting one end of the hook-up wire into the push-button female header. The other end of the hook-up wire should be inserted into the appropriate female header assigned to the I/O pin of the EPM7128S device.

MAX_SW1 & MAX_SW2 Switches

MAX_SW1 and MAX_SW2 each contain eight switches that provide logic-level signals. These switches are pulled-up through 10-KΩ resistors. Connections to these signals are easily made by inserting one end of the hook-up wire into the female header aligned with the appropriate switch. The other end of the hook-up wire should be inserted into the appropriate female header assigned to the I/O pin of the EPM7128S device. The switch output is set to logic 1 when the switch is open and set to logic 0 when the switch is closed.

D1 through D16 LEDs

The UP 1 Education Board contains 16 LEDs that are pulled-up with a 330-Ω resistor. An LED is illuminated when a logic 0 is applied to the female header associated with the LED. LEDs D1 through D8 are connected in the same sequence to the female headers (i.e., D1 is connected to position 1, and D2 is connected to position 2). LEDs D9 through D16 are connected in the same sequence to the female headers (i.e., D9 is connected to position 1, and D10 is connected to position 2). See Figure 3.

Figure 3. LED Positions

<table>
<thead>
<tr>
<th>Female Header Position</th>
<th>LEDs</th>
<th>Female Header Position</th>
<th>LEDs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D1</td>
<td>1</td>
<td>D9</td>
</tr>
<tr>
<td>2</td>
<td>D5</td>
<td>2</td>
<td>D13</td>
</tr>
<tr>
<td>3</td>
<td>D2</td>
<td>3</td>
<td>D10</td>
</tr>
<tr>
<td>4</td>
<td>D6</td>
<td>4</td>
<td>D14</td>
</tr>
<tr>
<td>5</td>
<td>D3</td>
<td>5</td>
<td>D11</td>
</tr>
<tr>
<td>6</td>
<td>D7</td>
<td>6</td>
<td>D15</td>
</tr>
<tr>
<td>7</td>
<td>D4</td>
<td>7</td>
<td>D12</td>
</tr>
<tr>
<td>8</td>
<td>D8</td>
<td>8</td>
<td>D16</td>
</tr>
</tbody>
</table>
MAX_DIGIT Display

MAX_DIGIT is a dual-digit seven-segment display connected directly to the EPM7128S device. Each LED segment of the display can be illuminated by driving the connected EPM7128S device I/O pin with a logic 0. Figure 4 shows the name of each segment.

Table 4 lists the pin assignments for each segment.

<table>
<thead>
<tr>
<th>Display Segment</th>
<th>Pin for Digit 1</th>
<th>Pin for Digit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>58</td>
<td>69</td>
</tr>
<tr>
<td>b</td>
<td>60</td>
<td>70</td>
</tr>
<tr>
<td>c</td>
<td>61</td>
<td>73</td>
</tr>
<tr>
<td>d</td>
<td>63</td>
<td>74</td>
</tr>
<tr>
<td>e</td>
<td>64</td>
<td>76</td>
</tr>
<tr>
<td>f</td>
<td>65</td>
<td>75</td>
</tr>
<tr>
<td>g</td>
<td>67</td>
<td>77</td>
</tr>
<tr>
<td>Decimal point</td>
<td>68</td>
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</tbody>
</table>

MAX_EXPANSION

MAX_EXPANSION is a dual row of 0.1-inch spaced holes for accessing signal I/O pins and global signals on the EPM7128S device, power, and ground. Figure 5 shows the numbering convention for the holes.
Table 5 lists the signal names and the EPM7128S device pins connected to each hole.

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>Signal/Pin</th>
<th>Hole Number</th>
<th>Signal/Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RAW</td>
<td>2</td>
<td>GND</td>
</tr>
<tr>
<td>3</td>
<td>VCC</td>
<td>4</td>
<td>GND</td>
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<td>5</td>
<td>VCC</td>
<td>6</td>
<td>GND</td>
</tr>
<tr>
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<td>No Connect</td>
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<td>OE2/GCLK2/2</td>
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<td>16</td>
<td>5</td>
</tr>
<tr>
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<td>18</td>
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</tr>
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</table>
Table 5. MAX_EXPANSION Signal Names & Device Connections (Part 2 of 2)

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<thead>
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<th>Hole Number</th>
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<td>57</td>
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<tr>
<td>59</td>
<td>VCC</td>
<td>60</td>
<td>GND</td>
</tr>
</tbody>
</table>

**EPF10K20 Device**

The UP 1 Education Board provides the following resources for the EPF10K20 device. The pins from the EPF10K20 device are pre-assigned to switches and LEDs on the board.

- JTAG chain connection for the ByteBlaster
- Socket for an EPC1 Configuration EPROM
- 2 momentary push-button switches
- 1 octal dipswitch
- Dual-digit 7-segment display
- On-board oscillator (25.175 MHz)
- VGA port
- Mouse port
- 3 expansion ports, each with 42 I/O pins and 7 global pins

**FLEX_PB1 & FLEX_PB2 Push-Buttons**

FLEX_PB1 and FLEX_PB2 are two push-buttons that provide active-low signals to two general-purpose I/O pins on the EPF10K20 device. FLEX_PB1 is connected to pin 28, and FLEX_PB2 is connected to pin 29. Each push-button is pulled-up through a 10-KΩ resistor.

**FLEX_SW1 Switches**

FLEX_SW1 contains eight switches that provide logic-level signals to eight general-purpose I/O pins on the EPF10K20 device. An input pin is set to logic 1 when the switch is open and set to logic 0 when the switch is closed. Table 6 lists the pin assignment for each switch.
**Table 6. FLEX_SW1 Pin Assignments**

<table>
<thead>
<tr>
<th>Switch</th>
<th>EPF10K20 Pin</th>
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</thead>
<tbody>
<tr>
<td>FLEX_SWITCH-1</td>
<td>41</td>
</tr>
<tr>
<td>FLEX_SWITCH-2</td>
<td>40</td>
</tr>
<tr>
<td>FLEX_SWITCH-3</td>
<td>39</td>
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<tr>
<td>FLEX_SWITCH-4</td>
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</tr>
<tr>
<td>FLEX_SWITCH-5</td>
<td>36</td>
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<tr>
<td>FLEX_SWITCH-6</td>
<td>35</td>
</tr>
<tr>
<td>FLEX_SWITCH-7</td>
<td>34</td>
</tr>
<tr>
<td>FLEX_SWITCH-8</td>
<td>33</td>
</tr>
</tbody>
</table>

**FLEX_DIGIT Display**

FLEX_DIGIT is a dual-digit seven-segment display connected directly to the EPF10K20 device. Each LED segment on the display can be illuminated by driving the connected EPF10K20 device I/O pin with a logic 0. See Figure 4 on page 9 for the name of each segment. Table 7 lists the pin assignment for each segment.

**Table 7. FLEX_DIGIT Segment I/O Connections**

<table>
<thead>
<tr>
<th>Display Segment</th>
<th>Pin for Digit 1</th>
<th>Pin for Digit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>6</td>
<td>17</td>
</tr>
<tr>
<td>b</td>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td>c</td>
<td>8</td>
<td>19</td>
</tr>
<tr>
<td>d</td>
<td>9</td>
<td>20</td>
</tr>
<tr>
<td>e</td>
<td>11</td>
<td>21</td>
</tr>
<tr>
<td>f</td>
<td>12</td>
<td>23</td>
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<tr>
<td>g</td>
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<td>24</td>
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<tr>
<td>Decimal point</td>
<td>14</td>
<td>25</td>
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</tbody>
</table>

**VGA Interface**

The VGA interface allows the EPF10K20 device to control an external video monitor. This interface is composed of a simple diode-resistor network and a 15-pin D-sub connector (labeled VGA), where the monitor can plug into the UP 1 Education Board. The diode-resistor network and D-sub connector are designed to generate voltages that conform to the VGA standard.
Information about the color of the screen, and the row and column indexing of the screen, are sent from the EPF10K20 device to the monitor via five signals. Three VGA signals are red, green, and blue, while the other two signals are horizontal and vertical synchronization. Manipulating these signals allows images to be written to the monitor's screen.

See “VGA Driver Operation” on page 25 for details on how the VGA interface operates.

Table 8 lists the D-sub connector and the EPF10K20 device connections.

<table>
<thead>
<tr>
<th>Signal</th>
<th>D-Sub Connector Pin</th>
<th>EPF10K20 Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>RED</td>
<td>1</td>
<td>236</td>
</tr>
<tr>
<td>GREEN</td>
<td>2</td>
<td>237</td>
</tr>
<tr>
<td>BLUE</td>
<td>3</td>
<td>238</td>
</tr>
<tr>
<td>GND</td>
<td>6, 7, 8, 10, 11</td>
<td>–</td>
</tr>
<tr>
<td>HORIZ_SYNC</td>
<td>13</td>
<td>240</td>
</tr>
<tr>
<td>VERT_SYNC</td>
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<td>239</td>
</tr>
<tr>
<td>No Connect</td>
<td>4, 5, 9, 15</td>
<td>–</td>
</tr>
</tbody>
</table>

**MOUSE Connector**

The MOUSE interface, which consists of a 6-pin mini-DIN connector, allows the EPF10K20 device to receive data from a PS/2 mouse or a PS/2 keyboard. The UP 1 Education Board provides power and ground to the attached mouse or keyboard. The EPF10K20 device outputs the DATA_CLOCK signal to the mouse and inputs the data signal from the mouse. Table 9 lists the signal names and the mini-DIN and EPF10K20 pin connections.

See “MOUSE Interface Operation” on page 27 for details on how the MOUSE interface operates.

<table>
<thead>
<tr>
<th>Mouse Signal</th>
<th>Mini-DIN Pin</th>
<th>EPF10K20 Pin</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOUSE_CLK</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>MOUSE_DATA</td>
<td>3</td>
<td>31</td>
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<tr>
<td>VCC</td>
<td>5</td>
<td>–</td>
</tr>
<tr>
<td>GND</td>
<td>2</td>
<td>–</td>
</tr>
</tbody>
</table>
FLEX_EXPAN_A, FLEX_EXPAN_B, and FLEX_EXPAN_C are dual rows of 0.1-inch spaced holes for accessing signal I/O pins and global signals on the EPF10K20 device, power, and ground. Figure 6 shows the numbering convention for these holes.

Figure 6. FLEX_EXPAN_A, FLEX_EXPAN_B & FLEX_EXPAN_C Numbering Convention

Tables 10, 11, and 12 list the signal name and the EPF10K20 device pin connected to each hole.

Table 10. FLEX_EXPAN_A Signal Names & Device Connections (Part 1 of 2)

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>Signal/Pin</th>
<th>Hole Number</th>
<th>Signal/Pin</th>
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<tbody>
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<td>3</td>
<td>VCC</td>
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<td>GND</td>
</tr>
<tr>
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<td>DI1/99</td>
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<td>DI2/92</td>
<td>10</td>
<td>DI3/210</td>
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<td>DI4/212</td>
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<td>DEV_CLR/209</td>
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<td>DEV_OB/213</td>
<td>14</td>
<td>DEV_CLK/211</td>
</tr>
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### Table 10. FLEX_EXPAN_A Signal Names & Device Connections (Part 2 of 2)

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### Table 11. FLEX_EXPAN_B Signal Names & Device Connections (Part 1 of 2)

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<th>Hole Number</th>
<th>Signal/Pin</th>
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<tbody>
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<td>1</td>
<td>RAW</td>
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<td>VCC</td>
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<td>DI3/210</td>
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<td>DEV_CLK/213</td>
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<td>DEV_CLK/211</td>
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### Table 11. FLEX_EXPAN_B Signal Names & Device Connections (Part 2 of 2)

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### Table 12. FLEX_EXPAN_C Signal Names & Device Connections (Part 1 of 2)

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<th>Hole Number</th>
<th>Signal/Pin</th>
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<tbody>
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<tr>
<td>3</td>
<td>VCC</td>
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<td>GND</td>
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<td>5</td>
<td>VCC</td>
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<td>GND</td>
</tr>
<tr>
<td>7</td>
<td>No Connect</td>
<td>8</td>
<td>D11/99</td>
</tr>
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<td>9</td>
<td>D12/92</td>
<td>10</td>
<td>D13/210</td>
</tr>
<tr>
<td>11</td>
<td>D14/212</td>
<td>12</td>
<td>DEV_CLR/209</td>
</tr>
<tr>
<td>13</td>
<td>DEV_OE/213</td>
<td>14</td>
<td>DEV_CLK/211</td>
</tr>
<tr>
<td>15</td>
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<td>29</td>
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<td>196</td>
</tr>
</tbody>
</table>
This section describes how to install the MAX+PLUS II version 7.21 Student Edition software for the following operating systems:

- Windows 3.11 and Windows NT 3.51
- Windows 95 and Windows NT 4.0

After installation, students can register to obtain an authorization code via the Altera world-wide web site at the following URL:

For complete installation instructions, refer to the read.me file on the MAX+PLUS II 7.21 Student Edition CD-ROM or go to the MAX+PLUS II Getting Started manual.

### Table 12. FLEX_EXPAN_C Signal Names & Device Connections (Part 2 of 2)

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>Signal/Pin</th>
<th>Hole Number</th>
<th>Signal/Pin</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>59</td>
<td>VCC</td>
<td>60</td>
<td>GND</td>
</tr>
</tbody>
</table>
Windows 3.1 & Windows NT 3.51

Follow the steps shown below to install the MAX+PLUS II version 7.21 Student Edition software on your PC.

1. Insert the MAX+PLUS II 7.21 Student Edition CD-ROM into your CD-ROM drive.
2. In the Windows Program Manager, choose Run (File menu).
3. Type: `<CD-ROM drive>:\pc\maxplus2\install` and choose OK. You are guided through the installation procedure.

Windows 95 & Windows NT 4.0

Follow the steps shown below to install the MAX+PLUS II version 7.21 Student Edition software on your PC.

1. Insert the MAX+PLUS II 7.21 Student Edition CD-ROM into your CD-ROM drive.
2. Choose Run (Start menu).
3. Type: `<CD-ROM drive>:\pc\maxplus2\install` and choose OK. You are guided through the installation procedure.

Programming or Configuring Devices

Programming or configuring the devices on the UP 1 Education Board requires setting the on-board jumpers and the JTAG programming options in MAX+PLUS II, and connecting the ByteBlaster download cable to the PC's parallel port and to the JTAG_IN connector on the UP 1 Education Board. This section describes how to set these options to perform the following actions:

- Program only the EPM7128S device
- Configure only the EPF10K20 device
- Configure/program both devices
- Connect multiple UP 1 Education Boards together in a chain

EPM7128S Programming

This section describes the procedures for programming only the EPM7128S device, (i.e., how to set the on-board jumpers, connect the ByteBlaster download cable, and set options in the MAX+PLUS II software).
Setting the On-Board Jumpers for EPM7128S Programming

To program only the EPM7128S device in a JTAG chain, set the jumpers TDI, TDO, DEVICE, and BOARD as shown in Figure 7.

Figure 7. Jumper Settings for Programming Only the EPM7128S Device

<table>
<thead>
<tr>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE</th>
<th>BOARD</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>C3</td>
<td>C3</td>
<td>C3</td>
<td>C3</td>
</tr>
</tbody>
</table>

Connecting the ByteBlaster Download Cable for EPM7128S Programming

Attach the ByteBlaster directly to the PC’s parallel port and to the JTAG_IN connector on the UP 1 Education Board. For more information on setting up the ByteBlaster, go to the ByteBlaster Parallel Port Download Cable Data Sheet.

Setting the JTAG Options in MAX+PLUS II for EPM7128S Programming

The following steps describe how to use the MAX+PLUS II software to program the EPM7128S device in a JTAG chain. For more information on how to use MAX+PLUS II, see MAX+PLUS II Help.

1. To program more than one EPM7128S device, turn on the Multi-Device JTAG Chain command (JTAG menu) in the MAX+PLUS II Programmer.
2. Choose Multi-Device JTAG Chain Setup (JTAG menu).
3. In the Multi-Device JTAG Chain Setup dialog box, select EPM7128S in the Device Name drop-down list box.
4. Type the name of the programming file for the EPM7128S device in the Programming File Name box. The Select Programming File button can also be used to browse your computer’s directory structure to locate the appropriate programming file.
5. Choose **Add** to add the device and associated programming file to the *Device Names & Programming File Names* box. The number to the left of the device name shows the order of the device in the JTAG chain. The device’s associated programming file is displayed on the same line as the device name. If no programming file is associated with a device, "<none>" is displayed next to the device name.

6. Choose **Detect JTAG Chain Info** to have the ByteBlaster check the device count, JTAG ID code, and total instruction length of the JTAG chain. A message just above the **Detect JTAG Chain Info** button reports the information detected by the ByteBlaster. You must manually verify that this message matches the information in the *Device Names & Programming File Names* box.

7. To save the current settings to a JTAG Chain File (.jcf) for future use, choose **Save JCF**. In the **Save JCF** dialog box, type the name of the file in the *File Name* box and then select the desired directory in the *Directories* box. Choose **OK**.

8. Choose **OK** to save your changes.

9. In the MAX+PLUS II Programmer, choose **Program**.

**EPF10K20 Configuration**

This section describes the procedures for configuring only the EPF10K20 device, (i.e., how to set the on-board jumpers, connect the ByteBlaster download cable, and set options in the MAX+PLUS II software).

**Setting the On-Board Jumpers for EPF10K20 Configuration**

To configure only the EPF10K20 device in a JTAG chain, set the jumpers TDI, TDO, DEVICE, and BOARD as shown in Figure 8.

**Figure 8. Jumper Settings for Configuring Only the EPF10K20 Device**

<table>
<thead>
<tr>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE</th>
<th>BOARD</th>
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<td>C1</td>
<td>C1</td>
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<td>C2</td>
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<td>C2</td>
</tr>
<tr>
<td>C3</td>
<td>C3</td>
<td>C3</td>
<td>C3</td>
</tr>
</tbody>
</table>
Connecting the ByteBlaster Download Cable for EPF10K20 Configuration

Attach the ByteBlaster directly to the PC’s parallel port and to the JTAG_IN connector on the UP 1 Education Board. For more information on setting up the ByteBlaster, go to the ByteBlaster Parallel Port Download Cable Data Sheet.

Setting the JTAG Options in MAX+PLUS II for EPF10K20 Configuration

The following steps describe how to use MAX+PLUS II to configure the EPF10K20 device in a JTAG chain. For more information on how to configure a device, see MAX+PLUS II Help.

1. To configure more than one EPF10K20 device, turn on the Multi-Device JTAG Chain command (JTAG menu) in the MAX+PLUS II Programmer.
2. Choose Multi-Device JTAG Chain Setup (JTAG menu).
3. In the Multi-Device JTAG Chain Setup dialog box, select EPF10K20 in the Device Name drop-down list box.
4. Type the name of the programming file for the EPF10K20 device in the Programming File Name box. The Select Programming File button can also be used to browse your computer’s directory structure to locate the appropriate programming file.
5. Choose Add to add the device and associated programming file to the Device Names & Programming File Names box. The number to the left of the device name shows the order of the device in the JTAG chain. The device’s associated programming file is displayed on the same line as the device name. If no programming file is associated with a device, “<none>” is displayed next to the device name.
6. Choose Detect JTAG Chain Info to have the ByteBlaster check the device count, JTAG ID code, and total instruction length of the JTAG chain. A message just above the Detect JTAG Chain Info button reports the information detected by the ByteBlaster. You must manually verify that this message matches the information in the Device Names & Programming File Names box.
7. To save the current settings to a JCF for future use, choose Save JCF. In the Save JCF dialog box, type the name of the file in the File Name box and then select the desired directory in the Directories box. Choose OK.
8. Choose **OK** to save your changes.

9. In the MAX+PLUS II Programmer, choose **Configure**.

**Configure/Program Both Devices**

This section describes the procedures for configuring/programming both the EPF10K20 and EPM7128S devices in a JTAG chain, (i.e., how to set the on-board jumpers, connect the ByteBlaster download cable, and set options in the MAX+PLUS II software).

**Setting the On-Board Jumpers for Configuring/Programming Both Devices**

To configure/program both the EPF10K20 and EPM7128S devices in a multi-device JTAG chain, set the jumpers TDI, TDO, DEVICE, and BOARD as shown in Figure 9.

**Figure 9. Jumper Settings for Configuring/Programming Both Devices**

<table>
<thead>
<tr>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE</th>
<th>BOARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>C1</td>
<td>C1</td>
<td>C1</td>
</tr>
<tr>
<td>C2</td>
<td>C2</td>
<td>C2</td>
<td>C2</td>
</tr>
<tr>
<td>C3</td>
<td>C3</td>
<td>C3</td>
<td>C3</td>
</tr>
</tbody>
</table>

**Connecting the ByteBlaster Download Cable for Configuring/Programming Both Devices**

Attach the ByteBlaster directly to the PC’s parallel port and to the JTAG_IN connector on the UP 1 Education Board. For more information on setting up the ByteBlaster, go to the *ByteBlaster Parallel Port Download Cable Data Sheet*.

**Setting the JTAG Options in MAX+PLUS II for Configuring/Programming Both Devices**

The following steps describe how to use MAX+PLUS II to configure/program both devices in a multi-device JTAG chain. For more information on how to program or configure a device, see MAX+PLUS II Help.
1. Turn on the *Multi-Device JTAG Chain* command (JTAG menu).

2. Choose *Multi-Device JTAG Chain Setup* (JTAG menu).

3. In the *Multi-Device JTAG Chain Setup* dialog box, select the first target device name in the *Device Name* drop-down list box.

4. In the *Programming File Name* box, type the name of the programming file for the device listed in the *Device Name* box. The *Select Programming File* button can also be used to browse your computer’s directory structure to locate the appropriate programming file.

5. Choose *Add* to add the device and associated programming file to the *Device Names & Programming File Name* box. The number to the left of the device name shows the device’s order in the JTAG chain. The device’s associated programming file is displayed on the same line as the device name. If no programming file is associated with a device, "<none>" is displayed next to the device name.

6. Repeat steps 3 through 5 to add information for each device in the JTAG chain.

7. Choose *Detect JTAG Chain Info* to have the ByteBlaster check the device count, JTAG ID code, and total instruction length of the multi-device JTAG chain. A message just above the *Detect JTAG Chain Info* button reports the information detected by the ByteBlaster. You must manually verify that this message matches the information in the *Device Names & Programming File Names* box.

8. To save the current settings to a JCF for future use, choose *Save JCF*. In the *Save JCF* dialog box, type the name of the file in the *File Name* box and then select the desired directory in the *Directories* box. Choose *OK*.

9. Choose *OK* to save the changes.

10. In the MAX+PLUS II Programmer, choose *Configure* to configure all the EPF10K20 devices in the JTAG chain. Then, choose *Program* to program all the EPM7128S devices in the JTAG chain.

**Connect Multiple UP 1 Education Boards Together in a Chain**

This section describes the procedures for connecting multiple UP 1 Education Boards together, (i.e., how to set the on-board jumpers, connect the ByteBlaster download cable, and set options in the MAX+PLUS II software).
Setting the On-Board Jumpers for Connecting Multiple UP 1 Education Boards Together

To configure/program EPM7128S and EPF10K20 devices on multiple UP 1 Education Boards connected in a multi-device JTAG chain, set the jumpers TDI, TDO, DEVICE, and BOARD for all boards except the last board in the chain as shown in Figure 10.

Figure 10. Jumper Settings for All Boards Except the Last Board in the Chain

<table>
<thead>
<tr>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE BOARD</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>C1</td>
<td>C1</td>
</tr>
<tr>
<td>C2</td>
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<td>C2</td>
</tr>
<tr>
<td>C3</td>
<td>C3</td>
<td>C3</td>
</tr>
</tbody>
</table>

The last UP 1 Education Board in the chain can configure/program one or both devices. However, the BOARD jumper must be set as shown in Figure 11.

Figure 11. Jumper Settings for the Last Board in the Chain

The TDI, TDO, and DEVICE settings depend on which configuration you use.

<table>
<thead>
<tr>
<th>TDI</th>
<th>TDO</th>
<th>DEVICE BOARD</th>
</tr>
</thead>
<tbody>
<tr>
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<td>C2</td>
</tr>
<tr>
<td>C3</td>
<td>C3</td>
<td>C3</td>
</tr>
</tbody>
</table>

Connecting the ByteBlaster Download Cable for Connecting Multiple UP 1 Education Boards Together

Attach the ByteBlaster directly to your PC’s parallel port and to the JTAG_IN connector on the UP 1 Education Board. For more information on setting up the ByteBlaster, go to the ByteBlaster Parallel Port Download Cable Data Sheet.

Setting the JTAG Options in MAX+PLUS II for Connecting Multiple UP 1 Education Boards Together

For information on how to set the JTAG Options in MAX+PLUS II, see “Setting the JTAG Options in MAX+PLUS II for Configuring/Programming Both Devices” on page 22.
VGA Driver Operation

A standard VGA monitor consists of a grid of pixels that can be divided into rows and columns. A VGA monitor typically contains 480 rows, with 640 pixels per row, as shown in Figure 12. Each pixel can display various colors, depending on the state of the red, green, and blue signals.

Figure 12. VGA Monitor

Each VGA monitor has an internal clock that determines when each pixel is updated. This clock operates at the VGA-specified frequency of 25.175 MHz. The monitor refreshes the screen in a prescribed manner that is partially controlled by the horizontal and vertical synchronization signals. The monitor starts each refresh cycle by updating the pixel in the top left-hand corner of the screen, which can be treated as the origin of an X–Y plane (see Figure 12). After the first pixel is refreshed, the monitor refreshes the remaining pixels in the row. When the monitor receives a pulse on the horizontal synchronization, it refreshes the next row of pixels. This process is repeated until the monitor reaches the bottom of the screen. When the monitor reaches the bottom of the screen, the vertical synchronization pulses, causing the monitor to begin refreshing pixels at the top of the screen (i.e., at [0,0]).

VGA Timing

For the VGA monitor to work properly, it must receive data at specific times with specific pulses. Horizontal and vertical synchronization pulses must occur at specified times to synchronize the monitor while it is receiving color data. Figures 13 and 14 show the timing waveforms for the color information with respect to the horizontal and vertical synchronization signals.
The frequency of operation and the number of pixels that the monitor must update determines the time required to update each pixel, and the time required to update the whole screen. The following equations roughly calculate the time required for the monitor to perform all of its functions.
\[
T_{\text{pixel}} = \frac{1}{f_{\text{CLK}}}
\]
\[
= 40 \text{ ns}
\]

\[
T_{\text{ROW}} = A
\]
\[
= (T_{\text{pixel}} \times 640 \text{ pixels})/(\text{row} + \text{guard bands})
\]
\[
= 25 \mu s + B + C + E
\]
\[
= 31.77 \mu s
\]

\[
T_{\text{screen}} = (T_{\text{ROW}} \times 480 \text{ rows}) + \text{guard bands}
\]
\[
= 15.5 \text{ ms} + P + Q + S
\]
\[
= 16.6 \text{ ms}
\]

\[
f_{\text{RR}} = \frac{1}{T_{\text{ROW}}}
\]
\[
= 31.5 \text{ KHz}
\]

\[
f_{\text{SR}} = \frac{1}{T_{\text{screen}}}
\]
\[
= 60 \text{ Hz}
\]

Where:
- \( T_{\text{pixel}} \): Time required to update a pixel
- \( f_{\text{CLK}} \): 25.175 MHz
- \( T_{\text{ROW}} \): Time required to update one row
- \( T_{\text{screen}} \): Time required to update the screen
- \( f_{\text{RR}} \): Row refresh frequency
- \( f_{\text{SR}} \): Screen refresh frequency

The monitor writes to the screen by sending red, green, blue, horizontal sync, and vertical synchronization signals when the screen is at the expected location. Once the timing of the horizontal and vertical synchronization signals is accurate, the monitor only needs to keep track of the current location, so it can send the correct color data to the pixel.

**MOUSE Interface Operation**

You can connect a mouse to the UP 1 Education Board via the 6-pin mini-DIN connector. The data is sent using a synchronous serial protocol, and the transmission is controlled by the \( CLK \) and \( DATA \) signals. During non-transmission, \( CLK \) is at logic 1 and \( DATA \) can be either logic 0 or logic 1.

Each transmission contains one start bit, eight data bits, odd parity, and one stop bit. Data transmission starts from the least significant bit (LSB), i.e., the sequence of transmission is start bit, \( DATA0 \) through \( DATA7 \), parity, stop bit. Start bits are logic 0, and stop bits are logic 1. Each clock period is 30 to 50 \( \mu \text{sec} \); the data transition to the falling edge of the clock is 5 to 25 \( \mu \text{sec} \). Table 13 shows the data packet format.
### Table 13. Data Packet Format

<table>
<thead>
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<th>Packet Number</th>
<th>D7</th>
<th>D6</th>
<th>D5</th>
<th>D4</th>
<th>D3</th>
<th>D2</th>
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<th>D0</th>
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<tbody>
<tr>
<td>1</td>
<td>YV</td>
<td>XV</td>
<td>YS</td>
<td>XS</td>
<td>1</td>
<td>0</td>
<td>R</td>
<td>L</td>
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<td>X6</td>
<td>X5</td>
<td>X4</td>
<td>X3</td>
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<td>3</td>
<td>Y7</td>
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<td>Y4</td>
<td>Y3</td>
<td>Y2</td>
<td>Y1</td>
<td>Y0</td>
</tr>
</tbody>
</table>

**Note:**

(1) where:
- **L** = Left button state (1 = left mouse button is pressed down)
- **R** = Right button state (1 = right mouse button is pressed down)
- **X0 – X7** = Movement in X direction
- **Y0 – Y7** = Movement in Y direction
- **XS, YS** = Movement data sign (1 = negative)
- **XV, YV** = Movement data overflow (1 = overflow has occurred)

The mouse operates on a Cartesian coordinate system (i.e., moving to the right is positive, moving to the left is negative, moving up is positive, and moving down is negative). The magnitude of the movement is a function of the mouse’s rate of movement. The faster the mouse moves, the greater the magnitude.
This user guide supplement provides updated pin-out and timing information for the UP 1 Education Board. This supplement should be used together with the University Program Design Laboratory Package User Guide.

Table 1 provides updated pin-out information for the MAX_EXPANSION port on the UP 1 Education Board.

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
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</thead>
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<td>VCC</td>
<td>4</td>
<td>GND</td>
</tr>
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<td>VCC</td>
<td>6</td>
<td>GND</td>
</tr>
<tr>
<td>7</td>
<td>No Connect</td>
<td>8</td>
<td>No Connect</td>
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<td>9</td>
<td>No Connect</td>
<td>10</td>
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</tr>
<tr>
<td>11</td>
<td>No Connect</td>
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</table>
Table 1. MAX_EXPANSION Signal Names & Device Connections (Part 2 of 2)

<table>
<thead>
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<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
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<td>60</td>
<td>GND</td>
</tr>
</tbody>
</table>

Note:
(1) The updated pin numbers are highlighted in gray.

Table 2 shows updated pin-out information for the FLEX_EXPAN_C port on the UP 1 Education Board.

Table 2. FLEX_EXPAN_C Signal Names & Device Connections (Part 1 of 2)

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>RAW</td>
<td>2</td>
<td>GND</td>
</tr>
<tr>
<td>3</td>
<td>VCC</td>
<td>4</td>
<td>GND</td>
</tr>
<tr>
<td>5</td>
<td>VCC</td>
<td>6</td>
<td>GND</td>
</tr>
<tr>
<td>7</td>
<td>No Connect</td>
<td>8</td>
<td>D11/99</td>
</tr>
<tr>
<td>9</td>
<td>D12/92</td>
<td>10</td>
<td>D13/210</td>
</tr>
<tr>
<td>11</td>
<td>D14/212</td>
<td>12</td>
<td>DEV_CLR/209</td>
</tr>
<tr>
<td>13</td>
<td>DEV_OE/213</td>
<td>14</td>
<td>DEV_CLK2/211</td>
</tr>
<tr>
<td>15</td>
<td>175</td>
<td>16</td>
<td>181</td>
</tr>
<tr>
<td>17</td>
<td>182</td>
<td>18</td>
<td>183</td>
</tr>
<tr>
<td>19</td>
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<td>20</td>
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</tr>
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<td>21</td>
<td>186</td>
<td>22</td>
<td>187</td>
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<td>23</td>
<td>188</td>
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<td>25</td>
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<tr>
<td>43</td>
<td>217</td>
<td>44</td>
<td>218</td>
</tr>
</tbody>
</table>
Table 2. FLEX_EXPAN_C Signal Names & Device Connections (Part 2 of 2)

<table>
<thead>
<tr>
<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
<th>Hole Number</th>
<th>Signal/Pin, Note (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>219</td>
<td>46</td>
<td>220</td>
</tr>
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<td>47</td>
<td>221</td>
<td>48</td>
<td>222</td>
</tr>
<tr>
<td>49</td>
<td>223</td>
<td>50</td>
<td>225</td>
</tr>
<tr>
<td>51</td>
<td>226</td>
<td>52</td>
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<td>53</td>
<td>228</td>
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<td>55</td>
<td>230</td>
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<td>231</td>
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<tr>
<td>57</td>
<td>VCC</td>
<td>58</td>
<td>GND</td>
</tr>
<tr>
<td>59</td>
<td>VCC</td>
<td>60</td>
<td>GND</td>
</tr>
</tbody>
</table>

Note:
(1) The updated pin numbers are highlighted in gray.

VGA Timing Information

For the VGA monitor to work properly, it must receive data at specific times with specific pulses. Horizontal and vertical synchronization pulses must occur at specified times to synchronize the monitor while it is receiving color data. Figures 1 and 2 show updated timing waveforms for color information with respect to horizontal and vertical synchronization signals.

Figure 1. Horizontal Refresh Cycle

<table>
<thead>
<tr>
<th>Parameters</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>31.77 µs</td>
<td>3.77 µs</td>
<td>1.89 µs</td>
<td>25.17 µs</td>
<td>0.94 µs</td>
</tr>
</tbody>
</table>
Figure 2. Vertical Refresh Cycle

![Diagram showing vertical refresh cycle with parameters O, P, Q, R, S.]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>O</th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>16.6 ms</td>
<td>64 µs</td>
<td>1.02 ms</td>
<td>15.25 ms</td>
<td>0.35 ms</td>
</tr>
</tbody>
</table>

The following updated equations determine the time required for a monitor to update each pixel and to update a whole screen.

\[
T_{\text{pixel}} = \frac{1}{f_{\text{CLK}}} = 40 \text{ ns}
\]

\[
T_{\text{ROW}} = A = B + C + D + E = (T_{\text{pixel}} \times 640 \text{ pixels}) + \text{row + guard bands} = 31.77 \mu\text{s}
\]

\[
T_{\text{screen}} = O = P + Q + R + S = (T_{\text{ROW}} \times 480 \text{ rows}) + \text{guard bands} = 16.6 \text{ ms}
\]

Where:
- \(T_{\text{pixel}}\) = Time required to update a pixel
- \(f_{\text{CLK}}\) = 25.175 MHz
- \(T_{\text{ROW}}\) = Time required to update one row
- \(T_{\text{screen}}\) = Time required to update the screen
- B, C, E, P, Q, S = Guard bands
APPENDIX B

VHDL Language Reference

1. VHDL Basics
   1.1. Valid Names
   1.2. Comments
   1.3. Entity and Architecture
   1.4. Ports
   1.5. Signals and Variables
   1.6. Type
      1.6.1. STD_LOGIC
      1.6.2. Enumerated Type
   1.7. Libraries and Packages

1.1 Valid Names

A valid name in VHDL consists of a letter followed by any number of letters or numbers, without spaces. VHDL is not case sensitive. An underscore may be used within a name, but may not begin or end the name. Two consecutive underscores are not permitted.

| EXAMPLES | Valid names: | decode4 |
|          |             | just_in_time |
|          |             | What_4 |
| Invalid names: | 4decode (begins with a digit) | in_time (two consecutive underscores) |
|             | _What_4 (begins with underscore) | my design (space inside name) |
|             | your_words? (special character ? not allowed) |

1.2 Comments

A comment is explanatory text that is ignored by the VHDL compiler. It is indicated by two consecutive hyphens.

| EXAMPLE | -- This is a comment. |
1.3 Entity and Architecture

All VHDL files require an entity declaration and an architecture body. The entity declaration indicates the input and output ports of the design. The architecture body details the internal relationship between inputs and outputs. The VHDL file name must be the same as the entity name.

Syntax:

```
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY __entity_name IS
  GENERIC(define parameters);
  PORT(define inputs and outputs);
END __entity_name;

ARCHITECTURE a OF __entity_name IS
  SIGNAL and COMPONENT declarations;
BEGIN
  statements;
END a;
```

EXAMPLES:

--- Majority vote circuit (majority.vhd)
```
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY majority IS
  PORT(
    a, b, c: IN   STD_LOGIC;
    y      : OUT  STD_LOGIC);
END majority;

ARCHITECTURE a OF majority IS
BEGIN
  y <= (a and b) or (b and c) or (a and c);
END a;
```

--- 2-line-to-4-line decoder with active-HIGH outputs (decoder.vhd)
```
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY decoder IS
  PORT(
    d : IN  STD_LOGIC_VECTOR (1 downto 0);
    y : OUT STD_LOGIC_VECTOR (3 downto 0));
END decoder;

ARCHITECTURE a OF decoder IS
BEGIN
  WITH d SELECT
  y <=   "0001" WHEN "00",
        "0010" WHEN "01",
        "0100" WHEN "10",
        "1000" WHEN "11",
        "0000" WHEN others;
END a;
```
1.4 Ports

A port in VHDL is a connection from a VHDL design entity to the outside world. The direction or directions in which a port may operate is called its mode. A VHDL port may have one of four modes: IN (input only), OUT (output only), INOUT (bidirectional), and BUFFER (output, with feedback from the output back into the design entity). The mode of a port is declared in the port statement of an entity declaration or component declaration.

EXAMPLES:

ENTITY mux IS
    PORT(
        s1, s0     : IN STD_LOGIC;
        y0, y1, y2, y3 : OUT STD_LOGIC);
END mux;

ENTITY srg8 IS
    PORT(
        clock, reset   : IN       STD_LOGIC;
        q              : BUFFER   STD_LOGIC_VECTOR (7 downto 0));
END srg8;

1.5 Signals and Variables

A signal is like a wire connecting two or more points inside an architecture body. It is declared before the BEGIN statement of an architecture body and is global to the architecture. Its value is assigned with the := operator.

A variable is an piece of working memory, local to a specific process. It is declared before the BEGIN statement of a process and is assigned using the := operator.

EXAMPLE:

ARCHITECTURE a OF design4 IS
    SIGNAL connect : STD_LOGIC_VECTOR ( 7 downto 0);
BEGIN
    PROCESS check IS
        VARIABLE count : INTEGER RANGE 0 TO 255;
        BEGIN
            IF (clock'EVENT and clock = '1') THEN
                count := count + 1;     —— Variable assignment statement
            END IF;
        END PROCESS;
    connect <= a and b;            —— Signal assignment statement
END a;

1.6 Type

The type of a port, signal, or variable determines the values it can have. For example, a signal of type BIT can only have values '0' and '1'. A signal of type INTEGER can have any
The STD_LOGIC (standard logic) type, also called IEEE Std.1164 Multi-Valued Logic, gives a broader range of output values than just ‘0’ and ‘1’. Any port, signal, or variable of type STD_LOGIC or STD_LOGIC_VECTOR can have any of the following values.

- ‘U’, —— Uninitialized
- ‘X’, —— Forcing Unknown
- ‘0’, —— Forcing 0
- ‘1’, —— Forcing 1
- ‘Z’, —— High Impedance
- ‘W’, —— Weak Unknown
- ‘L’, —— Weak 0
- ‘H’, —— Weak 1
- ‘-’, —— Don’t care

“Forcing” levels are deemed to be the equivalent of a gate output. “Weak” levels are specified by a pull-up or pull-down resistor. The ‘Z’ state is used as the high-impedance state of a tristate buffer.

The majority of applications can be handled by ‘X’, ‘0’, ‘1’, and ‘Z’ values.

To use STD_LOGIC in a VHDL file, you must include the following reference to the VHDL library called ieee and the std_logic_1164 package before the entity declaration.

```vhdl
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
```

### 1.6.2 Enumerated Type

An enumerated type is a user-defined type that lists all possible values for a port, signal, or variable. One use of an enumerated type is to list the states of a state machine.

**EXAMPLE:**

```vhdl
TYPE STATE_TYPE IS (idle, start, pulse, read);
SIGNAL state: STATE_TYPE;
```

### 1.7 Libraries and Packages

A library is a collection of previously compiled VHDL constructs that can be used in a design entity. A package is an uncompiled collection of VHDL constructs that can be used in multiple design entities. Library names must be included at the beginning of a VHDL file, before the entity declaration, to use certain types or functions. The most obvious is the library ieee, which in the package std_logic_1164, defines the STD_LOGIC (standard logic)
2. Concurrent Structures

2.1 Concurrent Signal Assignment Statement

2.2 Selected Signal Assignment Statement

2.3 Conditional Signal Assignment Statements

2.4 Components

2.4.1 Component Declaration

2.4.2 Component Instantiation

2.4.3 Generic Clause

2.5 Generate Statement

2.6 Process Statement

A concurrent structure in VHDL acts as a separate component. A change applied to multiple concurrent structures acts on all affected structures at the same time. This is similar to a signal applied to multiple components in a circuit; a change in the signal will operate on all the components simultaneously.

2.1 Concurrent Signal Assignment Statement

A concurrent signal assignment statement assigns a port or signal the value of a Boolean expression or constant. This statement is useful for encoding a Boolean equation. Since the operators and, or, not, and xor have equal precedence in VHDL, the order of precedence must be made explicit by parentheses.

Syntax:

```vhdl
__signal <= __expression;
```

**EXAMPLES:**

```vhdl
sum <= (a xor b) xor c;
c_out <= ((a xor b) and c_in) or (a and b);
```

2.2 Selected Signal Assignment Statement

A selected signal assignment statement assigns one of several alternative values to a port or
signal, based on the value of a selecting signal. It can be used to implement a truth table or a selecting circuit like a multiplexer.

**Syntax:**

```
label: WITH _expression SELECT
   _signal <= _expression WHEN _constant_value,
           _expression WHEN _constant_value,
           _expression WHEN _constant_value,
           _expression WHEN _constant_value;
```

**EXAMPLES:**

--- decoder implemented as a truth table (2 inputs, 4 outputs)
--- d has been defined as STD_LOGIC_VECTOR (1 downto 0)
--- y has been defined as STD_LOGIC_VECTOR (3 downto 0)

WITH d SELECT
   y <=   "0001" WHEN "00",
         "0010" WHEN "01",
         "0100" WHEN "10",
         "1000" WHEN "11",
         "0000" WHEN others;

--- multiplexer
--- input signal assigned to y, depending on states of s1, s0

M: WITH s SELECT
   y <=   d0 WHEN "00",
          d1 WHEN "01",
          d2 WHEN "10",
          d3 WHEN "11";

### 2.3 Conditional Signal Assignment Statement

A conditional signal assignment statement assigns a value to a port or signal based on a series of linked conditions. The basic structure assigns a value if the first condition is true. If not, another value is assigned if a second condition is true, and so on, until a default condition is reached. This is an ideal structure for a priority encoder.

**Syntax:**

```
__label:
__signal <= __expression WHEN __boolean_expression ELSE
            __expression WHEN __boolean_expression ELSE
            __expression;
```

**EXAMPLE:**

--- priority encoder
--- q defined as INTEGER RANGE 0 TO 7
--- d defined as STD_LOGIC_VECTOR (7 downto 0)

encoder:
```
q  <=  7 WHEN d(7)='1' ELSE
      6 WHEN d(6)='1' ELSE
      5 WHEN d(5)='1' ELSE
      4 WHEN d(4)='1' ELSE
      3 WHEN d(3)='1' ELSE
      2 WHEN d(2)='1' ELSE
      1 WHEN d(1)='1' ELSE
      0;
```
2.4 Components

A VHDL file can use another VHDL file as a component. The general form of a design entity using components is:

ENTITY entity_name IS
  PORT ( input and output definitions);
END entity_name;

ARCHITECTURE arch_name OF entity_name IS
  component declaration(s);
  signal declaration(s);
BEGIN
  Component instantiation(s);
  Other statements;
END arch_name;

2.4.1 Component Declaration

A component declaration is similar in form to an entity declaration, in that it includes the required ports and parameters of the component. The difference is that it refers to a design described in a separate VHDL file. The ports and parameters in the component declaration may be a subset of those in the component file, but they must have the same names.

Syntax:

COMPONENT __component_name
  GENERIC(__parameter_name : string := __default_value;
          __parameter_name : integer := __default_value);
  PORT(
    __input name, __input_name      : IN STD_LOGIC;
    __bidir name, __bidir_name      : INOUT STD_LOGIC;
    __output name, __output_name     : OUT  STD_LOGIC);
END COMPONENT;

EXAMPLE:

ARCHITECTURE adder OF add4pa IS
  COMPONENT full_add
    PORT(
      a, b, c_in : IN BIT;
      c_out, sum : OUT BIT);
  END COMPONENT;
  SIGNAL c : BIT_VECTOR (3 downto 1);
BEGIN
  statements
END adder;

2.4.2 Component Instantiation

Each instance of a component requires a component instantiation statement. Ports can be
assigned explicitly with the \( => \) operator, or implicitly by inserting the user port name in the position of the corresponding port name within the component declaration.

**Syntax:**

\[
\begin{align*}
\text{\_instance\_name: \_component\_name} \\
\text{GENERIC MAP (\_parameter\_name => \_parameter\_value,} \\
\text{\_parameter\_name => \_parameter\_value)} \\
\text{PORT MAP (\_component\_port => \_connect\_port,} \\
\text{\_component\_port => \_connect\_port);}
\end{align*}
\]

**EXAMPLES:**

--- Four component instantiation statements

--- Explicit port assignments

adder1: full_add

PORT MAP (a => a(1),
          b => b(1),
          c\_in => c0,
          c\_out => c(1),
          sum => sum(1));

adder2: full_add

PORT MAP (a => a(2),
          b => b(2),
          c\_in => c(1),
          c\_out => c(2),
          sum => sum(2));

adder3: full_add

PORT MAP (a => a(3),
          b => b(3),
          c\_in => c(2),
          c\_out => c(3),
          sum => sum(3));

adder4: full_add

PORT MAP (a => a(4),
          b => b(4),
          c\_in => c(3),
          c\_out => c4,
          sum => sum(4));

--- Four component instantiations

--- Implicit port assignments

adder1: full_add PORT MAP (a(1), b(1), c0, c(1), sum(1));

adder2: full_add PORT MAP (a(2), b(2), c(1), c(2), sum(2));

adder3: full_add PORT MAP (a(3), b(3), c(2), c(3), sum(3));

adder4: full_add PORT MAP (a(4), b(4), c(3), c4, sum(4));

--- 2.4.3 Generic Clause

A generic clause allows a component to be designed with one or more unspecified properties (“parameters”) that are specified when the component is instantiated. A parameter specified in a generic clause must be given a default value with the := operator.

**Syntax:**

--- Parameters defined in entity declaration of component file

ENTITY entity\_name IS
    GENERIC (\_parameter\_name : type := \_default\_value;
--- COMPONENT component_name IS
   GENERIC(__parameter_name : type;
   __parameter_name : type);
   PORT (port declarations);
END COMPONENT;

--- Parameters specified in generic map in component instantiation
__instance_name: __component_name
GENERIC MAP  (__parameter_name => __parameter_value,
   __parameter_name => __parameter_value)
PORT MAP (port instantiations);

EXAMPLE:
--- Component: behaviorally defined shift register
--- with default width of 4.
ENTITY srt_bhv IS
   GENERIC (width : POSITIVE := 4);
   PORT(
      serial_in, clk : IN         STD_LOGIC;
      q              : BUFFER     STD_LOGIC_VECTOR(width-1 downto 0));
END srt_bhv;
ARCHITECTURE right_shift of srt_bhv IS
BEGIN
   PROCESS (clk)
   BEGIN
      IF (clk'EVENT and clk = '1') THEN
         q(width-1 downto 0) <= serial_in & q(width-1 downto 1);
      END IF;
   END PROCESS;
END right_shift;

--- srt8_bhv.vhd
--- 8-bit shift register that instantiates srt_bhv
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY srt8_bhv IS
   PORT(
      data_in, clock : IN       STD_LOGIC;
      qo          : BUFFER   STD_LOGIC_VECTOR(7 downto 0));
END srt8_bhv;
ARCHITECTURE right_shift of srt8_bhv IS
--- component declaration
COMPONENT srt_bhv
   GENERIC (width : POSITIVE);
   PORT(
      serial_in, clk : IN  STD_LOGIC;
      q              : OUT STD_LOGIC_VECTOR(7 downto 0));
END COMPONENT;

(example continues)
BEGIN
    -- component instantiation
    Shift_right_8: srt_bhv
    GENERIC MAP  (width=> 8)
    PORT MAP  (serial_in => data_in,
                clk       => clock,
                q         => qo);
END right_shift;

2.5 Generate Statement

A generate statement is used to create multiple instances of a particular hardware structure. It relies on the value of one or more index variables to create the required number of repetitions.

Syntax:

__generate_label:
FOR __index_variable IN __range GENERATE
    __statement;
END GENERATE;

EXAMPLES:

-- Instantiate four full adders
adders:
FOR i IN 1 to 4 GENERATE
    adder: full_add PORT MAP (a(i), b(i), c(i-1), c(i), sum(i));
END GENERATE;

-- Instantiate four latches from MAX+PLUS II primitives
-- Requires the statements LIBRARY altera; and
-- USE altera.maxplus.ALL;
latch4:
FOR i IN 3 downto 0 GENERATE
    latch_primitive: latch
    PORT MAP (d => d_in(i), ena => enable, q => q_out (i));
END GENERATE;

2.6 Process Statement

A process is a concurrent statement, but the statements inside the process are sequential. For example, a process can define a flip-flop, a separate component whose ports are affected concurrently, but the inside of the flip-flop acts sequentially. A process executes all statements inside it when there is a change of a signal in its sensitivity list. The process label is optional.

Syntax:

__process_label:
PROCESS (sensitivity list)
    variable declarations
BEGIN
    sequential statements
3. Sequential Structures

3.1. If Statement

3.1.1. Evaluating Clock Functions

3.2. Case Statement

A sequential structure in VHDL is one in which the order of statements affects the operation of the circuit. It can be used to implement combinational circuits, but is primarily used to implement sequential circuits such as latches, counters, shift registers, and state machines. Sequential statements must be contained within a process.

3.1 If Statement

An IF statement executes one or more statements if a Boolean condition is satisfied.

Syntax:

IF __expression THEN
  __statement;
  __statement;
ELSIF __expression THEN
  __statement;
  __statement;
ELSE
  __statement;
  __statement;
END IF;

EXAMPLE:

PROCESS (reset, load, clock)
  VARIABLE count INTEGER RANGE 0 TO 255;
BEGIN
  IF (reset = '0') THEN
    q <= 0;
  ELSIF (reset = '1' and load = '0') THEN
    q <= p;
  ELSIF (clock'EVENT and clock = '1') THEN
    count := count + 1;
    q <= count;
END IF;
3.1.1 Evaluating Clock Functions

As implied in previous examples, the state of a system clock can be checked with an IF statement using the predefined attribute called EVENT. The clause `clock'EVENT` ("clock tick EVENT") is true if there has been activity on the signal called clock. Thus `(clock'EVENT and clock = '1')` is true just after a positive edge on clock.

3.2 Case Statement

A case statement is used to execute one of several sets of statements, based on the evaluation of a signal.

**Syntax:**

```
CASE __expression IS
    WHEN __constant_value =>
        __statement;
        __statement;
    WHEN __constant_value =>
        __statement;
        __statement;
    WHEN OTHERS =>
        __statement;
        __statement;
END CASE;
```

**EXAMPLES:**

```vhdl
-- Case evaluates 2-bit value of s and assigns 4-bit values of x and y accordingly
-- Default case (others) required if using STD_LOGIC
CASE s IS
    WHEN "00" =>
        y <= "0001";
        x <= "1110";
    WHEN "01" =>
        y <= "0010";
        x <= "1101";
    WHEN "10" =>
        y <= "0100";
        x <= "1011";
    WHEN "11" =>
        y <= "1000";
        x <= "0111";
    WHEN others =>
        y <= "0000";
```

x <= "1111";
END CASE;

-- This case evaluates the state variable "sequence"
-- that can have two possible values: "start" and "continue"
-- Values of out1 and out2 are also assigned for each case.
CASE sequence IS
  WHEN start =>
    IF in1 = '1' THEN
      sequence <= start;
      out1 <= '0';
      out2 <= '0';
    ELSE
      sequence <= continue;
      out1 <= '1';
      out2 <= '0';
    END IF;
  WHEN continue =>
    sequence <= start;
    out1 <= '0';
    out2 <= '1';
END CASE;
## Manufacturers’ Data Sheets

### Data Sheet List

<table>
<thead>
<tr>
<th>Device</th>
<th>Description</th>
<th>Source/File Name</th>
<th>Pages</th>
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<tbody>
<tr>
<td>74LS00</td>
<td>Quad 2-input NAND Gate</td>
<td>Motorola/sn74ls00rev6.pdf</td>
<td>703</td>
</tr>
<tr>
<td>74LS02</td>
<td>Quad 2-input NOR Gate</td>
<td>Motorola/sn74ls02rev5.pdf</td>
<td>705</td>
</tr>
<tr>
<td>74LS04</td>
<td>Hex Inverter</td>
<td>Motorola/sn74ls04rev6.pdf</td>
<td>707</td>
</tr>
<tr>
<td>74LS05</td>
<td>Hex Inverter (Open Collector)</td>
<td>Motorola/sn74ls05rev6.pdf</td>
<td>709</td>
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<tr>
<td>74LS06/16</td>
<td>Hex Inverting Buffer (Open Collector)</td>
<td>Texas Instruments/sdls020a.pdf</td>
<td>711</td>
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<tr>
<td>75LS07</td>
<td>Hex Noninverting Buffer (Open Collector)</td>
<td>Texas Instruments/sdls021a.pdf</td>
<td>714</td>
</tr>
<tr>
<td>74LS86</td>
<td>Quad 2-input XOR Gate</td>
<td>Motorola/sn74ls86rev6.pdf</td>
<td>721</td>
</tr>
<tr>
<td>74F00</td>
<td>Quad 2-input NAND Gate</td>
<td>Texas Instruments/sdfs035a.pdf</td>
<td>723</td>
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<tr>
<td>74AS/ALS00</td>
<td>Quad 2-input NAND Gate</td>
<td>Texas Instruments/sdas187a.pdf</td>
<td>726</td>
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<tr>
<td>74HC00</td>
<td>Quad 2-input NAND Gate</td>
<td>Motorola/mc74hc00arev7a.pdf</td>
<td>731</td>
</tr>
<tr>
<td>74HCT00</td>
<td>Quad 2-input NAND Gate (TTL Input Levels)</td>
<td>Motorola/mc74hct00arev6.pdf</td>
<td>735</td>
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<tr>
<td>74VHC00</td>
<td>Quad 2-input NAND Gate</td>
<td>Motorola/mc74vhc00arev0.pdf</td>
<td>738</td>
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<td>74VHCT00</td>
<td>Quad 2-input NAND Gate (TTL Input Levels)</td>
<td>Motorola/mc74vhct00arev0.pdf</td>
<td>741</td>
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<tr>
<td>74HCU04</td>
<td>Hex Inverter (Unbuffered)</td>
<td>Motorola/mc74hcu04arev1.pdf</td>
<td>744</td>
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<tr>
<td>74HC4049/4050</td>
<td>Hex Buffer</td>
<td>Motorola/mc74hc4049rev6.pdf</td>
<td>749</td>
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<tr>
<td>74LVX00</td>
<td>Quad 2-input NAND Gate</td>
<td>Motorola/mc74lvx00rev0b.pdf</td>
<td>753</td>
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<tr>
<td>74LCX00</td>
<td>Quad 2-input NAND Gate</td>
<td>Motorola/mc74lcx00rev1.pdf</td>
<td>756</td>
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<tr>
<td>74LCX00</td>
<td>Quad 2-input NAND Gate</td>
<td>Motorola/mc14001brev3.pdf</td>
<td>759</td>
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</tbody>
</table>
SN74LS00

Quad 2-Input NAND Gate

- ESD > 3500 Volts

GUARANTEED OPERATING RANGES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>Supply Voltage</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
<td>V</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Ambient Temperature Range</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>°C</td>
</tr>
<tr>
<td>ICH</td>
<td>Output Current – High</td>
<td></td>
<td></td>
<td>−0.4</td>
<td>mA</td>
</tr>
<tr>
<td>IOL</td>
<td>Output Current – Low</td>
<td></td>
<td></td>
<td>8.0</td>
<td>mA</td>
</tr>
</tbody>
</table>

ORDERING INFORMATION

<table>
<thead>
<tr>
<th>Device</th>
<th>Package</th>
<th>Shipping</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN74LS00N</td>
<td>14 Pin DIP</td>
<td>2000 Units/Box</td>
</tr>
<tr>
<td>SN74LS00D</td>
<td>14 Pin</td>
<td>2500/Tape &amp; Reel</td>
</tr>
</tbody>
</table>
## SN74LS00

### DC CHARACTERISTICS OVER OPERATING TEMPERATURE RANGE

(Unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_H$</td>
<td>Input HIGH Voltage</td>
<td>2.0</td>
<td>Guaranteed Input HIGH Voltage for All Inputs</td>
</tr>
<tr>
<td>$V_L$</td>
<td>Input LOW Voltage</td>
<td>0.8</td>
<td>Guaranteed Input LOW Voltage for All Inputs</td>
</tr>
<tr>
<td>$V_{IC}$</td>
<td>Input Clamp Diode Voltage</td>
<td>$-0.65$</td>
<td>$V_{CC} = \text{MIN}, I_{IN} = -18 \text{ mA}$</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>Output HIGH Voltage</td>
<td>2.7</td>
<td>$V_{CC} = \text{MIN}, I_{OH} = \text{MAX}, V_{IN} = V_H$</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Output LOW Voltage</td>
<td>0.25</td>
<td>$I_{OL} = 4.0 \text{ mA}$ $V_{CC} = V_{CC} \text{ MIN, } V_{IN} = V_L$ or $V_{IH}$ per Truth Table</td>
</tr>
<tr>
<td>$I_H$</td>
<td>Input HIGH Current</td>
<td>0.35</td>
<td>$I_{OL} = 0.1 \text{ mA}$ $V_{CC} = \text{MAX, } V_{IN} = 2.7 \text{ V}$</td>
</tr>
<tr>
<td>$I_L$</td>
<td>Input LOW Current</td>
<td>0.1</td>
<td>$V_{CC} = \text{MAX, } V_{IN} = 7.0 \text{ V}$</td>
</tr>
<tr>
<td>$I_{OS}$</td>
<td>Short Circuit Current (Note 1)</td>
<td>$-0.4$</td>
<td>$V_{CC} = \text{MAX, } V_{IN} = 0.4 \text{ V}$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Power Supply Current</td>
<td>20</td>
<td>$V_{CC} = \text{MAX}$</td>
</tr>
<tr>
<td></td>
<td>Total, Output HIGH</td>
<td>1.5</td>
<td>$V_{CC} = \text{MAX}$</td>
</tr>
<tr>
<td></td>
<td>Total, Output LOW</td>
<td>4.4</td>
<td>$V_{CC} = \text{MAX}$</td>
</tr>
</tbody>
</table>

Note 1: Not more than one output should be shorted at a time, nor for more than 1 second.

### AC CHARACTERISTICS ($T_A = 25^\circ\text{C}$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{PLH}$</td>
<td>Turn–Off Delay, Input to Output</td>
<td>9.0</td>
<td>$V_{CC} = 5.0 \text{ V}$</td>
</tr>
<tr>
<td>$t_{PHL}$</td>
<td>Turn–On Delay, Input to Output</td>
<td>10</td>
<td>$C_L = 15 \text{ pF}$</td>
</tr>
</tbody>
</table>
QUAD 2-INPUT NOR GATE

Guaranteed Operating Ranges

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>Supply Voltage</td>
<td>54</td>
<td>4.5</td>
<td>5.0</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td>4.75</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>TA</td>
<td>Operating Ambient Temperature</td>
<td>54</td>
<td>-55</td>
<td>25</td>
<td>°C</td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>74</td>
<td>0</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>IOH</td>
<td>Output Current — High</td>
<td>54</td>
<td></td>
<td>-0.4</td>
<td>mA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td></td>
<td>8.0</td>
<td></td>
</tr>
<tr>
<td>IOL</td>
<td>Output Current — Low</td>
<td>54</td>
<td></td>
<td>4.0</td>
<td>mA</td>
</tr>
<tr>
<td></td>
<td></td>
<td>74</td>
<td></td>
<td>8.0</td>
<td></td>
</tr>
</tbody>
</table>
### DC Characteristics Over Operating Temperature Range (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
</tr>
<tr>
<td>$V_{IH}$</td>
<td>Input HIGH Voltage</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Input LOW Voltage</td>
<td>54</td>
<td>0.7</td>
</tr>
<tr>
<td>$V_{IK}$</td>
<td>Input Clamp Diode Voltage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>Output HIGH Voltage</td>
<td>54</td>
<td>2.5</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Output LOW Voltage</td>
<td>54, 74</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{IH}$</td>
<td>Input HIGH Current</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>$I_{IL}$</td>
<td>Input LOW Current</td>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>$I_{OS}$</td>
<td>Short Circuit Current (Note 1)</td>
<td>-20</td>
<td></td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Power Supply Current</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total, Output HIGH</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total, Output LOW</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note 1:** Not more than one output should be shorted at a time, nor for more than 1 second.

### AC Characteristics ($T_A = 25^\circ C$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
</tr>
<tr>
<td>$\tau_{PLH}$</td>
<td>Turn-Off Delay, Input to Output</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>$\tau_{PHL}$</td>
<td>Turn-On Delay, Input to Output</td>
<td>10</td>
<td>15</td>
</tr>
</tbody>
</table>
### GUARANTEED OPERATING RANGES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>Supply Voltage</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
<td>V</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Operating Ambient Temperature Range</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>°C</td>
</tr>
<tr>
<td>$I_{OH}$</td>
<td>Output Current ± High</td>
<td>±0.4</td>
<td>mA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{OL}$</td>
<td>Output Current ± Low</td>
<td>8.0</td>
<td>mA</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### ORDERING INFORMATION

<table>
<thead>
<tr>
<th>Device</th>
<th>Package</th>
<th>Shipping</th>
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</thead>
<tbody>
<tr>
<td>SN74LS04N</td>
<td>14 Pin DIP</td>
<td>2000 Units/Box</td>
</tr>
<tr>
<td>SN74LS04D</td>
<td>14 Pin</td>
<td>2500/Tape &amp; Reel</td>
</tr>
</tbody>
</table>
### SN74LS04

**DC CHARACTERISTICS OVER OPERATING TEMPERATURE RANGE** (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{IH}$</td>
<td>Input HIGH Voltage</td>
<td>Min 2.0</td>
<td>V</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Input LOW Voltage</td>
<td>Typ 0.8</td>
<td>V</td>
</tr>
<tr>
<td>$V_{IK}$</td>
<td>Input Clamp Diode Voltage</td>
<td>± 0.65</td>
<td>± 1.5</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>Output HIGH Voltage</td>
<td>Min 2.7</td>
<td>Typ 3.5</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Output LOW Voltage</td>
<td>Min 0.25</td>
<td>Typ 0.4</td>
</tr>
<tr>
<td>$I_{IH}$</td>
<td>Input HIGH Current</td>
<td>Min 20</td>
<td>µA</td>
</tr>
<tr>
<td>$I_{IL}$</td>
<td>Input LOW Current</td>
<td>Min 0.1</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{OS}$</td>
<td>Short Circuit Current (Note 1)</td>
<td>Min ±20</td>
<td>Typ ±100</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Power Supply Current</td>
<td>Total, Output HIGH</td>
<td>Min 2.4</td>
</tr>
<tr>
<td></td>
<td>Total, Output LOW</td>
<td>Min 6.6</td>
<td></td>
</tr>
</tbody>
</table>

Note 1: Not more than one output should be shorted at a time, nor for more than 1 second.

**AC CHARACTERISTICS** ($T_A = 25^\circ\text{C}$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{PLH}$</td>
<td>Turn-Off Delay, Input to Output</td>
<td>Min 9.0</td>
<td>Typ 15</td>
</tr>
<tr>
<td>$t_{PHL}$</td>
<td>Turn-On Delay, Input to Output</td>
<td>Min 10</td>
<td>Typ 15</td>
</tr>
</tbody>
</table>
**SN74LS05**

### Hex Inverter

![Diagram of SN74LS05](image)

*OPEN COLLECTOR OUTPUTS*

### Guaranteed Operating Ranges

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>Supply Voltage</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
<td>V</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Operating Ambient</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>°C</td>
</tr>
<tr>
<td></td>
<td>Temperature Range</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>Output Voltage – High</td>
<td></td>
<td>5.5</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>$I_{CL}$</td>
<td>Output Current – Low</td>
<td></td>
<td>8.0</td>
<td></td>
<td>mA</td>
</tr>
</tbody>
</table>

### Ordering Information

<table>
<thead>
<tr>
<th>Device</th>
<th>Package</th>
<th>Shipping</th>
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</thead>
<tbody>
<tr>
<td>SN74LS05N</td>
<td>14 Pin DIP</td>
<td>2000 Units/Box</td>
</tr>
<tr>
<td>SN74LS05D</td>
<td>14 Pin</td>
<td>2500/Tape &amp; Reel</td>
</tr>
</tbody>
</table>
### DC CHARACTERISTICS OVER OPERATING TEMPERATURE RANGE (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{IH}$</td>
<td>Input HIGH Voltage</td>
<td>2.0</td>
<td>Guaranteed Input HIGH Voltage for All Inputs</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Input LOW Voltage</td>
<td>0.8</td>
<td>Guaranteed Input LOW Voltage for All Inputs</td>
</tr>
<tr>
<td>$V_{IK}$</td>
<td>Input Clamp Diode Voltage</td>
<td>$-0.65$</td>
<td>$V_{CC} = \text{MIN}, \text{I}_{IN} = -18 \text{ mA}$</td>
</tr>
<tr>
<td>$I_{OH}$</td>
<td>Output HIGH Current</td>
<td>100</td>
<td>$V_{CC} = \text{MIN}, V_{OH} = \text{MAX}$</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Output LOW Voltage</td>
<td>0.25</td>
<td>$I_{OL} = 4.0 \text{ mA}$, $V_{CC} = V_{CC} \text{ MIN}$, $V_{IN} = V_{IL}$ or $V_{IH}$ per Truth Table</td>
</tr>
<tr>
<td>$I_{IH}$</td>
<td>Input HIGH Current</td>
<td>0.35</td>
<td>$I_{OL} = 8.0 \text{ mA}$, $V_{CC} = \text{MAX}$</td>
</tr>
<tr>
<td>$I_{IL}$</td>
<td>Input LOW Current</td>
<td></td>
<td>$V_{CC} = \text{MAX}$, $V_{IN} = 2.7 \text{ V}$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Power Supply Current</td>
<td></td>
<td>$V_{CC} = \text{MAX}$</td>
</tr>
<tr>
<td></td>
<td>Total, Output HIGH</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total, Output LOW</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

### AC CHARACTERISTICS ($T_A = 25^\circ \text{C}$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{PH}$</td>
<td>Turn–Off Delay, Input to Output</td>
<td>17</td>
<td>$V_{CC} = 5.0 \text{ V}$</td>
</tr>
<tr>
<td>$I_{PHL}$</td>
<td>Turn–On Delay, Input to Output</td>
<td>15</td>
<td>$C_L = 15 \text{ pF}, R_L = 2.0 \text{ k}\Omega$</td>
</tr>
</tbody>
</table>
SN54LS06, SN54LS16, SN74LS06, SN74LS16
HEX INVERTER BUFFERS/DRIVERS WITH
OPEN-COLLECTOR HIGH-VOLTAGE OUTPUTS

- Converts TTL Voltage Levels to MOS Levels
- High Sink-Current Capability
- Input Clamping Diodes Simplify System Design
- Open-Collector Driver for Indicator Lamps and Relays
- Package Options Include "Small Outline" Packages, Ceramic Chip Carriers, and Standard Plastic and Ceramic 300-mil DIPs

**description**

These monolithic hex inverter buffers/drivers feature high-voltage open-collector outputs to interface with high-level circuits (such as MOS), or for driving high-current loads, and are also characterized for use as inverter buffers for driving TTL inputs. The 'LS06 has a rated output voltage of 30 V and the 'LS16 has a rated output voltage of 15 V. The maximum sink current for the SN54LS06 and SN54LS16 is 30 mA and the SN74LS06 and SN74LS16 is 40 mA.

These circuits are compatible with most TTL families. Inputs are diode-clamped to minimize transmission-effects, which simplifies design. Typical power dissipation is 175 mW and average propagation delay time is 8 ns.

The SN54LS06 and SN54LS16 are characterized over the full military temperature range of −55°C to 125°C. The SN74LS06 and SN74LS16 are characterized for operation from 0°C to 70°C.

**logic symbol**

![Logic symbol diagram](image)

† This symbol is in accordance with ANSI/IEEE Std 91-1984 and IEC Publication 817-12.

Pin numbers shown are for D, J, and N packages.

**logic diagram (positive logic)**

![Logic diagram diagram](image)
SN54LS06, SN54LS16, SN74LS06, SN74LS16
HEX INVERTER BUFFERS/DRIVERS WITH
OPEN-COLLECTOR HIGH-VOLTAGE OUTPUTS

schematic (each gate)

absolute maximum ratings over operating free-air temperature range (unless otherwise noted)†

Supply voltage, \( V_{CC} \) ........................................ 7 V
Input voltage, \( V_i \) (see Note 1) ................................ 5.5 V
Output voltage, \( V_O \) (see Notes 1 and 2): SN54LS06, SN74LS06 .................. 30 V
SN54LS16, SN74LS16 ....................................... 15 V
Operating free-air temperature range: SN54LS06, SN54LS16 .............. \(-55^\circ C\) to \(125^\circ C\)
SN74LS06, SN74LS16 ....................................... \(0^\circ C\) to \(70^\circ C\)
Storage temperature range ...................................... \(-65^\circ C\) to \(150^\circ C\)

† Stresses beyond those listed under “absolute maximum ratings” may cause permanent damage to the device. This are stress ratings only, and functional operation of the device at these or any other conditions beyond those indicated under “recommended operating conditions” is not implied. Exposure to absolute-maximum-rated conditions for extended periods may affect device reliability.

NOTES: 1. Voltage values are with respect to network ground terminal.
2. This is the maximum voltage that should be applied to any output when it is in the off state.

recommended operating conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54LS06</th>
<th>SN54LS16</th>
<th>SN74LS06</th>
<th>SN74LS16</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{CC} ) Supply voltage</td>
<td>MIN</td>
<td>NOM</td>
<td>MAX</td>
<td>MIN</td>
<td>NOM</td>
</tr>
<tr>
<td>( V_{IH} ) High-level input voltage</td>
<td>4.5</td>
<td>5</td>
<td>5.5</td>
<td>4.75</td>
<td>5</td>
</tr>
<tr>
<td>( V_{IL} ) Low-level input voltage</td>
<td>2</td>
<td>2</td>
<td>V</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>( V_{OH} ) High-level output voltage</td>
<td>LS06</td>
<td>30</td>
<td>30</td>
<td>LS16</td>
<td>15</td>
</tr>
<tr>
<td>( I_{OL} ) Low-level output current</td>
<td>30</td>
<td>40</td>
<td>mA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_A ) Operating free-air temperature</td>
<td>(-55)</td>
<td>125</td>
<td>(0)</td>
<td>70</td>
<td>°C</td>
</tr>
</tbody>
</table>
## Appendix C • Manufacturers’ Data Sheets

### SN54LS06, SN54LS16, SN74LS06, SN74LS16
**Hex Inverter Buffers/Drivers with Open-Collector High-Voltage Outputs**

**SDL5020A • May 1990**

#### Electrical Characteristics

- **Over Recommended Operating Free-Air Temperature Range (Unless Otherwise Noted)**

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>TEST CONDITIONS†</th>
<th>SN54LS06</th>
<th>SN54LS16</th>
<th>SN74LS06</th>
<th>SN74LS16</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MIN</td>
<td>TYP†</td>
<td>MAX</td>
<td>MIN</td>
</tr>
<tr>
<td>( V_{IH} )</td>
<td>( V_{CC} = \text{MIN.} ), ( I_I = -12 \text{ mA} )</td>
<td>-1.5</td>
<td>-1.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_{OH} )</td>
<td>( V_{CC} = \text{MIN.} ), ( V_{IL} = 0.8 \text{ V} )</td>
<td>( \text{LS06: } V_{OH} = 30 \text{ V} )</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \text{LS16: } V_{OH} = 15 \text{ V} )</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>( I_{OL} )</td>
<td>( V_{CC} = \text{MIN.} ), ( V_{IH} = 2 \text{ V} )</td>
<td>( I_{OL} = 16 \text{ mA} )</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( I_{OL} = 30 \text{ mA} )</td>
<td>0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( I_{OL} = 40 \text{ mA} )</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( I_I )</td>
<td>( V_{CC} = \text{MAX.} ), ( V_{I} = 7 \text{ V} )</td>
<td></td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( I_{IH} )</td>
<td>( V_{CC} = \text{MAX.} ), ( V_{I} = 2.4 \text{ V} )</td>
<td></td>
<td>20</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>( I_{IL} )</td>
<td>( V_{CC} = \text{MAX.} ), ( V_{I} = 0.4 \text{ V} )</td>
<td></td>
<td>-0.2</td>
<td></td>
<td>-0.2</td>
</tr>
<tr>
<td>( I_{CCH} )</td>
<td>( V_{CC} = \text{MAX.} )</td>
<td></td>
<td>18</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>( I_{CCL} )</td>
<td>( V_{CC} = \text{MAX.} )</td>
<td></td>
<td>60</td>
<td></td>
<td>60</td>
</tr>
</tbody>
</table>

† For conditions shown as MIN or MAX, use the appropriate value specified under recommended operating conditions.

‡ All typical values are at \( V_{CC} = 5 \text{ V} \), and \( T_A = 25^\circ \text{C} \).

### Switching Characteristics

- **\( V_{CC} = 5 \text{ V} \), \( T_A = 25^\circ \text{C} \) (see Note 3)**

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>FROM (INPUT)</th>
<th>TO (OUTPUT)</th>
<th>TEST CONDITIONS</th>
<th>MIN</th>
<th>TYP</th>
<th>MAX</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{PLH} )</td>
<td>( A )</td>
<td>( Y )</td>
<td>( R_L = 110 \text{ \Omega} ), ( C_L = 15 \text{ pF} )</td>
<td>7</td>
<td>15</td>
<td></td>
<td>ns</td>
</tr>
<tr>
<td>( I_{PHL} )</td>
<td>( A )</td>
<td>( Y )</td>
<td></td>
<td>10</td>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note 3:** Load circuit and voltage waveforms are shown in Section 1 of 74L Logic Data Book, 1988.
SN54LS07, SN74LS07, SN74LS17
HEX BUFFERS/DRIVERS WITH
OPEN-COLLECTOR HIGH-VOLTAGE OUTPUTS
SDL5021A, D3517, MAY 1990–REVISED AUGUST 1991

- Converts TTL-Voltage Levels to MOS Levels
- High Sink-Current Capability
- Input Clamping Diodes Simplify System Design
- Open-Collector Driver for Indicator Lamps and Relays
- Package Options Include “Small Outline” Packages, Ceramic Chip Carriers, and Standard and Ceramic 300-mil DIPs

description

These monolithic hex buffers/drivers feature high-voltage open-collector outputs to interface with high-level circuits or for driving high-current loads. They are also characterized for use as buffers for driving TTL inputs. The 'LS07 has a rated output voltage of 30 V and the 'LS17 has a rated output voltage of 15 V. The maximum sink current is 30 mA for the SN54LS07 and 40 mA for the SN74LS07 and SN74LS17.

These circuits are compatible with most TTL families. Inputs are diode-clamped to minimize transmission-line effects, which simplifies design. Typical power dissipation is 140 mW and average propagation delay time is 12 ns.

The SN54LS07 is characterized over the full military temperature range of −55°C to 125°C. The SN74LS07 and SN74LS17 are characterized for operation from 0°C to 70°C.

logic symbol†

logic diagram (positive logic)

† This symbol is in accordance with ANSI/IEEE Std 91-1984 and IEC Publication 617-12.
Pin numbers shown are for D, J, and N packages.
SN54LS07, SN74LS07, SN74LS17
HEX BUFFERS/DRIVERS WITH
OPEN-COLLECTOR HIGH-VOLTAGE OUTPUTS

schematic (each gate)

Resistor values shown are nominal.

absolute maximum ratings over operating free-air temperature range (unless otherwise noted)†

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54LS07</th>
<th>SN74LS07</th>
<th>SN74LS17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage, ( V_{CC} )</td>
<td>5.5 V</td>
<td>5.5 V</td>
<td>30 V</td>
</tr>
<tr>
<td>Input voltage, ( V_{i} ) (see Note 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output voltage, ( V_{O} ) (see Notes 1 and 2):</td>
<td>SN54LS07</td>
<td>SN74LS07</td>
<td></td>
</tr>
<tr>
<td>Operating free-air temperature range:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Storage temperature range</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† Stresses beyond those listed under “absolute maximum ratings” may cause permanent damage to the device. These are stress ratings only, and functional operation of the device at these or any other conditions beyond those indicated under “recommended operating conditions” is not implied. Exposure to absolute-maximum-rated conditions for extended periods may affect device reliability.

NOTES: 1. Voltage values are with respect to network ground terminal.
2. This is the maximum voltage that should be applied to any output when it is in the off state.

recommended operating conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54LS07</th>
<th>SN74LS07</th>
<th>SN74LS17</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{CC} ) Supply voltage</td>
<td>4.5 V</td>
<td>4.75 V</td>
<td>15 V</td>
</tr>
<tr>
<td>( V_{IH} ) High-level input voltage</td>
<td>5.5 V</td>
<td>5.25 V</td>
<td></td>
</tr>
<tr>
<td>( V_{IL} ) Low-level input voltage</td>
<td>2 V</td>
<td>2 V</td>
<td></td>
</tr>
<tr>
<td>( V_{OH} ) High-level output voltage</td>
<td></td>
<td>30 V</td>
<td>15 V</td>
</tr>
<tr>
<td>( I_{OL} ) Low-level output current</td>
<td></td>
<td></td>
<td>40 mA</td>
</tr>
<tr>
<td>( T_{A} ) Operating free-air temperature</td>
<td>-55°C</td>
<td>125°C</td>
<td>0°C to 70°C</td>
</tr>
</tbody>
</table>
### SN54LS07, SN74LS07, SN74LS17

**Hex Buffers/Drivers with Open-Collector High-Voltage Outputs**

#### Electrical Characteristics

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>TEST CONDITIONS†</th>
<th>SN54LS07</th>
<th></th>
<th>SN74LS07</th>
<th></th>
<th>SN74LS17</th>
<th></th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{IL} )</td>
<td>( V_{CC} = \text{MIN} ), ( V_{IH} = 2 \text{ V} )</td>
<td></td>
<td>0.25</td>
<td></td>
<td>0.25</td>
<td></td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>( V_{OL} )</td>
<td>( V_{CC} = \text{MIN} ), ( V_{IL} = 0.8 \text{ V} )</td>
<td></td>
<td>0.4</td>
<td></td>
<td>0.4</td>
<td></td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>( I_{L} )</td>
<td>( V_{CC} = \text{MAX} ), ( V_{I} = 7 \text{ V} )</td>
<td></td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>( I_{1} )</td>
<td>( V_{CC} = \text{MAX} ), ( V_{1} = 2.4 \text{ V} )</td>
<td></td>
<td>20</td>
<td></td>
<td>20</td>
<td></td>
<td>( \mu \text{A} )</td>
<td></td>
</tr>
<tr>
<td>( I_{1L} )</td>
<td>( V_{CC} = \text{MAX} ), ( V_{1} = 0.4 \text{ V} )</td>
<td></td>
<td>-0.2</td>
<td></td>
<td>-0.2</td>
<td></td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>( I_{CCH} )</td>
<td>( V_{CC} = \text{MAX} )</td>
<td></td>
<td>14</td>
<td></td>
<td>14</td>
<td></td>
<td>mA</td>
<td></td>
</tr>
<tr>
<td>( I_{CCCL} )</td>
<td>( V_{CC} = \text{MAX} )</td>
<td></td>
<td>45</td>
<td></td>
<td>45</td>
<td></td>
<td>mA</td>
<td></td>
</tr>
</tbody>
</table>

† For conditions shown as \( \text{MIN} \) or \( \text{MAX} \), use the appropriate value specified under recommended operating conditions.

‡ All typical values are at \( V_{CC} = 5 \text{ V} \), \( T_{A} = 25^\circ \text{C} \).

§ \( I_{OL} \) = 30 mA for SN54 series parts and 40 mA for SN74 series parts.

#### Switching Characteristics, \( V_{CC} = 5 \text{ V}, T_{A} = 25^\circ \text{C} \) (see Note 3)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>FROM (INPUT)</th>
<th>TO (OUTPUT)</th>
<th>TEST CONDITIONS</th>
<th>MIN</th>
<th>TYP</th>
<th>MAX</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{PLH} )</td>
<td>A</td>
<td>Y</td>
<td>( R_{L} = 110 \Omega ), ( C_{L} = 15 \text{ pF} )</td>
<td>6</td>
<td>10</td>
<td></td>
<td>ns</td>
</tr>
</tbody>
</table>

Note 3: Load circuit and voltage waveforms are shown in Section 1 of TTL Logic Data Book, 1988.
SN74LS08
Quad 2-Input AND Gate

GUARANTEED OPERATING RANGES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>Supply Voltage</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
<td>V</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Ambient Temperature Range</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>°C</td>
</tr>
<tr>
<td>IOH</td>
<td>Output Current – High</td>
<td>−0.4</td>
<td></td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>IOL</td>
<td>Output Current – Low</td>
<td>8.0</td>
<td></td>
<td></td>
<td>mA</td>
</tr>
</tbody>
</table>

ORDERING INFORMATION

<table>
<thead>
<tr>
<th>Device</th>
<th>Package</th>
<th>Shipping</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN74LS08N</td>
<td>14 Pin DIP</td>
<td>2000 Units/Box</td>
</tr>
<tr>
<td>SN74LS08D</td>
<td>14 Pin</td>
<td>2500/Tape &amp; Reel</td>
</tr>
</tbody>
</table>
### DC CHARACTERISTICS OVER OPERATING TEMPERATURE RANGE (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Unit</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
<td>Max</td>
</tr>
<tr>
<td></td>
<td>V_{IH}</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>V_{IL}</td>
<td>0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>V_{IK}</td>
<td>-0.65</td>
<td>-1.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>V_{CH}</td>
<td>2.7</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>V_{OL}</td>
<td>0.25</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_{IH}</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_{IL}</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_{OS}</td>
<td>-40</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I_{CC}</td>
<td>4.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note 1: Not more than one output should be shorted at a time, nor for more than 1 second.

### AC CHARACTERISTICS (T_{A} = 25°C)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Unit</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
<td>Max</td>
</tr>
<tr>
<td>I_{PLH}</td>
<td>Turn–Off Delay, Input to Output</td>
<td>8.0</td>
<td>15</td>
<td>ns</td>
</tr>
<tr>
<td>I_{PHL}</td>
<td>Turn–On Delay, Input to Output</td>
<td>10</td>
<td>20</td>
<td>ns</td>
</tr>
</tbody>
</table>
SN74LS32

Quad 2-Input OR Gate

GUARANTEED OPERATING RANGES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>Supply Voltage</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
<td>V</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Operating Ambient Temperature Range</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>°C</td>
</tr>
<tr>
<td>$I_{OH}$</td>
<td>Output Current – High</td>
<td></td>
<td></td>
<td>$-0.4$</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{OL}$</td>
<td>Output Current – Low</td>
<td></td>
<td></td>
<td>8.0</td>
<td>mA</td>
</tr>
</tbody>
</table>

ORDERING INFORMATION

<table>
<thead>
<tr>
<th>Device</th>
<th>Package</th>
<th>Shipping</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN74LS32N</td>
<td>14 Pin DIP</td>
<td>2000 Units/Box</td>
</tr>
<tr>
<td>SN74LS32D</td>
<td>14 Pin</td>
<td>2500/Tape &amp; Reel</td>
</tr>
</tbody>
</table>
### DC CHARACTERISTICS OVER OPERATING TEMPERATURE RANGE (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Unit</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
<td>Max</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_{IH})</td>
<td>Input HIGH Voltage</td>
<td>2.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_{IL})</td>
<td>Input LOW Voltage</td>
<td>0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V_{IK})</td>
<td>Input Clamp Diode Voltage</td>
<td>-0.65</td>
<td>-1.5</td>
<td>V</td>
</tr>
<tr>
<td>(V_{OH})</td>
<td>Output HIGH Voltage</td>
<td>2.7</td>
<td>3.5</td>
<td>V</td>
</tr>
<tr>
<td>(V_{OL})</td>
<td>Output LOW Voltage</td>
<td>0.25</td>
<td>0.4</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.35</td>
<td>0.5</td>
<td>V</td>
</tr>
<tr>
<td>(I_{H})</td>
<td>Input HIGH Current</td>
<td>20</td>
<td></td>
<td>(\mu\text{A})</td>
</tr>
<tr>
<td>(I_{L})</td>
<td>Input LOW Current</td>
<td>0.1</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>(I_{OS})</td>
<td>Short Circuit Current (Note 1)</td>
<td>-20</td>
<td>-100</td>
<td>mA</td>
</tr>
<tr>
<td>(I_{CC})</td>
<td>Power Supply Current Total, Output HIGH</td>
<td>6.2</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td></td>
<td>Total, Output LOW</td>
<td>9.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note 1:** Not more than one output should be shorted at a time, nor for more than 1 second.

### AC CHARACTERISTICS (\(T_{A} = 25^\circ\text{C}\))

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Unit</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Min</td>
<td>Typ</td>
<td>Max</td>
</tr>
<tr>
<td>(t_{PLH})</td>
<td>Turn-Off Delay, Input to Output</td>
<td>14</td>
<td>22</td>
<td>ns</td>
</tr>
<tr>
<td>(t_{PHL})</td>
<td>Turn-On Delay, Input to Output</td>
<td>14</td>
<td>22</td>
<td>ns</td>
</tr>
</tbody>
</table>
SN74LS86

Quad 2-Input Exclusive OR Gate

TRUTH TABLE

<table>
<thead>
<tr>
<th>IN</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

GUARANTEED OPERATING RANGES

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>Supply Voltage</td>
<td>4.75</td>
<td>5.0</td>
<td>5.25</td>
<td>V</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Operating Ambient Temperature Range</td>
<td>0</td>
<td>25</td>
<td>70</td>
<td>°C</td>
</tr>
<tr>
<td>$I_{OH}$</td>
<td>Output Current – High</td>
<td>-0.4</td>
<td>mA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{OL}$</td>
<td>Output Current – Low</td>
<td>8.0</td>
<td>mA</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ORDERING INFORMATION

<table>
<thead>
<tr>
<th>Device</th>
<th>Package</th>
<th>Shipping</th>
</tr>
</thead>
<tbody>
<tr>
<td>SN74LS86N</td>
<td>14 Pin DIP</td>
<td>2000 Units/Box</td>
</tr>
<tr>
<td>SN74LS86D</td>
<td>14 Pin</td>
<td>2500/Tape &amp; Reel</td>
</tr>
</tbody>
</table>
### SN74LS86

#### DC CHARACTERISTICS OVER OPERATING TEMPERATURE RANGE (unless otherwise specified)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{IH}$</td>
<td>Input HIGH Voltage</td>
<td>2.0</td>
<td>Guaranteed Input HIGH Voltage for All Inputs</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Input LOW Voltage</td>
<td>0.8</td>
<td>Guaranteed Input LOW Voltage for All Inputs</td>
</tr>
<tr>
<td>$V_{IN}$</td>
<td>Input Clamp Diode Voltage</td>
<td>-0.65</td>
<td>$V_{CC} = MIN, I_{IN} = -18 mA$</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>Output HIGH Voltage</td>
<td>2.7</td>
<td>$V_{CC} = MIN, I_{OH} = MAX, V_{IN} = V_{IH}$ or $V_{IL}$ per Truth Table</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Output LOW Voltage</td>
<td>0.25</td>
<td>$I_{OL} = 4.0 mA$</td>
</tr>
<tr>
<td>$I_{IH}$</td>
<td>Input HIGH Current</td>
<td>0.35</td>
<td>$V_{CC} = V_{CC} MIN, V_{IN} = V_{IL}$ or $V_{IH}$ per Truth Table</td>
</tr>
<tr>
<td>$I_{IL}$</td>
<td>Input LOW Current</td>
<td>0.4</td>
<td>$V_{CC} = MAX, V_{IN} = 2.7 V$</td>
</tr>
<tr>
<td>$I_{OS}$</td>
<td>Short Circuit Current (Note 1)</td>
<td>0.2</td>
<td>$V_{CC} = MAX, V_{IN} = 7.0 V$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Power Supply Current</td>
<td>0.35</td>
<td>$V_{CC} = MAX, V_{IN} = 0.4 V$</td>
</tr>
</tbody>
</table>

Note 1: Not more than one output should be shorted at a time, nor for more than 1 second.

#### AC CHARACTERISTICS ($T_A = 25^\circ C$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter, Other Input LOW</th>
<th>Limits</th>
<th>Test Conditions</th>
</tr>
</thead>
</table>
| $\tau_{PLH}$ | Propagation Delay | 12     | 10 23 ns
| $\tau_{PHL}$ | Other Input LOW          | 10     | 17 17 ns
| $\tau_{PLH}$ | Propagation Delay        | 20     | 20 30 ns
| $\tau_{PHL}$ | Other Input HIGH         | 13     | 13 22 ns

$V_{CC} = 5.0 \text{ V}$

$C_L = 15 \text{ pF}$
Package Options Include Plastic Small-Outline Packages, Ceramic Chip Carriers, and Standard Plastic and Ceramic 300-mil DIPs

**description**

These devices contain four independent 2-input NAND gates. They perform the Boolean functions $Y = \overline{A} \cdot \overline{B}$ or $Y = \overline{A} + \overline{B}$ in positive logic.

The SN54F00 is characterized for operation over the full military temperature range of $-55^\circ C$ to $125^\circ C$. The SN74F00 is characterized for operation from $0^\circ C$ to $70^\circ C$.

**FUNCTION TABLE**

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>L</td>
<td>X</td>
</tr>
<tr>
<td>X</td>
<td>L</td>
</tr>
</tbody>
</table>

**logic symbol†**

†This symbol is in accordance with ANSI/IEEE Std 91-1984 and IEC Publication 617-12.

**logic diagram (positive logic)**

Pin numbers shown are for the D, J, and N packages.
## SN54F00, SN74F00
### Quadruple 2-Input Positive-NAND Gates

**SDS3035A – March 1987 – Revised October 1993**

### Absolute Maximum Ratings over Operating Free-Air Temperature Range (unless otherwise noted)†

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54F00</th>
<th>SN74F00</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage range, $V_{CC}$</td>
<td>-0.5 V to 7 V</td>
<td>-0.5 V to 7 V</td>
<td>V</td>
</tr>
<tr>
<td>Input voltage range, $V_I$ (see Note 1)</td>
<td>-1.2 V to 7 V</td>
<td>-1.2 V to 7 V</td>
<td>V</td>
</tr>
<tr>
<td>Input current range</td>
<td>-30 mA to 5 mA</td>
<td>-30 mA to 5 mA</td>
<td>mA</td>
</tr>
<tr>
<td>Voltage range applied to any output in the high state</td>
<td>-0.5 V to $V_{CC}$</td>
<td>-0.5 V to $V_{CC}$</td>
<td>V</td>
</tr>
<tr>
<td>Current into any output in the low state</td>
<td>40 mA</td>
<td>40 mA</td>
<td>mA</td>
</tr>
<tr>
<td>Operating free-air temperature range: SN54F00</td>
<td>-55°C to 125°C</td>
<td>-55°C to 125°C</td>
<td>°C</td>
</tr>
<tr>
<td>SN74F00</td>
<td>0°C to 70°C</td>
<td>0°C to 70°C</td>
<td>°C</td>
</tr>
<tr>
<td>Storage temperature range</td>
<td>-65°C to 150°C</td>
<td>-65°C to 150°C</td>
<td>°C</td>
</tr>
</tbody>
</table>

† Stresses beyond those listed under “absolute maximum ratings” may cause permanent damage to the device. These are stress ratings only and functional operation of the device at these or any other conditions beyond those indicated under “recommended operating conditions” is not implied. Exposure to absolute-maximum-rated conditions for extended periods may affect device reliability.

**NOTE 1:** The input voltage ratings may be exceeded provided the input current ratings are observed.

### Recommended Operating Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54F00</th>
<th>SN74F00</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>MIN</td>
<td>NOM</td>
<td>MAX</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>4.5</td>
<td>5</td>
<td>5.5</td>
</tr>
<tr>
<td>$V_IH$</td>
<td>High-level input voltage</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Low-level input voltage</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$I_{IK}$</td>
<td>Input clamp current</td>
<td>-18</td>
<td>-18</td>
</tr>
<tr>
<td>$I_{OH}$</td>
<td>High-level output current</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$I_{OL}$</td>
<td>Low-level output current</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Operating free-air temperature</td>
<td>-55</td>
<td>125</td>
</tr>
</tbody>
</table>

### Electrical Characteristics over Recommended Operating Free-Air Temperature Range (unless otherwise noted)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test Conditions</th>
<th>SN54F00</th>
<th>SN74F00</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{IK}$</td>
<td>$V_{CC} = 4.5$ V, $I_I = -18$ mA</td>
<td>-1.2</td>
<td>-1.2</td>
<td>V</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>$V_{CC} = 4.5$ V, $I_{OH} = -1$ mA</td>
<td>2.5</td>
<td>3.4</td>
<td>V</td>
</tr>
<tr>
<td>$I_{OL}$</td>
<td>$V_{CC} = 4.5$ V, $I_{OL} = 20$ mA</td>
<td>0.3</td>
<td>0.5</td>
<td>mA</td>
</tr>
<tr>
<td>$I_I$</td>
<td>$V_{CC} = 5.5$ V, $V_I = 7$ V</td>
<td>0.1</td>
<td>0.1</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{IH}$</td>
<td>$V_{CC} = 5.5$ V, $V_I = 2.7$ V</td>
<td>20</td>
<td>20</td>
<td>µA</td>
</tr>
<tr>
<td>$I_{II}$</td>
<td>$V_{CC} = 5.5$ V, $V_I = 0.5$ V</td>
<td>-0.6</td>
<td>-0.6</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{OS}$</td>
<td>$V_{CC} = 5.5$ V, $V_O = 0$</td>
<td>-60</td>
<td>-150</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{CH}$</td>
<td>$V_{CC} = 5.5$ V, $V_I = 0$</td>
<td>1.9</td>
<td>2.8</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{CL}$</td>
<td>$V_{CC} = 5.5$ V, $V_I = 4.5$ V</td>
<td>6.8</td>
<td>10.2</td>
<td>mA</td>
</tr>
</tbody>
</table>

† All typical values are at $V_{CC} = 5$ V, $T_A = 25°C$.

§ Not more than one output should be shorted at a time, and the duration of the short circuit should not exceed one second.
## SN54F00, SN74F00
### QUADRUPLE 2-INPUT POSITIVE-NAND GATES

**switching characteristics (see Note 2)**

| PARAMETER | FROM (INPUT) | TO (OUTPUT) | \( V_{CC} = 5\, V, \)  
|           |             |             | \( C_L = 50\, pF, \)  
|           |             |             | \( R_L = 500\, \Omega, \)  
|           |             |             | \( T_A = 25^\circ C \)  
|           |             |             | \( V_{CC} = 4.5\, V \)  
|           |             |             | to \( 5.5\, V, \)  
|           |             |             | \( C_L = 50\, pF, \)  
|           |             |             | \( R_L = 500\, \Omega, \)  
|           |             |             | \( T_A = \text{MIN to MAX}\)  
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>MIN</th>
<th>TYP</th>
<th>MAX</th>
<th>MIN</th>
<th>MAX</th>
<th>MIN</th>
<th>MAX</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>t( \text{PLH} )</td>
<td>1.6</td>
<td>3.3</td>
<td>5</td>
<td>2</td>
<td>7</td>
<td>1.6</td>
<td>6</td>
<td>( \text{ns} )</td>
</tr>
<tr>
<td>t( \text{PHL} )</td>
<td>1</td>
<td>2.8</td>
<td>4.3</td>
<td>1.5</td>
<td>6.5</td>
<td>1</td>
<td>5.3</td>
<td>( \text{ns} )</td>
</tr>
</tbody>
</table>

\( t \) for conditions shown as \( \text{MIN} \) or \( \text{MAX} \), use the appropriate value specified under recommended operating conditions.

**NOTE 2:** Load circuits and waveforms are shown in Section 1.
SN54ALS00A, SN54AS00, SN74ALS00A, SN74AS00 QUADRUPLE 2-INPUT POSITIVE-NAND GATES

Package Options Include Plastic Small-Outline (D) Packages, Ceramic Chip Carriers (FK), and Standard Plastic (N) and Ceramic (J) 300-mil DIPs

description

These devices contain four independent 2-input positive-NAND gates. They perform the Boolean functions \( Y = \overline{A} \cdot \overline{B} \) or \( Y = \overline{A} + \overline{B} \) in positive logic. The SN54ALS00A and SN54AS00 are characterized for operation over the full military temperature range of \(-55^\circ C \) to \(125^\circ C\). The SN74ALS00A and SN74AS00 are characterized for operation from \(0^\circ C \) to \(70^\circ C\).

FUNCTION TABLE (each gate)

<table>
<thead>
<tr>
<th>INPUTS</th>
<th>OUTPUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \overline{H} )</td>
<td>( \overline{H} )</td>
</tr>
<tr>
<td>( \overline{L} )</td>
<td>( \overline{X} )</td>
</tr>
<tr>
<td>( X )</td>
<td>( \overline{L} )</td>
</tr>
</tbody>
</table>

logic symbol†

† This symbol is in accordance with ANSI/IEEE Std 91-1984 and IEC Publication 617-12.
Pin numbers shown are for the D, J, and N packages.
SN54ALS00A, SN54AS00, SN74ALS00A, SN74AS00
QUADRUPLE 2-INPUT POSITIVE-NAND GATES

logic diagram (positive logic)

Pin numbers shown are for the D, J, and N packages.

absolute maximum ratings over operating free-air temperature range (unless otherwise noted)†

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54ALS00A</th>
<th>SN74ALS00A</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage, ( V_{CC} )</td>
<td>4.5 5 5.5</td>
<td>4.5 5 5.5</td>
<td>V</td>
</tr>
<tr>
<td>Input voltage, ( V_I )</td>
<td>2</td>
<td>2</td>
<td>V</td>
</tr>
<tr>
<td>Operating free-air temperature range, ( T_A ):</td>
<td></td>
<td>55°C to 125°C</td>
<td></td>
</tr>
<tr>
<td>SN54ALS00A</td>
<td></td>
<td>0°C to 70°C</td>
<td></td>
</tr>
<tr>
<td>SN74ALS00A</td>
<td></td>
<td>−65°C to 150°C</td>
<td></td>
</tr>
<tr>
<td>Storage temperature range</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

† Stresses beyond those listed under “absolute maximum ratings” may cause permanent damage to the device. These are stress ratings only, and functional operation of the device at these or any other conditions beyond those indicated under “recommended operating conditions” is not implied. Exposure to absolute-maximum-rated conditions for extended periods may affect device reliability.

recommended operating conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54ALS00A</th>
<th>SN74ALS00A</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{CC} ) Supply voltage</td>
<td>4.5 5 5.5</td>
<td>4.5 5 5.5</td>
<td>V</td>
</tr>
<tr>
<td>( V_{IH} ) High-level input voltage</td>
<td>2</td>
<td>2</td>
<td>V</td>
</tr>
<tr>
<td>( V_{IL} ) Low-level input voltage</td>
<td>0.8†</td>
<td>0.8</td>
<td>V</td>
</tr>
<tr>
<td>( I_{OH} ) High-level output current</td>
<td>−0.4</td>
<td>−0.4</td>
<td>mA</td>
</tr>
<tr>
<td>( I_{OL} ) Low-level output current</td>
<td>4</td>
<td>8</td>
<td>mA</td>
</tr>
<tr>
<td>( T_A ) Operating free-air temperature</td>
<td>−55 125</td>
<td>0 70</td>
<td>°C</td>
</tr>
</tbody>
</table>

† Applies over temperature range −55°C to 70°C
§ Applies over temperature range 70°C to 125°C
## APPENDIX C • Manufacturers’ Data Sheets

### SN54ALS00A, SN54AS00, SN74ALS00A, SN74AS00

#### Quadruple 2-Input Positive-NAND Gates

**SDAS187A – APRIL 1982 – REVISED DECEMBER 1994**

### Electrical Characteristics Over Recommended Operating Free-Air Temperature Range (Unless Otherwise Noted)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>TEST CONDITIONS</th>
<th>SN54ALS00A</th>
<th>SN74ALS00A</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MIN</td>
<td>TYP†</td>
<td>MAX</td>
</tr>
<tr>
<td>( V_{IK} )</td>
<td>( V_{CC} = 4.5 \text{ V} ), ( I_L = -18 \text{ mA} )</td>
<td>-1.2</td>
<td>-1.2</td>
<td></td>
</tr>
<tr>
<td>( V_{OH} )</td>
<td>( V_{CC} = 4.5 \text{ V} ) to ( 5.5 \text{ V} ), ( I_{OH} = -0.4 \text{ mA} )</td>
<td>( V_{CC} )</td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>( V_{OL} )</td>
<td>( V_{CC} = 4.5 \text{ V} ), ( I_{OL} = 4 \text{ mA} )</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>( I_L )</td>
<td>( V_{CC} = 5.5 \text{ V} ), ( V_I = 7 \text{ V} )</td>
<td>0.1</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>( I_{IH} )</td>
<td>( V_{CC} = 5.5 \text{ V} ), ( V_I = 2.7 \text{ V} )</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>( I_{IL} )</td>
<td>( V_{CC} = 5.5 \text{ V} ), ( V_I = 0.4 \text{ V} )</td>
<td>-0.1</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>( I_{O} )</td>
<td>( V_{CC} = 5.5 \text{ V} ), ( V_O = 2.25 \text{ V} )</td>
<td>-20</td>
<td>-112</td>
<td>-30</td>
</tr>
<tr>
<td>( I_{CC} )</td>
<td>( V_{CC} = 5.5 \text{ V} ), ( V_I = 0 )</td>
<td>0.5</td>
<td>0.85</td>
<td>0.5</td>
</tr>
<tr>
<td>( I_{CL} )</td>
<td>( V_{CC} = 5.5 \text{ V} ), ( V_I = 4.5 \text{ V} )</td>
<td>1.5</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

† All typical values are at \( V_{CC} = 5 \text{ V}, T_A = 25^\circ \text{C} \).

‡ The output conditions have been chosen to produce a current that closely approximates one half of the true short-circuit output current, \( I_{OS} \).

### Switching Characteristics (See Figure 1)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>FROM (INPUT)</th>
<th>TO (OUTPUT)</th>
<th>SN54ALS00A</th>
<th>SN74ALS00A</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>MIN</td>
<td>MAX</td>
<td>MIN</td>
</tr>
<tr>
<td>( t_{PHL} )</td>
<td>A or B</td>
<td>Y</td>
<td>3</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>( t_{PHL} )</td>
<td></td>
<td></td>
<td>2</td>
<td>9</td>
<td>2</td>
</tr>
</tbody>
</table>

§ For conditions shown as MIN or MAX, use the appropriate value specified under recommended operating conditions.
# SN54ALS00A, SN54AS00, SN74ALS00A, SN74AS00 QUADRUPLE 2-INPUT POSITIVE-NAND GATES

## APPENDIX C • Manufacturers' Data Sheets

### absolute maximum ratings over operating free-air temperature range (unless otherwise noted)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54AS00</th>
<th>SN74AS00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supply voltage, $V_{CC}$</td>
<td>MIN</td>
<td>NOM</td>
</tr>
<tr>
<td></td>
<td>4.5</td>
<td>5</td>
</tr>
<tr>
<td>Input voltage, $V_i$</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Operating free-air temperature range, $T_A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SN54AS00</td>
<td>-55°C to 125°C</td>
<td></td>
</tr>
<tr>
<td>SN74AS00</td>
<td>0°C to 70°C</td>
<td></td>
</tr>
</tbody>
</table>

| Storage temperature range  | -65°C to 150°C |

† Stresses beyond those listed under "absolute maximum ratings" may cause permanent damage to the device. These are stress ratings only, and functional operation of the device at these or any other conditions beyond those indicated under "recommended operating conditions" is not implied. Exposure to absolute-maximum-rated conditions for extended periods may affect device reliability.

### recommended operating conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SN54AS00</th>
<th>SN74AS00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>MIN</td>
<td>NOM</td>
</tr>
<tr>
<td>Supply voltage</td>
<td>4.5</td>
<td>5</td>
</tr>
<tr>
<td>$V_{IH}$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>$I_{OH}$</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>$I_{OL}$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$T_A$</td>
<td>-55</td>
<td>125</td>
</tr>
</tbody>
</table>

### electrical characteristics over recommended operating free-air temperature range (unless otherwise noted)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test Conditions</th>
<th>SN54AS00</th>
<th>SN74AS00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{IK}$</td>
<td>$V_{CC} = 4.5, V$, $I_{I} = -18, mA$</td>
<td>-1.2</td>
<td>-1.2</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>$V_{CC} = 4.5, V$ to 5.5 V, $I_{OH} = -2, mA$</td>
<td>$V_{CC} = 2$</td>
<td>$V_{CC} = 2$</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>$V_{CC} = 4.5, V$</td>
<td>$0.35$</td>
<td>$0.35$</td>
</tr>
<tr>
<td>$I_{I}$</td>
<td>$V_{CC} = 5.5, V$, $V_i = 7, V$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$I_{H}$</td>
<td>$V_{CC} = 5.5, V$, $V_i = 2.7, V$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$I_{L}$</td>
<td>$V_{CC} = 5.5, V$, $V_i = 0.4, V$</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>$I_{O}$</td>
<td>$V_{CC} = 5.5, V$, $V_o = 2.25, V$</td>
<td>-30</td>
<td>-30</td>
</tr>
<tr>
<td>$I_{CCH}$</td>
<td>$V_{CC} = 5.5, V$, $V_i = 0$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$I_{CL}$</td>
<td>$V_{CC} = 5.5, V$, $V_i = 4.5, V$</td>
<td>10.8</td>
<td>17.4</td>
</tr>
</tbody>
</table>

† All typical values are at $V_{CC} = 5\, V$, $T_A = 25°C$.

‡ The output conditions have been chosen to produce a current that closely approximates one half of the true short-circuit output current, $I_{OS}$.

### switching characteristics (see Figure 1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FROM (INPUT)</th>
<th>TO (OUTPUT)</th>
<th>SN54AS00</th>
<th>SN74AS00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{PLH}$</td>
<td>A or B</td>
<td>Y</td>
<td>MIN</td>
<td>MAX</td>
</tr>
<tr>
<td>$t_{PHL}$</td>
<td></td>
<td></td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

† For conditions shown as MIN or MAX, use the appropriate value specified under recommended operating conditions.
SN54ALS00A, SN54AS00, SN74ALS00A, SN74AS00
QUADRUPLE 2-INPUT POSITIVE-NAND GATES

PARAMETER MEASUREMENT INFORMATION
SERIES 54ALS/74ALS AND 54AS/74AS DEVICES

LOAD CIRCUIT FOR
BI-STATE
TOTEM-POLE OUTPUTS

LOAD CIRCUIT
FOR OPEN-COLLECTOR OUTPUTS

LOAD CIRCUIT
FOR 3-STATE OUTPUTS

VOLTAGE WAVEFORMS
SETUP AND HOLD TIMES

VOLTAGE WAVEFORMS
PULSE DURATIONS

VOLTAGE WAVEFORMS
ENABLE AND DISABLE TIMES, 3-STATE OUTPUTS

VOLTAGE WAVEFORMS
PROPAGATION DELAY TIMES

NOTES:
A. C_L includes probe and jig capacitance.
B. Waveform 1 is for an output with internal conditions such that the output is low except when disabled by the output control.
   Waveform 2 is for an output with internal conditions such that the output is high except when disabled by the output control.
C. When measuring propagation delay items of 3-state outputs, switch S1 is open.
D. All input pulses have the following characteristics: PRR ≤ 1 MHz, t_f = t_r = 2 ns, duty cycle = 50%.
E. The outputs are measured one at a time with one transition per measurement.

Figure 1. Load Circuits and Voltage Waveforms
Quad 2-Input NAND Gate
High-Performance Silicon-Gate CMOS

The MC54/74HC00A is identical in pinout to the LS00. The device inputs are compatible with Standard CMOS outputs; with pullup resistors, they are compatible with LSTTL outputs.

- Output Drive Capability: 10 LSTTL Loads
- Outputs Directly Interface to CMOS, NMOS and TTL
- Operating Voltage Range: 2 to 6V
- Low Input Current: 1μA
- High Noise Immunity Characteristic of CMOS Devices
- In Compliance With the JEDEC Standard No. 7A Requirements
- Chip Complexity: 32 FETs or 8 Equivalent Gates

LOGIC DIAGRAM

<table>
<thead>
<tr>
<th>A1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>A2</td>
<td>4</td>
</tr>
<tr>
<td>B2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>A3</td>
<td>9</td>
</tr>
<tr>
<td>B3</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>12</td>
</tr>
<tr>
<td>B4</td>
<td>13</td>
</tr>
</tbody>
</table>

PIN 14 = VCC
PIN 7 = GND

ORDERING INFORMATION
- MC54HC00AJ Ceramic
- MC74HC00AN Plastic
- MC74HC00AD SOIC
- MC74HC00ADT TSSOP

FUNCTION TABLE

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

Pinout: 14-Lead Packages (Top View)
## MC54/74HC00A

### Maximum Ratings *

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>$-0.5 \text{ to } +7.0$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{IN}$</td>
<td>DC Input Voltage (Referenced to GND)</td>
<td>$-0.5 \text{ to } V_{CC} + 0.5$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{OUT}$</td>
<td>DC Output Voltage (Referenced to GND)</td>
<td>$-0.5 \text{ to } V_{CC} + 0.5$</td>
<td>V</td>
</tr>
<tr>
<td>$I_{IN}$</td>
<td>DC Input Current, per Pin</td>
<td>$\pm 20$</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{OUT}$</td>
<td>DC Output Current, per Pin</td>
<td>$\pm 25$</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>DC Supply Current, $V_{CC}$ and GND Pins</td>
<td>$\pm 50$</td>
<td>mA</td>
</tr>
<tr>
<td>$P_D$</td>
<td>Power Dissipation in Still Air, Plastic or Ceramic DIP†, SOIC Package†, TSSOP Package†</td>
<td>750</td>
<td>mW</td>
</tr>
<tr>
<td>$T_{STG}$</td>
<td>Storage Temperature</td>
<td>$-65 \text{ to } +150$</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{L}$</td>
<td>Lead Temperature, 1 mm from Case for 10 Seconds Plastic DIP, SOIC or TSSOP Package Ceramic DIP</td>
<td>260, 300</td>
<td>°C</td>
</tr>
</tbody>
</table>

* Maximum Ratings are those values beyond which damage to the device may occur. Functional operation should be restricted to the Recommended Operating Conditions.

† Derating — Plastic DIP: $-10 \text{ mW/°C}$ from $65^\circ$ to $125^\circ$C
Ceramic DIP: $-10 \text{ mW/°C}$ from $100^\circ$ to $125^\circ$C
SOIC Package: $-7 \text{ mW/°C}$ from $65^\circ$ to $125^\circ$C
TSSOP Package: $-6.1 \text{ mW/°C}$ from $65^\circ$ to $125^\circ$C

For high frequency or heavy load considerations, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DL129/D).

### Recommended Operating Conditions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>2.0</td>
<td>6.0</td>
<td>V</td>
</tr>
<tr>
<td>$V_{IN}$, $V_{OUT}$</td>
<td>DC Input Voltage, Output Voltage (Referenced to GND)</td>
<td>0</td>
<td>$V_{CC}$</td>
<td>V</td>
</tr>
<tr>
<td>$T_A$</td>
<td>Operating Temperature, All Package Types</td>
<td>$-55 \text{ to } +125$</td>
<td>°C</td>
<td></td>
</tr>
<tr>
<td>$I_T$, $I_{TR}$</td>
<td>Input Rise and Fall Time (Figure 1)</td>
<td>$V_{CC} = 2.0$ V, 0</td>
<td>1000</td>
<td>ns</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{CC} = 4.5$ V, 0</td>
<td>500</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_{CC} = 6.0$ V, 0</td>
<td>400</td>
<td></td>
</tr>
</tbody>
</table>
### DC CHARACTERISTICS (Voltages Referenced to GND)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Condition</th>
<th>( V_{CC} ) V</th>
<th>Guarantee Limit</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{IH} )</td>
<td>Minimum High-Level Input Voltage</td>
<td>( V_{OUT} = 0.1 \text{V} \text{ or } V_{CC} = -0.1 \text{V} ) (</td>
<td>I_{OUT}</td>
<td>\leq 20\mu A )</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.0</td>
<td>2.10</td>
<td>2.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.5</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.0</td>
<td>4.20</td>
<td>4.20</td>
</tr>
<tr>
<td>( V_{IL} )</td>
<td>Maximum Low-Level Input Voltage</td>
<td>( V_{OUT} = 0.1 \text{V} \text{ or } V_{CC} = -0.1 \text{V} ) (</td>
<td>I_{OUT}</td>
<td>\leq 20\mu A )</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>3.0</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.5</td>
<td>1.35</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.0</td>
<td>1.80</td>
<td>1.80</td>
</tr>
<tr>
<td>( V_{OH} )</td>
<td>Minimum High-Level Output Voltage</td>
<td>( V_{IN} = V_{IH} \text{ or } V_{IL} ) (</td>
<td>I_{OUT}</td>
<td>\leq 2.4 \text{mA} )</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.5</td>
<td>4.4</td>
<td>4.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.0</td>
<td>5.9</td>
<td>5.9</td>
</tr>
<tr>
<td>( V_{OL} )</td>
<td>Maximum Low-Level Output Voltage</td>
<td>( V_{IN} = V_{IH} \text{ or } V_{IL} ) (</td>
<td>I_{OUT}</td>
<td>\leq 5.2 \text{mA} )</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4.5</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>6.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( I_{in} )</td>
<td>Maximum Input Leakage Current</td>
<td>( V_{IN} = V_{CC} \text{ or } GND )</td>
<td>6.0</td>
<td>±0.1</td>
<td>±1.0</td>
</tr>
<tr>
<td>( I_{CC} )</td>
<td>Maximum Quiescent Supply Current (per Package)</td>
<td>( V_{IN} = V_{CC} \text{ or } GND ) (</td>
<td>I_{OUT}</td>
<td>= 0\mu A )</td>
<td>6.0</td>
</tr>
</tbody>
</table>

NOTE: Information on typical parametric values can be found in Chapter 2 of the Motorola High-Speed CMOS Data Book (DL129/D).

### AC CHARACTERISTICS \( (C_{L} = 50 \text{ pF}, \text{Input } I_{f} = I_{r} = 6 \text{ ns}) \)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>( V_{CC} ) V</th>
<th>Guarantee Limit</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{PLH}, t_{PHL} )</td>
<td>Maximum Propagation Delay, Input A or B to Output Y (Figures 1 and 2)</td>
<td>2.0</td>
<td>75</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.0</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.5</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.0</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>( t_{THL}, t_{TTL} )</td>
<td>Maximum Output Transition Time, Any Output (Figures 1 and 2)</td>
<td>2.0</td>
<td>75</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.0</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.5</td>
<td>15</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.0</td>
<td>13</td>
<td>16</td>
</tr>
<tr>
<td>( C_{in} )</td>
<td>Maximum Input Capacitance</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

NOTE: For propagation delays with loads other than 50 pF, and information on typical parametric values, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DL129/D).

### Power Dissipation Capacitance (Per Buffer)*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>( C_{PD} )</th>
<th>Typical @ 25°C, ( V_{CC} = 5.0 \text{V}, V_{EE} = 0 \text{V} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22</td>
<td>pF</td>
</tr>
</tbody>
</table>

* Used to determine the no–load dynamic power consumption: \( P_{D} = C_{PD} V_{CC}^{2} + I_{CC} V_{CC} \). For load considerations, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DL129/D).
Figure 1. Switching Waveforms

Figure 2. Test Circuit

*Includes all probe and jig capacitance

Figure 3. Expanded Logic Diagram
(1/4 of the Device)
Quad 2-Input NAND Gate with LSTTL-Compatible Inputs
High-Performance Silicon-Gate CMOS

The MC54/74HCT00A may be used as a level converter for interfacing TTL or NMOS outputs to high-speed CMOS inputs.
- Output Drive Capability: 12 LSTTL Loads
- TTL/NMOS Compatibility: Input Lows
- Outputs Directly Interface to CMOS, NMOS, and TTL
- Operating Voltage Range: 4.5 to 5.5 V
- Low Input Current: 1.0 µA
- In Compliance with the Requirements Defined by JEDEC Standard No. 7A
- Chip Complexity: 48 PE's or 12 Equivalent Gates

LOGIC DIAGRAM

PIN ASSIGNMENT

FUNCTION TABLE
# APPENDIX C • Manufacturers' Data Sheets

## MC54/74HCT00A

### MAXIMUM RATINGS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>-0.5 to +7.0</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage (Referenced to GND)</td>
<td>-0.5 to VCC + 0.5</td>
<td>V</td>
</tr>
<tr>
<td>VOLT</td>
<td>DC Output Voltage (Referenced to GND)</td>
<td>-0.5 to VCC - 1.0</td>
<td>V</td>
</tr>
<tr>
<td>IDIL</td>
<td>DC Input Current per Pin</td>
<td>&lt; 20</td>
<td>mA</td>
</tr>
<tr>
<td>IDOL</td>
<td>DC Output Current per Pin</td>
<td>&lt; 25</td>
<td>mA</td>
</tr>
<tr>
<td>ISC</td>
<td>DC Supply Current, VCC, and GND Pins</td>
<td>&lt; 50</td>
<td>mA</td>
</tr>
<tr>
<td>Pd</td>
<td>Power Dissipation in Single Plastic or Ceramic DIP</td>
<td>500</td>
<td>mW</td>
</tr>
<tr>
<td>Tstg</td>
<td>Storage Temperature</td>
<td>-65 to +150</td>
<td>°C</td>
</tr>
<tr>
<td>TIL</td>
<td>Lead Temperature, 1 mm from Case for 10 Second Soldering Time</td>
<td>260</td>
<td>°C</td>
</tr>
</tbody>
</table>

*Maximum Ratings are those values beyond which damage to the device may occur.*

**Caution:** Plastic DIP - 10 mW/°C from 65° to 125°C
Ceramic DIP - 10 mW/°C from 100° to 125°C
SOJ Package - 7 mW/°C from 65° to 125°C

For high-frequency or heavy-load considerations, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DL-1390).

### RECOMMENDED OPERATING CONDITIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>2.0</td>
<td>5.5</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage (Referenced to GND)</td>
<td>0</td>
<td>VCC</td>
<td>V</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Temperature All Package Types</td>
<td>35°</td>
<td>125°</td>
<td>°C</td>
</tr>
<tr>
<td>tIL</td>
<td>Input Rise and Fall Time (Figure 1)</td>
<td>0</td>
<td>500</td>
<td>ns</td>
</tr>
</tbody>
</table>

### DC CHARACTERISTICS FOR THE MC54/74HCT00A (Voltages Referenced to GND)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>VCC V</th>
<th>Guaranteed Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>IIL</td>
<td>Minimum Low-Level Input Voltage</td>
<td>VIL &gt; 0.4 V &lt; 0.8 V</td>
<td>15</td>
<td>4.0</td>
</tr>
<tr>
<td>IIL</td>
<td>Maximum Low-Level Input Voltage</td>
<td>VIL &gt; 0.4 V &lt; 0.8 V</td>
<td>40</td>
<td>2.4</td>
</tr>
<tr>
<td>VOH</td>
<td>Minimum High-Level Output Voltage</td>
<td>VIL &gt; 0.4 V &lt; 0.8 V</td>
<td>5.5</td>
<td>0.10</td>
</tr>
<tr>
<td>VOL</td>
<td>Maximum Low-Level Output Voltage</td>
<td>VIL &gt; 0.4 V &lt; 0.8 V</td>
<td>5.5</td>
<td>0.28</td>
</tr>
<tr>
<td>IDIL</td>
<td>Maximum Input Leakage Current</td>
<td>VIL &gt; 0.4 V &lt; 0.8 V</td>
<td>5.5</td>
<td>0.10</td>
</tr>
<tr>
<td>ISB</td>
<td>Maximum Supply Current (per Package)</td>
<td>VCC &gt; 5 V</td>
<td>5.5</td>
<td>10</td>
</tr>
<tr>
<td>ISCC</td>
<td>Additional Supply Current</td>
<td>VIL &gt; 0.4 V</td>
<td>5.5</td>
<td>2.9</td>
</tr>
</tbody>
</table>

**NOTE:** Information on typical parametric values can be found in Chapter 3 of the Motorola High-Speed CMOS Data Book (DL-1390).
MC5474HCT00A

**AC CHARACTERISTICS FOR THE MC5474HCT00A** 

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter Description</th>
<th>Fig.</th>
<th>Guaranteed Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>t\text{\textsubscript{PHL}}</td>
<td>Max Propagation Delay, Input \text{\textsubscript{A}} to Output \text{\textsubscript{B}}</td>
<td>1.7</td>
<td>Min</td>
</tr>
<tr>
<td>t\text{\textsubscript{PHL}}</td>
<td>Max Propagation Delay, Input \text{\textsubscript{A}} to Output \text{\textsubscript{L}}</td>
<td>1.7</td>
<td>19</td>
</tr>
<tr>
<td>t\text{\textsubscript{PHL}}</td>
<td>Max Propagation Delay, Input \text{\textsubscript{H}} to Output \text{\textsubscript{B}}</td>
<td>1.7</td>
<td>19</td>
</tr>
<tr>
<td>C\text{\textsubscript{in}}</td>
<td>Max Input Capacitance</td>
<td>—</td>
<td>10</td>
</tr>
</tbody>
</table>

**NOTE:** For propagation delays with inputs other than \text{\textsubscript{A}} or \text{\textsubscript{B}}, and information on typical parameter values, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DS-1236).

**C\text{\textsubscript{in}}**

<table>
<thead>
<tr>
<th>Power Dissipation Capacitance (Part C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical @ 25°C, \text{\textsubscript{VCC}} = 5.0 V</td>
</tr>
</tbody>
</table>

* Used to determine the on-chip dynamic power consumption: \text{\textsubscript{P}}\text{\textsubscript{D}} = C\text{\textsubscript{in}} \times \text{\textsubscript{VCC}} \times \text{\textsubscript{VDD}} \times \text{\textsubscript{f}}. For load considerations, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DS-1236).

**Figure 1. Switching Waveforms**

**Figure 2. Test Circuit**

**EXPANDED LOGIC DIAGRAM**

(1/4 OF THE DEVICE)
Quad 2-Input NAND Gate

The MC74VHC00 is an advanced high-speed CMOS 2-input NAND gate fabricated with silicon gate CMOS technology. It achieves high-speed operation similar to equivalent Bipolar Schottky TTL while maintaining CMOS low power dissipation.

The internal circuit is composed of three stages including a buffer output which provides high noise immunity and stable output. The inputs tolerate voltages up to 7V, allowing the interface of 5V systems to 3V systems.

- High Speed: \( t_{PD} = 3.7\,\text{ns} \) (Typ) \( V_{CC} = 5\,\text{V} \)
- Low Power Dissipation: \( I_{CC} = 2\mu\text{A} \) (Max) at \( T_A = 25^\circ\text{C} \)
- High Noise Immunity: \( V_{NH} - V_{NIL} = 2.5\% \, V_{CC} \)
- Power Down Protection Provided on Inputs
- Balanced Propagation Delay
- Designed for 2V to 5V Operating Range
- Low Noise, \( V_{OL} = 0.8\,\text{V} \) (Max)
- Pin and Function Compatible with Other Standard Logic Families
- Latchup Performance Exceeds 300mA
- ESD Performance: HBM > 2000V; Machine Mode > 200V
- Chip Complexity: 32 Gates or 6 Equivalent Gates

**LOGIC DIAGRAM**

**FUNCTION TABLE**

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>
### MAXIMUM RATINGS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage</td>
<td>-0.5 to -7.0</td>
<td>V</td>
</tr>
<tr>
<td>VDD</td>
<td>DC Output Voltage</td>
<td>-0.5 to -7.0</td>
<td>V</td>
</tr>
<tr>
<td>IpK</td>
<td>Input Node current</td>
<td>-20</td>
<td>mA</td>
</tr>
<tr>
<td>IOK</td>
<td>Output Node current</td>
<td>470</td>
<td>mA</td>
</tr>
<tr>
<td>VDD</td>
<td>DC Output Current per Pin</td>
<td>1.25</td>
<td>mA</td>
</tr>
<tr>
<td>VSS</td>
<td>DC Supply Voltage, VCC, and GND Pins</td>
<td>±500  mV</td>
<td></td>
</tr>
<tr>
<td>Ppp</td>
<td>Power Dissipation in SJ84 or TSSOP Package</td>
<td>500 mW</td>
<td></td>
</tr>
<tr>
<td>TSTG</td>
<td>Storage Temperature</td>
<td>65 to 150</td>
<td>°C</td>
</tr>
</tbody>
</table>

Absolute maximum continuous ratings are those values beyond which damage to the device may occur. Exposure to these conditions or conditions beyond those indicated may adversely affect reliability. Functional operation under absolute maximum rated conditions is not implied.

Junction Temperature: 
- SJ84 Package: -7 mW/°C from 85° to 125°
- TSSOP Package: 5 mW/°C from 65° to 125°

### RECOMMENDED OPERATING CONDITIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage</td>
<td>2.0</td>
<td>5.5</td>
<td>V</td>
</tr>
<tr>
<td>VDD</td>
<td>DC Input Voltage</td>
<td>0</td>
<td>5.5</td>
<td>V</td>
</tr>
<tr>
<td>VDD</td>
<td>DC Output Voltage</td>
<td>0</td>
<td>VCC</td>
<td>V</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Temperature, All Package Types</td>
<td>45</td>
<td>85</td>
<td>°C</td>
</tr>
<tr>
<td>tJtJ</td>
<td>Input Rise and Fall Time</td>
<td>0</td>
<td>100</td>
<td>ns/µs</td>
</tr>
</tbody>
</table>

### DC ELECTRICAL CHARACTERISTICS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>VCC (V)</th>
<th>TA = 25°C</th>
<th>TA = -40 to 85°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIL</td>
<td>High-Level Input Voltage</td>
<td>1.8 to 5.5 V</td>
<td>0.5 µs</td>
<td>VCC x 3</td>
<td>1.0 V</td>
</tr>
<tr>
<td>VIL</td>
<td>Low-Level Input Voltage</td>
<td>2.0 to 6.5 V</td>
<td>0.5 µs</td>
<td>VCC x 3</td>
<td>1.0 V</td>
</tr>
<tr>
<td>VOH</td>
<td>High-Level Output Voltage</td>
<td>2.6 to 6.5 V</td>
<td>0.5 µs</td>
<td>VCC x 3</td>
<td>1.0 V</td>
</tr>
<tr>
<td>VOUT</td>
<td>Low-Level Output Voltage</td>
<td>2.6 to 6.5 V</td>
<td>0.5 µs</td>
<td>VCC x 3</td>
<td>1.0 V</td>
</tr>
<tr>
<td>VIL</td>
<td>High-Level Output Voltage</td>
<td>2.6 to 6.5 V</td>
<td>0.5 µs</td>
<td>VCC x 3</td>
<td>1.0 V</td>
</tr>
<tr>
<td>VOUT</td>
<td>Low-Level Output Voltage</td>
<td>2.6 to 6.5 V</td>
<td>0.5 µs</td>
<td>VCC x 3</td>
<td>1.0 V</td>
</tr>
</tbody>
</table>

This device contains protection circuitry to guard against damage due to high static voltages or electrostatic fields. However, precautions must be observed during handling and applications of any voltage higher than maximum-rated voltages to the high-impedance input. For proper operation, VIL and VOH should be constrained to the range GND ≤ VIL ≤ VDD ≤ VCC ≤ VOH ≤ VCC.

Unused inputs must always be left open or at an appropriate voltage level (e.g., either GND or VCC). Unused outputs must be left open.
### DC Electrical Characteristics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>$V_{CC}$</th>
<th>$TA = 25^\circ$C</th>
<th>$TA = -40$ to $85^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{IN}$</td>
<td>Input leakage Current</td>
<td>$V_{IN} = 5$ V or GND</td>
<td>3 to 5.5</td>
<td>Min: 1</td>
<td>Min: 1.3</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Quiescent Supply Current</td>
<td>$V_{IN} = V_{CC}$ or GND</td>
<td>0.3</td>
<td>Typ: 0</td>
<td>Max: 20</td>
</tr>
</tbody>
</table>

### AC Electrical Characteristics (Input $f = 3$ MHz)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>$V_{CC}$</th>
<th>$TA = 25^\circ$C</th>
<th>$TA = -40$ to $85^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{PLH}$</td>
<td>Propagation Delay Time</td>
<td>$V_{CC}$ = 3.3 V</td>
<td>$G_{L} = 10$ pF</td>
<td>Min: 5</td>
<td>Min: 5.5</td>
</tr>
<tr>
<td>$I_{PLH}$</td>
<td>Propagation Delay Time</td>
<td>$V_{CC}$ = 5.0 V</td>
<td>$G_{L} = 10$ pF</td>
<td>Min: 3.2</td>
<td>Min: 3.3</td>
</tr>
</tbody>
</table>

### Noise Characteristics (Input $f = 5$ MHz, $G_{L} = 50$ pF, $V_{CC} = 6$ V, Measured at SOIC Package)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Characteristics</th>
<th>$TA = 25^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{OLD}$</td>
<td>Quiet Output Dynamic Voltage</td>
<td>Min: 0.3</td>
</tr>
<tr>
<td>$V_{OHV}$</td>
<td>Quiet Output High Level Voltage</td>
<td>Min: 3.5</td>
</tr>
<tr>
<td>$V_{HLD}$</td>
<td>Minimum High Level Dynamic Input Voltage</td>
<td>1.5</td>
</tr>
<tr>
<td>$V_{ILD}$</td>
<td>Maximum Low Level Dynamic Input Voltage</td>
<td>0.3</td>
</tr>
</tbody>
</table>

---

**Figure 1.** Switching Waveforms

**Figure 2.** Test Circuit

**Figure 3.** Input Equivalent Circuit

---

1. $G_{PD}$ is defined as the value of the internal equivalent resistance which is calculated from the operating current consumption without ac coupling. The equation $G_{PD} = \frac{V_{CC}}{I_{PD}}$ is used to determine the dynamic power consumption.

2. $C_{PD}$ includes all probe and jig capacitance.
Quad 2-Input NAND Gate

The MC74VHCT00A is an advanced high-speed CMOS, 2-input NAND gate fabricated with silicon gate CMOS technology. It achieves high-speed operation similar to equivalent Bipolar Schottky TTL, while maintaining CMOS low power dissipation.

The VHCT inputs are compatible with TTL levels. This device can be used as a level converter or interfacing 3.3V to 5.0V, because it has full 5V CMOS level output ratings.

The VHCT00A input structures provide protection when voltages between 0V and 6V are applied, regardless of the supply voltage. The output structures also provide protection when VCC = 0V. These input and output structures help prevent device destruction caused by supply voltage - input/output voltage mismatch, battery backup, hot insertion, etc.

- **High-Speed**: tpd = 5.0ns (Typ) at VCC = 5V
- **Low Power Dissipation**: ICC = 2mA (Max) at TA = 25°C
- **TTL Compatible Inputs**: VIL = 0.8V, VIH = 2.0V
- **Power Down Protection Provided on Inputs and Outputs
- **Balanced Propagation Delays
- **Designed for 4.5V to 5.5V Operating Range
- **Low Noise: VMN = 0.3V (Max)
- **Pin and Function Compatible with Other Standard Logic Families
- **Latchup Performance**: Exceeds 300mA
- **ESD Performance**: HBM = 2000V, Machine Model > 200V
- **Chip Complexity**: 16 PES or 12 Equivalent Gates

**LOGIC DIAGRAM**

**FUNCTION TABLE**

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

**Pinout: 14-Lead Packages (Top View)**

- **VCC**
- **Pin 41**
- **Pin 42**
- **Pin 43**
- **Pin 44**
- **Pin 34**
- **Pin 35**
- **Pin 36**
- **Pin 37**
- **Pin 38**
- **Pin 39**
- **Pin 40**
- **Pin 2**
- **Pin 1**
### MAXIMUM RATINGS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage</td>
<td>$-15.5$ to $+7.0$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{in}$</td>
<td>DC Input Voltage</td>
<td>$-0.5$ to $+7.0$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>DC Output Voltage</td>
<td>$-0.5$ to $+7.0$</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>$V_{CC} = 3$</td>
<td>$-0.6$ to $+0.5$</td>
<td>V</td>
</tr>
<tr>
<td>$I_{in}$</td>
<td>Input Diode Current</td>
<td>30</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{o}$</td>
<td>Output Diode Current</td>
<td>20</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{out}$</td>
<td>DC Output Current, per Pin</td>
<td>$25$</td>
<td>mA</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>DC Supply Current, $V_{CC}$ and GND Pins</td>
<td>$50$</td>
<td>mA</td>
</tr>
<tr>
<td>$P_{DP}$</td>
<td>Power Dissipation in SOIC Package</td>
<td>1.0</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td>Power Dissipation in 8-Pin TSSOP Package</td>
<td>1.0</td>
<td>W</td>
</tr>
<tr>
<td>$T_{stg}$</td>
<td>Storage Temperature</td>
<td>$-65$ to $+150$</td>
<td>°C</td>
</tr>
</tbody>
</table>

*Absolute maximum ratings are those values beyond which damage to the device may occur. Exposure to conditions exceeding those indicated may adversely affect reliability. Functional operation under absolute maximum rated conditions is not implied.*

### RECOMMENDED OPERATING CONDITIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CC}$</td>
<td>DC Supply Voltage</td>
<td>$3.3$</td>
<td>$5.5$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{in}$</td>
<td>DC Input Voltage</td>
<td>$0$</td>
<td>$5.5$</td>
<td>V</td>
</tr>
<tr>
<td>$V_{out}$</td>
<td>DC Output Voltage</td>
<td>$0$</td>
<td>$5.5$</td>
<td>V</td>
</tr>
<tr>
<td>$T_{A}$</td>
<td>Operating Temperature</td>
<td>$-40$</td>
<td>$+85$</td>
<td>°C</td>
</tr>
</tbody>
</table>

### DC ELECTRICAL CHARACTERISTICS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>$V_{CC}$</th>
<th>$T_A = 25$°C</th>
<th>$T_A = -40$ to $85$°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{IH}$</td>
<td>Minimum High-Level Input Voltage</td>
<td></td>
<td>$4.5$</td>
<td>$20$</td>
<td>$2.6$</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Maximum Low-Level Input Voltage</td>
<td></td>
<td>$4.5$</td>
<td>$0.8$</td>
<td>$3.8$</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>Minimum High-Level Output Voltage</td>
<td>$V_{in} = V_{CC}$ or $GND$</td>
<td>$4.5$</td>
<td>$0.8$</td>
<td>$3.8$</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Maximum Low-Level Output Voltage</td>
<td>$V_{in} = V_{CC}$ or $GND$</td>
<td>$4.5$</td>
<td>$0.8$</td>
<td>$3.8$</td>
</tr>
<tr>
<td>$I_{in}$</td>
<td>Maximum Input Leakage Current</td>
<td></td>
<td>$0.65$</td>
<td>$1.0$</td>
<td>$1.3$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Maximum Quiescent Supply Current</td>
<td></td>
<td>$5.5$</td>
<td>$2.0$</td>
<td>$20.0$</td>
</tr>
<tr>
<td>$I_{CCST}$</td>
<td>Quiescent Supply Current</td>
<td>$P_{in} = 1.5$ W</td>
<td>$5.5$</td>
<td>$1.35$</td>
<td>$1.53$</td>
</tr>
<tr>
<td>$I_{PS}$</td>
<td>Output Leakage Current</td>
<td>$V_{out} = 1$ V</td>
<td>$5$</td>
<td>$5$</td>
<td>$5$</td>
</tr>
</tbody>
</table>
APPENDIX C • Manufacturers' Data Sheets

MC74VHCT00A

AC ELECTRICAL CHARACTERISTICS (Input I = 3 mA)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>TA = 25°C</th>
<th>TA = -40 to 85°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHH</td>
<td>Propagation Delay, A to B &lt; Y</td>
<td>VCC = 5.0 ± 0.5V, CL = 15pF, CL = 40pF</td>
<td>Min</td>
<td>Typ</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>IPH</td>
<td>Input Capacitance</td>
<td></td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Power Dissipation (In Heatsink) @ 25°C, VDC = 5.0V

1. Power is defined as the value of the internal equivalent resistance calculated from the operating current consumption of the circuit.

2. Power dissipation (in Heatsink) is calculated by the equation: PDP = VCC * IOP + IOP * VCC (IOP = IOP + IOP)

NOISE CHARACTERISTICS (Input I = 3 mA, CL = 50pF, VCC = 5.0V, Measured SOIC Package)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Characteristic</th>
<th>Test Conditions</th>
<th>Typ</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>VODL</td>
<td>Output Dynamic VOH</td>
<td></td>
<td>0.4</td>
<td>0.8</td>
</tr>
<tr>
<td>VODH</td>
<td>Output Dynamic VOL</td>
<td></td>
<td>-0.4</td>
<td>-0.8</td>
</tr>
<tr>
<td>VILH</td>
<td>Minimum High Level Dynamic Input Voltage</td>
<td></td>
<td>2.4</td>
<td>V</td>
</tr>
<tr>
<td>VNIL</td>
<td>Maximum Low Level Dynamic Input Voltage</td>
<td></td>
<td>0.5</td>
<td>V</td>
</tr>
</tbody>
</table>

Figure 1. Switching Waveforms

Figure 2. Test Circuit

Includes all probe and jig capacitance
Hex Unbuffered Inverter
High-Performance Silicon-Gate CMOS

The MC74HCU04A is identical in pinout to the LS04 and the MC14062UB. The device inputs are compatible with standard CMOS outputs, with pull-up resistors, they are compatible with LS TTL outputs.

This device consists of six single-stage inverters. These inverters are well suited for use as oscillators, pulse shapers, and in many other applications requiring a high-input impedance amplifier. For digital applications, the HC04A is recommended.

- Output Drive Capability: 10 LS TTL Loads
- Outputs Directly Interface to CMOS, NMOS, and TL
- Operating Voltage Range: 2 to 6 V, ±5 to 6 V in Oscillator Configurations
- Low Input Current: 1 µA
- High Noise Immunity Characteristic of CMOS Devices
- Compliance with the Requirements Defined by JEDEC Standard No. 7A
- Chip Complexity: 12 FETs or 3 Equivalent Gates

LOGIC DIAGRAM

```
  A1  1  2  Y1
  A2  3  4  Y2
  A3  5  6  Y3
  A4  7  8  Y4
  A5  9  10 Y5
  A6 11 12  Y6

PIN 1 = A1
PIN 7 = GND

FUNCTION TABLE

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Y</td>
</tr>
<tr>
<td>M</td>
<td>H</td>
</tr>
</tbody>
</table>
```

MC74HCU04A
PLASTIC PACKAGE CASE 640-810

ORDERING INFORMATION
MC74HCU4AAB Plastic
MC74HCU4AAQ SOIC
MC74HCU4ADT TSOP
### APPENDIX C • Manufacturers' Data Sheets

#### MC74HC04A

#### MAXIMUM RATINGS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>2.65 to 5.5 V</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage (Referenced to GND)</td>
<td>-0.5 to VIL + 0.5 V</td>
<td></td>
</tr>
<tr>
<td>VIL</td>
<td>DC Output Voltage (Referenced to GND)</td>
<td>≤ 0.6 V</td>
<td></td>
</tr>
<tr>
<td>ILH</td>
<td>DC Input Current, per Pin</td>
<td>≤ 20 mA</td>
<td></td>
</tr>
<tr>
<td>IOH</td>
<td>DC Output Current, per Pin</td>
<td>≤ 20 mA</td>
<td></td>
</tr>
<tr>
<td>ICC</td>
<td>DC Supply Current, VCC and GND Pins</td>
<td>≤ 30 mA</td>
<td></td>
</tr>
<tr>
<td>PD</td>
<td>Power Dissipation in Still Air</td>
<td>753 mW</td>
<td></td>
</tr>
<tr>
<td>Tstg</td>
<td>Storage Temperature</td>
<td>-65 to +150 °C</td>
<td></td>
</tr>
<tr>
<td>TOL</td>
<td>Lead Temperature 1 min from case for 15 seconds</td>
<td>-55 to 125 °C</td>
<td></td>
</tr>
</tbody>
</table>

*Maximum Ratings are those values beyond which damage to the device may occur.*

For high frequency or heavy load conditions, see Chapter 2 of the Micrel's High-Speed CMOS Data Book (DL-2161D).

#### RECOMMENDED OPERATING CONDITIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>2.0 V</td>
<td>5.5 V</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage (Referenced to GND)</td>
<td>0.3 V</td>
<td>VCC</td>
<td>V</td>
</tr>
<tr>
<td>TO</td>
<td>Operating Temperature, All Package Types</td>
<td>-55 °C to 125 °C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TSH</td>
<td>Input Rise and Fall Time (Figure 1)</td>
<td>Unlimited</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### DC ELECTRICAL CHARACTERISTICS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>VCC (V)</th>
<th>Guaranteed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIL</td>
<td>Minimum High-Level Input Voltage</td>
<td>2.4 V</td>
<td>17 17 17 17</td>
</tr>
<tr>
<td></td>
<td>VIL = 5.5 V - IIL</td>
<td>3.0 V</td>
<td>25 25 25 25</td>
</tr>
<tr>
<td></td>
<td>IIL = 20 μA</td>
<td>4.0 V</td>
<td>35 35 35 35</td>
</tr>
<tr>
<td></td>
<td>IIL = 40 μA</td>
<td>6.0 V</td>
<td>48 48 48 48</td>
</tr>
<tr>
<td>VIL</td>
<td>Maximum Low-Level Input Voltage</td>
<td>0.03 V</td>
<td>0.03 0.03 0.03 0.03</td>
</tr>
<tr>
<td></td>
<td>VIL = VCC - 0.6 V IIL = 20 μA</td>
<td>0.05 V</td>
<td>0.05 0.05 0.05 0.05</td>
</tr>
<tr>
<td></td>
<td>IIL = 40 μA</td>
<td>0.07 V</td>
<td>0.07 0.07 0.07 0.07</td>
</tr>
<tr>
<td></td>
<td>IIL = 60 μA</td>
<td>0.11 V</td>
<td>0.11 0.11 0.11 0.11</td>
</tr>
<tr>
<td>VCM</td>
<td>Minimum High-Level Output Voltage</td>
<td>0.18 V</td>
<td>0.3 0.3 0.3 0.3</td>
</tr>
<tr>
<td></td>
<td>VCM = GND</td>
<td>0.18 V</td>
<td>0.3 0.3 0.3 0.3</td>
</tr>
<tr>
<td></td>
<td>IOL = 24 mA</td>
<td>0.18 V</td>
<td>0.3 0.3 0.3 0.3</td>
</tr>
<tr>
<td></td>
<td>IOL = 48 mA</td>
<td>0.18 V</td>
<td>0.3 0.3 0.3 0.3</td>
</tr>
<tr>
<td></td>
<td>IOL = 52 mA</td>
<td>0.18 V</td>
<td>0.3 0.3 0.3 0.3</td>
</tr>
<tr>
<td>VCL</td>
<td>Maximum Low-Level Output Voltage</td>
<td>0.02 V</td>
<td>0.02 0.02 0.02 0.02</td>
</tr>
<tr>
<td></td>
<td>VCL = VCC</td>
<td>0.02 V</td>
<td>0.02 0.02 0.02 0.02</td>
</tr>
<tr>
<td></td>
<td>IOL = 24 mA</td>
<td>0.02 V</td>
<td>0.02 0.02 0.02 0.02</td>
</tr>
<tr>
<td></td>
<td>IOL = 48 mA</td>
<td>0.02 V</td>
<td>0.02 0.02 0.02 0.02</td>
</tr>
<tr>
<td></td>
<td>IOL = 52 mA</td>
<td>0.02 V</td>
<td>0.02 0.02 0.02 0.02</td>
</tr>
</tbody>
</table>
### DC Electrical Characteristics (Voltages Referenced to GND)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>$V_{CC}$</th>
<th>Guaranteed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{IN}$</td>
<td>Maximum Input Leakage Current</td>
<td>$V_{IN} = V_{CC}$ or GND</td>
<td>$= 0$</td>
<td>$-6.0$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Maximum Quiescent Supply Current (per Package)</td>
<td>$V_{IN} = V_{CC}$ or GND</td>
<td>$= 4.0$</td>
<td>$1.0$</td>
</tr>
</tbody>
</table>

**Note:** Information on typical parameter values can be found in Chapter 2 of the Motorola High-Speed CMOS Data Book (DL13829).

For $V_{CC} = 2.3V$, $V_{CC} = 0.2V$ or $V_{CC} = 0.2V$.

### AC Electrical Characteristics ($C_{IN} = 50pF$, Input $f = 10kHz$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>$V_{CC}$</th>
<th>Guaranteed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{PLH}$</td>
<td>Maximum Propagation Delay, Input A to Output Y</td>
<td>$= 2.0$</td>
<td>$= 3.6$</td>
</tr>
<tr>
<td>$t_{PHL}$</td>
<td>Maximum Propagation Delay, Input B to Output Y</td>
<td>$= 2.0$</td>
<td>$= 3.6$</td>
</tr>
<tr>
<td>$t_{TCH}$</td>
<td>Maximum Output Transition Time, Any Output</td>
<td>$= 2.0$</td>
<td>$= 3.0$</td>
</tr>
<tr>
<td>$C_{IN}$</td>
<td>Maximum Input Capacitance</td>
<td>$= 10.0$</td>
<td>$= 10$</td>
</tr>
</tbody>
</table>

**Notes:**
1. For propagation delays with loads other than $50pF$, see Chapter 2 of the Motorola High-Speed CMOS Data Book (DL13829).
2. Information on typical parameter values can be found in Chapter 2 of the Motorola High-Speed CMOS Data Book (DL13829).

### Power Dissipation Characteristic

<table>
<thead>
<tr>
<th>$I_{PD}$</th>
<th>Power Dissipation Characteristic (Per Package)</th>
<th>Typical $@25^\circ C$, $V_{CC} = 5.0V$</th>
</tr>
</thead>
</table>

*Used to determine the most dynamic power dissipation in $P_{D}$: $P_{D} = G_{D} \times V_{CC}^2 = I_{CC} \times V_{CC}$.*
MC74HC04A

Figure 1. Switching Waveforms

Figure 2. Test Circuit

Logic Detail
(1/16 of Device Shown)
**TYPICAL APPLICATIONS**

**Crystal Oscillator**

\[ R_2 \]
\[ C_1 \]
\[ C_2 \]

**Stable RC Oscillator**

\[ 1 \mu F \]
\[ 1 \mu F \]
\[ C \]

**Schmitt Trigger**

\[ R_2 \]
\[ R_1 \]
\[ 1 \mu F \]
\[ 1 \mu F \]

**High Input Impedance Single-Stage Amplifier with a 2 to 6 V Supply Range**

**Multi-Stage Amplifier**

\[ 1 \mu F \]
\[ 1 \mu F \]

**LED Driver**

For reduced power supply current, use high-efficiency LEDs such as the Hewlett-Packard HJL series or equivalent.
Hex Buffers/Logic-Level Down Converters
High-Performance Silicon-Gate CMOS

The MC54/74H4049 consists of six inverting buffers, and the MC54/74H4050 consists of six noninverting buffers. They are identical in pinout to the MC144044UB and MC144050UB metal-gate CMOS buffers. The device inputs are compatible with standard CMOS outputs with pullup resistors. They are compatible with LSSTL outputs.

The input protection circuitry on these devices has been modified by eliminating the VCC device to allow the use of input voltages up to 15 volts. Thus, the devices may be used as logic-dual translators that convert from a high voltage to a low voltage while operating at the low-voltage power supply. Typically, MC144044UB-series CMOS operating up to 1.5 volts to be interfaced with high-speed CMOS at 2 to 5 volts. The protection diodes in CMOS are 'donor' diodes, which protect the inputs from both positive and negative voltage transients.

- Output Drive Capability: 10 LSSTL Loads
- Outputs Directly Interface to CMOS, NMOS, and TTL
- Operating Voltage Range: 2 to 6 V
- Low Supply Current: 5 µA
- High Noise Immunity Characteristic of CMOS Devices
- Complies with the Requirements Defined by JEDEC Standard
- 35 FETs or 3 Equivalent Gates (4049),
  24 FETs or 3 Equivalent Gates (4050)

LOGIC DIAGRAMS

MC54/74HC4049
MC54/74HC4050
## APPENDIX C • Manufacturers' Data Sheets

### MC54/74HC4049, MC54/74HC4050

**MAXIMUM RATINGS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>-1.5 V to 5.5 V</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage</td>
<td>-1.5 V to 0 V</td>
<td>V</td>
</tr>
<tr>
<td>VILH</td>
<td>DC Output Voltage (Referenced to GND)</td>
<td>-0.5 V to VCC + 0.5 V</td>
<td>V</td>
</tr>
<tr>
<td>IIL</td>
<td>DC Input Current, power</td>
<td>-25 mA</td>
<td>mA</td>
</tr>
<tr>
<td>IOL</td>
<td>DC Input Current, sink</td>
<td>25 mA</td>
<td>mA</td>
</tr>
<tr>
<td>ICC</td>
<td>DC Supply Current, VCC and GND Pins</td>
<td>3.5 mA</td>
<td>mA</td>
</tr>
<tr>
<td>BTJO</td>
<td>Storage Temperature, time from Case for 10 Seconds</td>
<td>-55°C to 155°C</td>
<td>°C</td>
</tr>
<tr>
<td>TJL</td>
<td>Lead Temperature, All Package Types</td>
<td>250°C</td>
<td>°C</td>
</tr>
</tbody>
</table>

*Maximum Ratings are those values beyond which damage to the device may occur. Rated specifications should be limited to the Recommended Operating Conditions specifying a DC input voltage of 5 V ± 5%. For high frequency or heavy load considerations, refer to Chapter 2 of the Motorola High-Speed CMOS Data Book (DL1250).*

**RECOMMENDED OPERATING CONDITIONS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage (Referenced to GND)</td>
<td>2.5 V</td>
<td>5.5 V</td>
<td>V</td>
</tr>
<tr>
<td>VR</td>
<td>DC input voltage</td>
<td>0.8 V to 5.5 V</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>VDD</td>
<td>DC Output Voltage (Referenced to GND)</td>
<td>0 V to VCC</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>IOL</td>
<td>Operating Temperature, All Package Types</td>
<td>-25°C to 85°C</td>
<td>°C</td>
<td></td>
</tr>
<tr>
<td>fMAX</td>
<td>Input Rise And Fall Time (Figure 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**DC ELECTRICAL CHARACTERISTICS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>VCC V</th>
<th>Guaranteed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIH</td>
<td>Minimum High Level Input Voltage</td>
<td>VCC ± 0.15 V</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>VIL</td>
<td>Maximum Low Level Input Voltage</td>
<td></td>
<td>2.0</td>
<td>0.3</td>
</tr>
<tr>
<td>VOM</td>
<td>Minimum High Level Output Voltage</td>
<td>2.0 mA</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>VOH</td>
<td>Maximum Low Level Output Voltage</td>
<td>2.0 mA</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>VILH</td>
<td>Maximum Low-Level Output Voltage</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:** Information on typical performance values can be found in Chapter 2 of the Motorola High-Speed CMOS Data Book (DL1250).
AC ELECTRICAL CHARACTERISTICS (\(t_{c} = 5 \, \mu F\), \(t_{r} = 6 \, \mu F\))

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Guaranteed Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_{pl})</td>
<td>Maximum Propagation Delay, Input A to Output X (Figures 1 and 2)</td>
<td>2.0, 4.5, 6.0 ns</td>
</tr>
<tr>
<td></td>
<td>Maximum Output Transition Time, Any Output (Figures 1 and 2)</td>
<td>7.0, 9.5, 11.0 ns</td>
</tr>
<tr>
<td>(C_{ip})</td>
<td>Maximum Input Capacitance</td>
<td>10, 10, 10 pF</td>
</tr>
</tbody>
</table>

**NOTES**

1. For propagation delays with loads other than 50 \(\Omega\), see Chapter 2 of the Motorola High-Speed CMOS Data Book (DL1290C).
2. Information on typical parametric values can be found in Chapter 2 of the Motorola High-Speed CMOS Data Book (DL1290C).

**Typical at 25°C, Vcc = 5 V**

---

![Figure 1a. Switching Waveforms (HC4049)](image1)

![Figure 1b. Switching Waveforms (HC4050)](image2)

![Figure 2. Test Circuit](image3)

*Includes all probe and jig capacitance.*
MC54/74HC4048 MC54/74HC4050

LOGIC DETAIL

MC4048
(1/8 of the Device)

A → B

MC4054
(1/8 of the Device)

A → B

TYPICAL APPLICATIONS

LS-TTL to Low-Voltage HSCMOS

High-Voltage CMOS to HSCMOS

NOTE. To determine the noise immunity of the LS-TTL to low-voltage configuration, use Eq. 1 and Eq. 2.

\[ V_{IH} - V_{IL} \text{ (TTL)}, V_{OH} - V_{OL} \text{ (CMOS)} \]

For the supply levels shown:

- For supply levels greater than 4.5 volts use the 74HC14A for direct interface to TTL outputs.

<table>
<thead>
<tr>
<th>Table 1. Supply Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VDD</strong></td>
</tr>
<tr>
<td>15 V</td>
</tr>
<tr>
<td>12 V</td>
</tr>
<tr>
<td>12 V</td>
</tr>
</tbody>
</table>
Quad 2-Input NAND Gate
With 5V-Tolerant Inputs

The MC74LVX00 is an advanced high speed CMOS 2-input NAND gate. The inputs tolerate voltages up to 7V, allowing the interface of 5V systems to 3V systems.

- High Speed, $t_{PD} = 4.1\text{ns}$ (Typ) at $V_{CC} = 3.3\text{V}$
- Low Power Dissipation: $I_{CC} = 2\mu A$ (Max) at $T_A = 25^\circ\text{C}$
- Power Down Protection Provided on Inputs
- Balanced Propagation Delays
- Low Noise $V_{OLP} = 0.05\text{V}$ (Max)
- Pin and Function Compatible with Other Standard Logic Families
- Latchup Performance: Exceeds 100mA
- ESD Performance: HBM > 2000V, Machine Mode > 200V

**MC74LVX00**

**LVX**
LOW-VOLTAGE CMOS

**Figure 1. 14-Lead Pinout (Top View)**

**Figure 2. Logic Diagram**

**PIN NAMES**

<table>
<thead>
<tr>
<th>PIN</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>A0</td>
<td>Data Inputs</td>
</tr>
<tr>
<td>Or</td>
<td>Output</td>
</tr>
</tbody>
</table>

**FUNCTION TABLE**

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>An</td>
<td>Bn</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>X</td>
<td>H</td>
</tr>
</tbody>
</table>
## MAXIMUM RATINGS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage</td>
<td>3.3 to 5.5</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage</td>
<td>-0.5 to $V_{CC}$</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Output Voltage</td>
<td>-1.6 to $V_{CC}+0.3$</td>
<td>V</td>
</tr>
<tr>
<td>ILH</td>
<td>Input Leakage Current</td>
<td>$-20$</td>
<td>mA</td>
</tr>
<tr>
<td>IOL</td>
<td>Output Leakage Current</td>
<td>$-20$</td>
<td>mA</td>
</tr>
<tr>
<td>IDH</td>
<td>DC Output Current, per Pin</td>
<td>$-75$</td>
<td>mA</td>
</tr>
<tr>
<td>IDH</td>
<td>DC Supply Current, $V_{CC}$ and GND Min.</td>
<td>$+50$</td>
<td>mA</td>
</tr>
<tr>
<td>PD</td>
<td>Power Dissipation</td>
<td>$180$</td>
<td>mW</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Temperature</td>
<td>$-40$ to $+85$</td>
<td>°C</td>
</tr>
</tbody>
</table>

Absolute maximum continuous ratings are those values beyond which damage to the device may occur. Exposure to these conditions or conditions beyond those indicated may adversely affect device reliability. Functional operation under absolute maximum rated conditions is not implied.

## RECOMMENDED OPERATING CONDITIONS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage</td>
<td>$2.7$</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Input Voltage</td>
<td>$0$</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>DC Output Voltage</td>
<td>$0$</td>
<td>V</td>
</tr>
<tr>
<td>TA</td>
<td>Operating Temperature All Package Types</td>
<td>$-40$ to $+85$</td>
<td>°C</td>
</tr>
<tr>
<td>JAT</td>
<td>Junction Temperature</td>
<td>$0$</td>
<td>°C</td>
</tr>
</tbody>
</table>

## DC ELECTRICAL CHARACTERISTICS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>$V_{CC}$</th>
<th>$T_{A} = 25^\circ$C</th>
<th>$T_{A} = -40$ to $+85^\circ$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{IH}$</td>
<td>High-Level Input Voltage</td>
<td>$2.0$</td>
<td>$1.5$</td>
<td>$1.5$</td>
<td>$V$</td>
</tr>
<tr>
<td>$V_{IL}$</td>
<td>Low-Level Input Voltage</td>
<td>$2.0$</td>
<td>$3.6$</td>
<td>$0$</td>
<td>$V$</td>
</tr>
<tr>
<td>$V_{OH}$</td>
<td>High-Level Output Voltage</td>
<td>$2.0$</td>
<td>$1.1$</td>
<td>$0.0$</td>
<td>$2.8$</td>
</tr>
<tr>
<td>$V_{OL}$</td>
<td>Low-Level Output Voltage</td>
<td>$2.0$</td>
<td>$1.1$</td>
<td>$0.0$</td>
<td>$2.8$</td>
</tr>
<tr>
<td>$I_{IN}$</td>
<td>Input Leakage Current</td>
<td>$3.8$</td>
<td>$0.1$</td>
<td>$1.8$</td>
<td>$10\mu A$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Quiescent Supply Current</td>
<td>$3.8$</td>
<td>$0.1$</td>
<td>$1.8$</td>
<td>$10\mu A$</td>
</tr>
</tbody>
</table>
APPENDIX C • Manufacturers' Data Sheets

AC ELECTRICAL CHARACTERISTICS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>TA = 25°C</th>
<th>TA = -40 to 85°C</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>HPLH</td>
<td>Propagation Delay Input/Output</td>
<td>VCC = 2.7V, CL = 15pF</td>
<td>7.4</td>
<td>11</td>
<td>17.5</td>
<td>ns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>VCC = 2.7V, CL = 50pF</td>
<td>9.2</td>
<td>12.5</td>
<td>18.6</td>
<td>ns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>VCC = 2.7V, CL = 50pF</td>
<td>4.4</td>
<td>6.2</td>
<td>10.5</td>
<td>ns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>VCC = 2.7V, CL = 50pF</td>
<td>5.8</td>
<td>7.9</td>
<td>11.8</td>
<td>ns</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Skew is defined as the absolute value of the difference between the actual propagation delay for any two separate outputs of the same device.

2. The specification applies to any outputs switching in the same direction, either HIGH to LOW (HOSLH) or LOW to HIGH (HOSLH). Parameter guaranteed by design.

CAPACITIVE CHARACTERISTICS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Test Conditions</th>
<th>TA = 25°C</th>
<th>TA = -40 to 85°C</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIN</td>
<td>Input Capacitance</td>
<td></td>
<td>4</td>
<td>12</td>
<td>18</td>
<td>pF</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDO</td>
<td>Power/Output Capacitance (Note 2)</td>
<td></td>
<td>19</td>
<td></td>
<td></td>
<td>pF</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. CDO is defined as the value of the internal equivalent capacitance which is calculated from the operating current consumption without load. Average operating current is obtained by dividing the test output current by the number of outputs used. CDO is used to determine the on-state dynamic power consumption. PD = CDO * VCC * f

3. Noise Characteristics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Characteristic</th>
<th>Test Conditions</th>
<th>TA = 25°C</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VOH</td>
<td>Output Maximum Dynamic Voltage</td>
<td></td>
<td>5.3</td>
<td>V</td>
</tr>
<tr>
<td>VOL</td>
<td>Output Minimum Dynamic Voltage</td>
<td></td>
<td>0.3</td>
<td>V</td>
</tr>
<tr>
<td>VHOL</td>
<td>Minimum High Level Dynamic Input Voltage</td>
<td></td>
<td>2.0</td>
<td>V</td>
</tr>
<tr>
<td>VILD</td>
<td>Maximum Low Level Dynamic Input Voltage</td>
<td></td>
<td>0.8</td>
<td>V</td>
</tr>
</tbody>
</table>

Figure 3. Switching Waveforms

Figure 4. Test Circuit

* Includes all probe and jig capacitance
Low-Voltage CMOS Quad
2-Input NAND Gate
With 5V-Tolerant Inputs

The MC74LCX00 is a high performance, quad 2-input NAND gate operating from a 2.7 to 3.6V supply. High impedance TTL compatible inputs significantly reduce current loading to input drivers while TTL compatible outputs offer improved switching noise performance. A Vih specification of 5.5V allows MC74LCX00 inputs to be safely driven from 5V devices.

Current drive capability is 24mA at the outputs.

- Designed for 3.3 V or 5 V Vcc Operation
- 5V Tolerant Inputs — Interface Capability With 5V TTL Logic
- LV TTL Compatible
- LC CMOS Compatible
- 24mA Balanced Output Sink and Source Capability
- Very Low Standby Current (10µA) Substantially Reduces System Power Requirements
- Latchup Performance Exceeds 500mA
- ESD Performance: Human Body Model >2000V; Machine Model >200V

Pinout: 14-Lead (Top View)

LOGIC DIAGRAM

FUNCTION TABLE

<table>
<thead>
<tr>
<th>An</th>
<th>Bn</th>
<th>f</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>L</td>
<td>L</td>
<td>L</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>L</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td>H</td>
<td></td>
</tr>
</tbody>
</table>
### Absolute Maximum Ratings

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Condition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>DC Supply Voltage</td>
<td>5.5 V to +7V</td>
<td></td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>ICH Input Current</td>
<td>-0.5 to +0.5 VCC</td>
<td></td>
<td>mA</td>
</tr>
<tr>
<td>VCC</td>
<td>IC Supply Voltage</td>
<td>-0.5 to +0.5 VCC</td>
<td>VCC &gt; 0.5 V</td>
<td>V</td>
</tr>
<tr>
<td>IOL</td>
<td>IC Output Current</td>
<td>-0.5 to +0.5 VCC</td>
<td>VCC = VCC</td>
<td>mA</td>
</tr>
<tr>
<td>VIL</td>
<td>IC Supply Current</td>
<td>-0.5 to +0.5 VCC</td>
<td>VCC = VCC</td>
<td>mA</td>
</tr>
<tr>
<td>VCC</td>
<td>IC Output Current</td>
<td>-0.5 to +0.5 VCC</td>
<td>VCC = VCC</td>
<td>mA</td>
</tr>
<tr>
<td>TSTG</td>
<td>Storage Temperature Range</td>
<td>E3 to +100°C</td>
<td></td>
<td>°C</td>
</tr>
</tbody>
</table>

* Absolute maximum ratings are those values beyond which damage to the device may occur. Operating to these conditions or conditions beyond those indicated may adversely affect device reliability. Functional operation under absolute-maximum-rated conditions is not implied.

### Recommended Operating Conditions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Operating Conditions</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VCC</td>
<td>Supply Voltage</td>
<td>Operating</td>
<td>2.0</td>
<td>3.3</td>
<td>5.5</td>
<td>V</td>
</tr>
<tr>
<td>VIL</td>
<td>Input Voltage</td>
<td>Data Retention Only</td>
<td>1.5</td>
<td>3.3</td>
<td>5.5</td>
<td>V</td>
</tr>
<tr>
<td>VIN</td>
<td>Output Voltage</td>
<td>0</td>
<td></td>
<td></td>
<td>VCC</td>
<td>V</td>
</tr>
<tr>
<td>IOL</td>
<td>HIGH Level Output Current</td>
<td>VCC = 3.3V to 3.6V</td>
<td></td>
<td></td>
<td>36</td>
<td>mA</td>
</tr>
<tr>
<td>ICH</td>
<td>LOW Level Output Current</td>
<td>VCC = 3.3V to 3.6V</td>
<td></td>
<td></td>
<td>24</td>
<td>mA</td>
</tr>
<tr>
<td>IL</td>
<td>HIGH Level Input Current</td>
<td>VCC = 3.3V to 3.6V</td>
<td></td>
<td></td>
<td>12</td>
<td>mA</td>
</tr>
<tr>
<td>VSS</td>
<td>Operating Free-Air Temperature</td>
<td>-40°C to +85°C</td>
<td></td>
<td></td>
<td>85</td>
<td>°C</td>
</tr>
<tr>
<td>IFN</td>
<td>Input Transition Rate or Fall Rate, VIL from 0V to 2.5V</td>
<td>VCC = 2.0V</td>
<td></td>
<td></td>
<td>10</td>
<td>mA</td>
</tr>
</tbody>
</table>

### DC Electrical Characteristics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Characteristic</th>
<th>Condition</th>
<th>T_A = -40°C to +85°C</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIL</td>
<td>HIGH Level Input Voltage (Note 2)</td>
<td>2.7V ≤ VCC ≤ 3.6V</td>
<td>0.4</td>
<td>0.8</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>IL</td>
<td>LOW Level Input Voltage (Note 2)</td>
<td>2.7V ≤ VCC ≤ 3.6V</td>
<td>0.8</td>
<td>1.6</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>VOH</td>
<td>HIGH Level Output Voltage</td>
<td>2.7V ≤ VCC ≤ 3.6V</td>
<td>4.1</td>
<td>22</td>
<td>V</td>
<td></td>
</tr>
<tr>
<td>VIL</td>
<td>LOW Level Output Voltage</td>
<td>2.7V ≤ VCC ≤ 3.6V</td>
<td>2.2</td>
<td>4.4</td>
<td>V</td>
<td></td>
</tr>
</tbody>
</table>

* Notes: (1) All currents measured at the terminals of the device, with all other inputs grounded.

* These values of V are used to test DC electrical characteristics only.
### DC Electrical Characteristics (continued)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Characteristic</th>
<th>Condition</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{IL}$</td>
<td>Input Leakage Current</td>
<td>$2.7V &lt; V_{CC} &lt; 3.3V$, $</td>
<td>V_{1}</td>
<td>&lt; 5.5V$</td>
<td>$\pm 50\mu A$</td>
</tr>
<tr>
<td>$I_{CC}$</td>
<td>Quiescent Supply Current</td>
<td>$2.7V &lt; V_{CC} &lt; 3.3V$, $V_{1} = 0V$, $V_{IH} = VCC$, $V_{IL} = 0V$</td>
<td>$10\mu A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_{LCC}$</td>
<td>Change in $I_{CC}$ per Input</td>
<td>$2.7V &lt; V_{CC} &lt; 3.3V$, $V_{IH} = VCC$, $V_{IL} = 0V$</td>
<td>$10\mu A$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$500\mu A$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### AC Characteristics ($\tau_{f} = 2.5ns$, $C_{L} = 80pF$, $R_{L} = 100\Omega$)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Waveform</th>
<th>Limits</th>
<th>Min</th>
<th>Max</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{PD}$</td>
<td>Propagation Delay</td>
<td>Input to Output</td>
<td>$\tau_{f}$</td>
<td>$1.5\mu s$</td>
<td>$2\mu s$</td>
<td>ns</td>
</tr>
<tr>
<td>$t_{OS}$</td>
<td>Output-Off-Output On</td>
<td>$\tau_{f}$</td>
<td>$1\mu s$</td>
<td>$1\mu s$</td>
<td>ns</td>
<td></td>
</tr>
</tbody>
</table>

$^2$ $\tau_{f}$ is defined as the absolute value of the difference between the actual propagation delay for any two separate output pulses in the same quiescent input state.

### Dynamic Switching Characteristics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Characteristic</th>
<th>Condition</th>
<th>$V_{CC} = 3.3V$, $V_{1} = 5V$, $V_{IH} = 3.3V$, $V_{IL} = 0V$</th>
<th>$V_{CC} = 3.3V$, $V_{1} = 5V$, $V_{IH} = 3.3V$, $V_{IL} = 0V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{dP}$</td>
<td>Dynamic LOW Output Voltage (Min 4.1)</td>
<td>$t_{dP}$</td>
<td>$0.8\mu s$</td>
<td>$1\mu s$</td>
</tr>
<tr>
<td>$t_{dL}$</td>
<td>Dynamic LOW Output Voltage (Min 4.1)</td>
<td>$t_{dP}$</td>
<td>$0.8\mu s$</td>
<td>$1\mu s$</td>
</tr>
</tbody>
</table>

$^4$ Number of outputs defined as "n" is measured with the n-th output switching from HIGH to LOW or LOW to HIGH. The remaining outputs are measured in the LOW state.

### Capacitive Characteristics

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Condition</th>
<th>Typical</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{IN}$</td>
<td>Input Capacitance</td>
<td>$V_{CC} = 3.3V$, $V_{1} = 5V$ or $V_{CC}$</td>
<td>$7pF$</td>
<td></td>
</tr>
<tr>
<td>$C_{OH}$</td>
<td>Output Capacitance</td>
<td>$V_{CC} = 3.3V$, $V_{1} = 5V$ or $V_{CC}$</td>
<td>$6pF$</td>
<td></td>
</tr>
<tr>
<td>$C_{DH}$</td>
<td>Dynamic Output Capacitance</td>
<td>$10V/Hz$, $V_{CC} = 3.3V$, $V_{1} = 0V$ or $V_{CC}$</td>
<td>$10pF$</td>
<td></td>
</tr>
</tbody>
</table>

$^5$ $V_{CC}$ is specified as $3.3V$, $V_{1}$ is specified as $5V$, and $V_{OH}$ is specified as $5V$ or $V_{CC}$. The capacitance measurements are taken with $V_{CC} = 3.3V$, $V_{1} = 5V$, and $V_{CC} = 3.3V$, $V_{1} = 5V$.
**B-Suffix Series CMOS Gates**

The B Series logic gates are constructed with P and N channel enhancement mode devices in a single monolithic structure (Complementary MOS). Their primary use is where low power dissipation and/or high noise immunity is desired.

- Supply Voltage Range = 3.0 Vdc to 18 Vdc
- All Outputs Buffered
- Capable of Driving Two Low-power TTL Loads or One Low-power Schottky TTL Load Over the Rated Temperature Range.
- Double Diode Protection on All Inputs Except: Triple Diode Protection on MC14011B and MC14081B
- Pin–for–Pin Replacements for Corresponding CD4000 Series B Suffix Devices (Exceptions: MC14068B and MC14078B)

**ORDERING INFORMATION**

<table>
<thead>
<tr>
<th>Model Code</th>
<th>Package</th>
<th>Package Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC14001B</td>
<td>Plastic</td>
<td>CASE 632</td>
</tr>
<tr>
<td>MC14002B</td>
<td>Ceramic</td>
<td>CASE 646</td>
</tr>
<tr>
<td>MC14011B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14012B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14023B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14025B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14068B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14071B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14072B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14073B</td>
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<td>CASE 751A</td>
</tr>
<tr>
<td>MC14075B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14078B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14081B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
<tr>
<td>MC14082B</td>
<td>SOIC</td>
<td>CASE 751A</td>
</tr>
</tbody>
</table>

**MAXIMUM RATINGS** *(Voltages Referenced to VSS)*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>VDD</td>
<td>DC Supply Voltage</td>
<td>– 0.5 to + 18.0</td>
<td>V</td>
</tr>
<tr>
<td>VIL, VOH</td>
<td>Input or Output Voltage (DC or Transient)</td>
<td>– 0.5 to VDD + 0.5</td>
<td>V</td>
</tr>
<tr>
<td>IIL, IOS</td>
<td>Input or Output Current (DC or Transient), per Pin</td>
<td>± 10</td>
<td>mA</td>
</tr>
<tr>
<td>Pd</td>
<td>Power Dissipation, per Package†</td>
<td>500</td>
<td>mW</td>
</tr>
<tr>
<td>Tstg</td>
<td>Storage Temperature</td>
<td>– 65 to + 150</td>
<td>°C</td>
</tr>
<tr>
<td>TLE</td>
<td>Lead Temperature (8–Second Soldering)</td>
<td>260</td>
<td>°C</td>
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*Maximum Ratings are those values beyond which damage to the device may occur.
†Temperature Derating:
- Plastic "P" and D/DW" Packages: – 7.0 mW/°C From 65°C To 125°C
- Ceramic "L" Packages: – 12.0 mW/°C From 100°C To 125°C

This device contains protection circuitry to guard against damage due to high static voltages or electric fields. However, precautions must be taken to avoid applications of any voltage higher than maximum rated voltages to this high-impedance circuit. For proper operation, VIL and VOH should be constrained to the range VSS ≤ (VIL or VOH) ≤ VDD.

Unused inputs must always be tied to an appropriate logic voltage level (e.g., either VSS or VDD). Unused outputs must be left open.
### Pin Assignments

<table>
<thead>
<tr>
<th>MC14001B</th>
<th>MC14002B</th>
<th>MC14001B</th>
<th>MC14002B</th>
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<tbody>
<tr>
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<td>Dual 4-Input NOR Gate</td>
<td>Quad 2-Input NAND Gate</td>
<td>Dual 4-Input NAND Gate</td>
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<tr>
<td>1k 1x</td>
<td>1k 1x</td>
<td>1k 1x</td>
<td>1k 1x</td>
</tr>
<tr>
<td>2k 2x</td>
<td>3k 3x</td>
<td>4k 4x</td>
<td>5k 5x</td>
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<tr>
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<td>OUTx</td>
<td>OUTx</td>
<td>OUTx</td>
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<td>VSS</td>
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<td>E-Input NAND Gate</td>
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<td>1k 1x</td>
<td>1k 1x</td>
<td>1k 1x</td>
</tr>
<tr>
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<td>4k 4x</td>
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<td>OUTx</td>
</tr>
<tr>
<td>INx</td>
<td>INx</td>
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<td>E-Input NOR Gate</td>
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<td>2k 2x</td>
<td>3k 3x</td>
<td>4k 4x</td>
<td>5k 5x</td>
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<td>1k 1x</td>
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<tr>
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*NC = NO CONNECTION*
## Electrical Characteristics (Voltages referenced to VDD)

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<tr>
<th>Characteristic</th>
<th>Symbol</th>
<th>VDD (Vdc)</th>
<th>-55°C</th>
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<th>95°C</th>
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<td>VIL (Max)</td>
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<td>14.95</td>
<td>14.95</td>
<td>14.95</td>
<td>14.95</td>
<td>Vdc</td>
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<tr>
<td>VIL (Typ)</td>
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<td>4.95</td>
<td>5.0</td>
<td>4.95</td>
<td>Vdc</td>
</tr>
<tr>
<td>VIL (Unit)</td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>VIL (Note)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VPP (Min)</td>
<td>5.0</td>
<td>4.95</td>
<td>4.95</td>
<td>5.0</td>
<td>4.95</td>
<td>Vdc</td>
</tr>
<tr>
<td>VPP (Max)</td>
<td>14.06</td>
<td>14.95</td>
<td>14.95</td>
<td>14.95</td>
<td>14.95</td>
<td>Vdc</td>
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<tr>
<td>VPP (Typ)</td>
<td>5.0</td>
<td>4.95</td>
<td>4.95</td>
<td>5.0</td>
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<td>VPP (Unit)</td>
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<td></td>
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<tr>
<td>VPP (Note)</td>
<td></td>
<td></td>
<td></td>
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**Notes:**
- "VIL" is not to be used for design purposes but is intended as an indication of the IC's potential performance.
- "VPP" is the voltage at which the input pin should be held to ensure proper operation.
- The values are given for the typical characteristics, except at 25°C.
- To calculate the total supply current, add: 1 = 15 µA typ, 1 = 35 µA max, 1 = 500 µA max.

**Formulas:**
- 1 = (0.5 mA per pin) x (number of gates)
- 1 = (2.5 mA per pin) x (number of gates)
- 1 = (4.0 mA per pin) x (number of gates)
## Appendix C • Manufacturers' Data Sheets

### Switching Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Symbol</th>
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<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
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</thead>
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<tr>
<td>Output Rise Time, All B-Serie Gates</td>
<td>( t_{RL} )</td>
<td>5.3</td>
<td>16</td>
<td>50</td>
<td>100</td>
<td>ns</td>
</tr>
<tr>
<td></td>
<td>( t_{LH} )</td>
<td>15</td>
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<td></td>
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<tr>
<td>Output Fall Time, All B-Serie Gates</td>
<td>( t_{FH} )</td>
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<td>50</td>
<td>100</td>
<td>ns</td>
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<tr>
<td></td>
<td>( t_{HL} )</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Propagation Delay Time</td>
<td>( t_{PLH}, t_{PHL} )</td>
<td>5.3</td>
<td>15</td>
<td>40</td>
<td>60</td>
<td>ns</td>
</tr>
</tbody>
</table>

*The format shown are for the given characteristics only at 25°C.*

*Data labeled "Typ" is not to be used for design purposes but is intended as an indication of the IC's potential performance.*

---

![Switching Time Test Circuit and Waveforms](image)

---

1. All unused inputs of AND, NAND gates must be connected to \( V_{SS} \).
2. All unused inputs of OR, NOR gates must be connected to \( V_{DD} \).
APPENDIX D

Handling Precautions for CMOS

HANDLING PRECAUTIONS

All CMOS devices are vulnerable to static damage. The gate oxide for MOSFET devices is about 40 Å thick and is prone to static damage unless the proper precautions are observed. CMOS devices require a high level of ESD (Electrostatic Sensitive) damage, so they must be carefully handled and stored. When handling CMOS devices, follow these precautions:

1. CMOS devices should be handled in a static-free environment or by using static-dissipative wrist straps.
2. CMOS devices should be stored in anti-static bags or containers.
3. CMOS devices should be handled with clean gloved hands or static-dissipative gloves.
4. CMOS devices should be grounded to the local grounded circuit before handling.

DESIGN GUIDELINES

CMOS devices are sensitive to electrostatic discharge (ESD). To protect CMOS devices from ESD damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid transporting CMOS devices in metal containers.
3. Store CMOS devices in a static-free environment.
4. Use proper handling techniques when handling CMOS devices.
5. Use proper test equipment when testing CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to electrical overstress (EOS) and electromechanical overstress (EMO). To protect CMOS devices from EOS and EMO damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to transient voltage (TVS) and transient current (TIV). To protect CMOS devices from TVS and TIV damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to power supply (PS) and power supply ripple (PSR). To protect CMOS devices from PS and PSR damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to thermal (TH) and thermal cycle (THC). To protect CMOS devices from TH and THC damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to mechanical (MCH) and mechanical shock (MSH). To protect CMOS devices from MCH and MSH damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to electrical (ELE) and electrical stress (EST). To protect CMOS devices from ELE and EST damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to environmental (ENV) and environmental stress (EST). To protect CMOS devices from ENV and EST damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.

CMOS devices are sensitive to mechanical (MCH) and mechanical shock (MSH). To protect CMOS devices from MCH and MSH damage, follow these guidelines:

1. Use proper packaging for CMOS devices.
2. Avoid exposing CMOS devices to high temperatures.
3. Use proper handling techniques when handling CMOS devices.
4. Use proper test equipment when testing CMOS devices.
5. Use proper grounding when using CMOS devices.
6. Use proper packaging when shipping CMOS devices.
7. Use proper grounding when using CMOS devices.
APPENDIX D • Handling Precautions for CMOS

12. The maximum current should be small during bonding or cutting operations.
13. Wipe areas around bonding pads with an approved cleaner.
14. Before attaching bonding wires, the bonding area should be cleaned with a suitable cleaner.
15. Do not use cleaning fluids that may cause corrosion or damage the CMOS devices.

RECOMMENDED FOR READING:


FIGURE 2 - ATTENTION WHEN HANDLING SMA AND SUBJECT TO SUSCEPTIBILITY

FIGURE 5 - TYPICAL MICROPLATING WORK STATION
# EPROM Data For A Digital Function Generator

Included in Appendix E:

- **E.1** A complete set of EPROM data for the EPROM-based digital function generator described in Section 13.4. The file can be used to program a standard EPROM or as an initialization file for an LPM_ROM component in MAX+PLUS II. (LPM_ROM can only be used with the FLEX 10K device on the Altera UP-1 board.)
- **E.2** A program written in ANSI C to generate an EPROM record file (Intel format).
- **E.3** A copy of the generated record file.

## E.1 EPROM Data

00 is the maximum negative voltage of a waveform, FF is maximum positive, and 80 is the zero-crossing point.

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### TRIANGLE

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### SAWTOOTH

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<tr>
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**E.2 C Program**

This program is also on the accompanying CD in the file
\`\`Student_Files\EPROM\EPROM.C.\`

```c
/*     Hex File Generator
*     Written by: Ronan Capina and Robert Dueck
*
*     This program is to create a hex file in a specific EPROM record format.
*     The EPROM is addressed in blocks of 256 bytes (8 address lines) by an 8 bit
*     counter to create a digital image of one of several waveform outputs.
*     Two additional address bits select one of four output functions.
*     When the EPROM data are run through a D/A Converter, they create an
*     analog waveform running at the frequency of the counter divided by 256.
*     The waveforms are sine, square, triangle, and sawtooth.
*     The record format is as follows. (Spaces are inserted only for clarity.
*     The actual record must have NO spaces.)
*
*     : 10 0080 00 AF5F67F0602703E0322CFA92007780C3 61
*     (:Record Length = 10hex = 16dec)
*     (Address = 0080hex; location in EPROM of first data byte in record)
*     (Record type = 00 = data)
*     (16 data bytes = 32 hex digits)
*     (Checksum; Record Length + Address High byte + Address Low byte
*     + Record type + data bytes + Checksum = 00, after discarding carry)
*
*     An END record is also required, having a similar format, except that
*     Record Type = 01 = END and there are no data bytes. Checksum is still
*     required.
*     eg. :00000001FF
*/
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <string.h>
#include <conio.h>

void Sawtooth(int);
void triangle(int);
void Square(int);
int AddrByte(int Addr);
int Chksum(int sum);
void sine(int);
char *HexString(int value);
char *Hex = HULL;
int Ampl;

const double Pi = 3.141592654;

int main(void)
{
    FILE *fp;
    int Fcn =1, Linenum, sum, Byte, Addr = 0;
    char Record[256];
    clrscr();
```

/*   Hex file is written to c:\eprom\eprom.hex
*    To change the file name, modify the following line to:
*     if (!(fp = fopen(“c:\\YourDirectoryName\\YourFile.hex”, “w+t”)))
*     {
*         printf(“Error opening output file.\r\n”);
*         exit(1);
*     } else
*     printf(“File opened successfully. \r\n”);
*/
while (Fcn != 5) {   /* Create records for 4 functions */
    /* Each function has 16 lines of data */
    for (Linenum = 1; Linenum <= 16; Linenum++) {   /* Create record and address information */
        if (Addr < 16)
            sprintf(Record, "010000%x", Addr);
        else
            if (Addr < 256)
                sprintf(Record, "0100%x", Addr);
        else
            sprintf(Record, "010%x", Addr);
    strcat(Record, "00");
    /* Accumulate sum for calculating checksum */
    sum = 16 + AddrByte(Addr);
    /* Calculate byte values for selected function */
    for (Byte = 1; Byte <= 16; Byte++) {
            if (Fcn == 1)
                sine(Addr % 256);
            else if (Fcn == 2)
                Square(Addr % 256);
            else if (Fcn == 3)
                triangle(Addr % 256);
            else
                Sawtooth(Addr % 256);
        /* Append calculated byte value (amplitude) to the record
         and update checksum accumulator */
        strcat (Record, HexString(Ampl));
    sum = sum + Ampl;
    Addr++;
    }
    strcat (Record, HexString(Chksum(sum)));
    fprintf(fp, "%s\r\n", Record);
    if (Fcn++ >= 5)
        fprintf(fp, ":000000001FP\r\n”);
    fclose(fp);
    return(0);}
int AddrByte(int Addr)
{
    if (Addr < 256)
        return Addr;
    else
        return (((int)(Addr / 256)) + (Addr % 256));
}

int Chksum(int sum)
{
    int IntRem = sum % 256;
    if (IntRem == 0)
        return 0;
    else
        return (256 - IntRem);
}

char *HexString(int value)
{
    if (value < 16) {
        sprintf(Hex, "0%x", value);
        return Hex;
    } else if (value > 255)
        return "FF";
    else {
        sprintf(Hex, "%x", value);
        return Hex;
    }
}

void Sawtooth(int Addr)
{
    Ampl = Addr;
    return;
}

void sine(int Addr)
{
    double angle = (((float)Addr) * 2 * Pi) / 256;
    Ampl = ((int)((sin(angle) * 128) + 128));
    if (Ampl > 255)
        Ampl = 255;
    return;
}

void Square(int Addr)
{
    if (Addr < 128)
        Ampl = 255;
    else
        Ampl = 0;
    return;
}
void triangle(int Addr)
{
    if (Addr < 64)
        Ampl = 128 + (2 * Addr);
    else if ((Addr >= 64) && (Addr < 192))
        Ampl = 256 - 2 * (Addr - 63);
    else
        Ampl = 2 * (Addr - 191);
    return;
}

E.3 Resultant Record File

:1000000808386898C8F9295989CFA2A5A8ABAE81
:10010000B0B3B69BCBFC1C4C7C9CCCED1D3D5D893
:10020000DADCEE0E2E4E6E8EABCEDEEFF0F2F3F54C
:10030000F6F7F8F9FAFBCFCFDFEFFFFFFFFFFF01
:10040000FFFFFFFFFFFFEFFDFDFFCFBFAF9F8F7E8
:10050000F6F5F3F2F0FEDECEAEAB8E6E4E2E0EDC00
:10060000DAD8D5D3D1CECC9C7C41BCBCB9B6B319
:10070000B0AEABA8A5A29F9C98959288C898681E1
:100800007FC797673706D6A6763605D5A575451EF
:10090004F4C494643403E3B38363332E2C2A27BD
:100A00002523211F1D1B1917515131210F0DC0AE4
:100B000090807605040303020101000000000F
:100C0000000000000000000101020303040506070808
:100D000090AC0D0F01012131517191B1D1F213D0
:100E000025272A2E313336383B3E404346494C97
:100F00004F51545755A5D6036767A6D707376797CAF
:10100000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
:10110000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFEF
:10120000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFDF
:10130000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFCF
:10140000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFBF
:10150000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFAF
:10160000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF9F
:10170000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF8F
:101800000000000000000000000000000006F
:101900000000000000000000000000000005F
:101A00000000000000000000000000000004F
:101B00000000000000000000000000000003F
:101C00000000000000000000000000000002F
:101D00000000000000000000000000000001F
:101E00000000000000000000000000000000F
:101F00000000000000000000000000000000F
:102000008082846888A8CE9092946989A9C98FE
:10210000A0A2A4A6A8AAACABE0B2B4B6B8BABCBEED
:10220000C0CC224C6C8CACCCED0D2D4D6D8DADCADED
:10230000BE2E4E6E8EABCEDEEF0F2F4F6F8FAPCFCFCE
:10240000FECPCAF8F6F4F2F0EECEAEAB8E6E4E2E0BE
:10250000DECDAD8D6D4D2D0CECCCCA8C6C4C20AE
:10260000EECBAB8B6B4B2B0AECAAAA8A6A42A09E
:10270000E9C9A98969492908E88A8886848280E
:10280007E7C7A78767472706E6C6A68666462607E
:1002A0003E3C3A38363432302E2C2A28262422205E
:1002B0001E1C1A181614121100E0C0A08060402004E
:1002C00020406080A0C0E10121416181A1C1E201E
:1002D000222232282A2C2E30323436383A3C3E400E
:1002E0004244446484A4C4E50525456585A5C5E60FE
:1002F000626466686A6C6E70727476787A7C7E80EE
:1003000000102030405060708090A0B0C0D0E0F75
:10031000101112131415161718191A1B1C1D1E1F65
:10032000202122232425262728292A2B2C2D2E2F55
:10033000303132333435363738393A3B3C3D3E3F45
:10034000404142434445464748494A4B4C4D4E4F35
:10035000505152535455565758595A5B5C5D5E5F25
:10036000606162636465666768696A6B6C6D6E6FF5
:10037000707172737475767778797A7B7C7D7E7F05
:10038000808182838485868788898A8B8C8D8E8FF5
:10039000909192939495969798999A9B9C9D9E9F5
:1003A000A0A1A2A3A4A5A6A7A8A9AABACAADAEDAFD5
:1003B000B0B1B2B3B4B5B6B7B8B9BABBCBDBBFC5
:1003C000C0C1C2C3C4C5C6C7C8C9CACCCDCCDCECFB5
:1003D000D0D1D2D3D4D5D6D7D8D9DADDDCDDDEDFA5
:1003E000E0E1E2E3E4E5E6E7E8E9EABEBCDEDEEEFF95
:1003F000F0F1F2F3F4F5F6F7F8F9FABFCFDFEFFF85
:00000001FF
Answers to Selected Odd-Numbered Problems

Chapter 1

1.1 Analog quantities:
   a. Water temperature at the beach;
   b. weight of a bucket of sand;
   e. height of a wave;
   Digital quantities:
   c. grains of sand in a bucket;
   d. waves hitting the beach in one hour;
   f. people in a square mile.
   Generally, any quantity that can be expressed as “the number of . . .” is digital.

1.3
   a. 4; b. 8; c. 25; d. 6; e. 21; f. 29; g. 59; h. 93; i. 33; j. 185

1.5 101, 110, 111, 100

1.7 16

1.9 a. 0.625; b. 0.375; c. 0.8125

1.11 a. 0.11; b. 0.101; c. 0.0011; d. 0.101001; e. 1.11; f. 11.111100;
g. 100011.1101011100001. . . (nonrepeating)

1.15 9F7, 9F8, 9F9, 9FA, 9FB, 9FC, 9FD, 9FE, 9FF, A00, A01, A02, A03

1.17 a. 2C5; b. 761; c. FFF; d. 1000; e. 2790; f. 7D00; g. 8000

1.19 a. 5E86; b. B6A; c. C5B; d. 6BC4; e. 15785; f. 198B7; g. 28000

1.21 Periodic: b., c., e. Each of these waveforms repeats itself in a fixed period of time. (Note that waveform b. may not immediately appear to be periodic. However, if we count the sequence of short pulse, short space, medium pulse, medium space, short pulse, long space, we will find that each repetition of this sequence takes the same time.) Aperiodic: a., d. Neither of these waveforms repeats in a fixed period of time. Waveform a. has three equally-spaced pulses of equal width, but this pattern does not repeat in the time shown. Waveform d. has pulses of equal duration, spaced at increasing (i.e. unequal) intervals.

1.23 From the graph in Figure 1.14, read the times corresponding to the 10%, 50%, and 90% values of the pulse on both leading and trailing edges.

Leading edge: 10%: 5 µs   Trailing edge: 90%: 40 µs
50%: 7.5 µs     50%: 45 µs
90%: 10 µs      10%: 50 µs

Pulse width: 50% of leading edge to 50% of trailing edge.

\[ t_w = 45 \mu s - 7.5 \mu s = 37.5 \mu s \]

Rise time: 10% of rising edge to 90% of rising edge.

\[ t_r = 10 \mu s - 5 \mu s = 5 \mu s \]

Fall time: 90% of falling edge to 10% of falling edge.

\[ t_f = 50 \mu s - 40 \mu s = 10 \mu s \]

Chapter 2

2.1 See Figure ANS2.1.

A
a. Distinctive Shape

\[ Y = \overline{A} \]

b. Rectangular Outline

\[ Y = \overline{A} \]

FIGURE ANS2.1
2.3 See Figure ANS2.3.

![Figure ANS2.3](image)

2.5 \( N \) is HIGH if \( J \) OR \( K \) OR \( L \) OR \( M \) IS HIGH. See Table ANS2.5.

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<th>( J )</th>
<th>( K )</th>
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2.7 The switches must be connected in parallel. See Figure ANS2.7.

![Figure ANS2.7](image)

2.9 Active LOW. When the switch is pressed, it generates a logic LOW.

2.11 The anode must be at a higher voltage than the cathode by a specified amount.

2.13 See Figure ANS2.13.

![Figure ANS2.13](image)

2.15 a. Output \( Y \) is LOW when \( A \) OR \( B \) OR \( C \) OR \( D \) are HIGH. The truth table is shown in Table ANS2.15.

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b. \( Y = A + B + C + D \)

c. See Figure ANS2.15

![Figure ANS2.15](image)
2.19 Required gate is a 2-input AND.

2.21 XNOR.

2.23 Output is HIGH if an odd number of inputs is HIGH.

2.25 a. and c. The attributes of shape, input level, and output level are all different between these two symbols.

2.27 See Figure ANS2.27.

2.29 See Figure ANS2.29.

2.31 A HIGH is required to enable the AND gate. This allows the lamp to flash.

2.33 No. An XOR gate has no inhibit state. The lamp always flashes.

2.35 Transistor-Transistor Logic (TTL) and Complementary Metal-Oxide-Semiconductor (CMOS). Typically, TTL can drive higher-current loads. CMOS has more flexible power supply requirements and uses less power.

2.37 Low power Schottky TTL: 74LS02; CMOS: 4001B; High-speed CMOS: 74HC02. NANDs and NORs are differentiated by the last two digits in their part numbers.

Chapter 3

3.1 a. \( Y = ABC; \)

b. \( X = PQ + RS; \)

c. \( M = HJKL; \)

d. \( A = W + X + Y + Z; \)

e. \( Y = (A + B)(C + D); \)

f. \( Y = (A + B)(C + D); \)

g. \( Y = (A + \overline{B})(C + \overline{D}); \)

h. \( X = \overline{P Q} + R \overline{S}; \)

i. \( X = \overline{P Q} + R \overline{S}; \)
3.3  
See Figure ANS3.3.

Boolean expressions:

a.  \( X = \overline{T} + \overline{U} + V + \overline{W} \);

e.  \( Y = AB + AC \);

f.  \( Y = \overline{(A + B)(A + C)} \);

h.  \( Y = AB + BC + AC \);

i.  \( Y = (A + B) + (B + C) + (\overline{A} + \overline{C}) = 1 \);

j.  \( Y = (A + B + C + D)\overline{AB} = AB\overline{C} \)

3.5  
\[
Y = D_3D_2D_1D_0 + D_3\overline{D_2D_1D_0} + D_1D_2\overline{D_1D_0} + D_1\overline{D_2D_1D_0} + D_1D_2\overline{D_1D_0} + D_1D_2D_0 + D_3D_2D_1.
\]

For a circuit that indicates that exactly three inputs are HIGH. If at least three inputs are HIGH, the equation simplifies to \( Y = D_2D_1D_0 + D_1D_1D_0 + D_1D_2D_0 + D_1D_2D_1 \). The latter circuit is shown in Figure ANS3.5.

3.7  
e.  \( Y = (A + C) + \overline{B}C \);

f.  \( Y = A\overline{B}C + C \);

g.  \( \overline{(ABD)}(B + \overline{C}) + \overline{A}C \);

h.  \( Y = (\overline{A})(\overline{A}\overline{C})(BC) \);

i.  \( Y = (A + \overline{B}) + (\overline{A}C)(BC) \)

All of the above equations could be simplified further with Boolean algebra.
3.9  a. | T | U | V | W | X
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3.11  SOP: \( Y = \overline{A} \overline{B} \overline{C} + \overline{A} \overline{B} C + \overline{A} B \overline{C} + \overline{A} B C \)
POS: \( Y = (A + B + C)(\overline{A} + B + \overline{C})(A + \overline{B} + C) \)
\((A + \overline{B} + C)\)

See Figure ANS3.11
3.13  SOP: \( Y = \overline{A} B C + \overline{A} B \overline{C} + A \overline{B} C + A B \overline{C} + A B C \)  
POS: \( Y = (A + B + C)(A + B + \overline{C})(\overline{A} + B + C) \)  
See Figure ANS3.13

3.15  \( Y = (A + B)(\overline{A} + \overline{B}) \)  
See Figure ANS3.15.

3.17  \( Y = (A + B + C)\overline{D} = A\overline{D} + BD + CD \)

3.19  a. \( Y = AB + C \); b. \( Y = C \);  
c. \( J = K \); d. \( S = 0 \);  
e. \( S = T \); f. \( Y = B \overline{C} D + A \overline{B} F + \overline{C} F \)

3.21  a. \( Y = \overline{A} + \overline{B} \);  
b. \( Y = C D + \overline{C} \overline{D} + A B \);  
c. \( K = MN + ML \)

3.23  SOP: \( Y = \overline{A} C + B \overline{C} \); POS: \( Y = (\overline{A} + B)\overline{C} \)

3.25  \( Y = AD + BC \)

3.27  \( Y = AD + CD + BCD \)

3.29  \( Y = \overline{A} B C D + A B C D + BC \)

3.31  \( Y = \overline{A} B C + A B \overline{C} + \overline{A} B D + A \overline{D} \)  
See Figure ANS3.31.

3.33  \( Y = \overline{A} + B + CD \)  
See Figure ANS3.33.

3.35  \( Y = AD + \overline{B}C \)  
See Figure ANS3.35.

3.37  \( Y = AB \overline{C} D + \overline{A}D + CD \)  
See Figure ANS3.37.
3.39 \( Y = \overline{AB} \overline{CD} + \overline{AB} + B\overline{D} \). See Figure ANS3.39.

3.41 \( Y = D \). See Figure ANS3.41.

3.43 \( Y = \overline{AC} + \overline{AB} + BCD \). See Figure ANS3.43.

3.45 \( Y = (\overline{A} + C)(A + \overline{C})(A + B + \overline{D}) \). See Figure ANS3.45.
Chapter 4

4.1 Advantages of programmable logic: User is not restricted to standard digital functions from a device manufacturer; only required functions need be implemented; package count can be reduced; design can be reprogrammed or reconfigured without changing the circuit board.

4.3 PAL (Programmable Array Logic); GAL (Generic Array Logic); EPLD (Erasable Programmable Logic Device); FPGA (Field-Programmable Gate Array)

4.5 A design file in MAX+PLUS II is a single file with descriptive information, such as a schematic or text in a hardware description language. A project is a collection of files associated with a design entered in MAX+PLUS II.

4.7 Primitives—Basic functional blocks, such as logic gates, used in PLD design files.

4.9 The gdf for the 4-channel demultiplexer circuit is shown in Figure ANS4.9.

4.11 The gdf for the half adder is shown in Figure ANS4.11a. The default symbol for the half adder is shown in Figure ANS4.11b.

4.13 The gdf for the full adder (hierarchical design) is shown in Figure ANS4.13.

4.15 AHDL—Altera Hardware Description Language

VHDL—VHSIC Hardware Description Language

VHSIC—Very High Speed Integrated Circuit

4.17 The two minimum VHDL structures are an entity declaration and an architecture body. The entity describes the input and output terminals of the design. The architecture defines the relationship between the inputs, outputs, and internal signals of the design.

4.19 A VHDL port of mode OUT can be used as an output only. A port of mode BUFFER is an output that can also be fed back into the design entity for use by other functions within the entity.
FIGURE ANS4.9

FIGURE ANS4.11

a. Half Adder Circuit

b. Symbol

FIGURE ANS4.13
4.21 — mux4.vhd
— 4-to-1 multiplexer
— Directs one of four input signals (d0 to d3) to output,
— depending on status of select bits (s1, s0).

— STD_LOGIC types
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

— Define inputs and outputs
ENTITY mux4 IS
PORT( d0, d1, d2, d3 : IN STD_LOGIC; — data inputs
      s: IN STD_LOGIC_VECTOR (1 downto 0); — select inputs
      y: OUT STD_LOGIC);
END mux4;

— Define i/o relationship
ARCHITECTURE mux4to1 OF mux4 IS
BEGIN
— Choose a signal assignment for y
— based on binary value of d
— Default case: output LOW
WITH s SELECT
  y <= d0 WHEN "00",
      d1 WHEN "01",
      d2 WHEN "10",
      d3 WHEN "11",
      '0' WHEN others;
END mux4to1;

4.23 — dmux4.vhd
— 4-channel demultiplexer
— Directs input to one of four outputs,
— depending on state of select inputs (s1, s0)

— Standard VHDL models
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

— Define inputs and outputs
ENTITY dmux4 IS
  PORT( d, s1, s0 : IN  STD_LOGIC;
        y0, y1, y2, y3 : OUT STD_LOGIC);
END dmux4;

— Define i/o relationship
ARCHITECTURE four_ch_dmux OF dmux4 IS
BEGIN
— Concurrent Signal Assignment
  y0  <=  (not s1) and (not s0) and d;
  y1  <=  (not s1) and ( s0) and d;
  y2  <=  ( s1) and (not s0) and d;
  y3  <=  ( s1) and ( s0) and d;
END four_ch_dmux;

4.25 — half_add.vhd
— Half Adder
— Adds two bits, A and B and produces SUM and CARRY outputs

— Standard VHDL models
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

-- Define inputs and outputs
ENTITY half_add IS
  PORT(
    a, b : IN STD_LOGIC;
    sum, carry : OUT STD_LOGIC);
END half_add;

-- Define relationship between A, B and SUM, CARRY
ARCHITECTURE half_adder OF half_add IS
BEGIN
  -- Concurrent Signal Assignment
  sum   <= a xor b;
  carry <= a and b;
END half_adder;

4.27 The gdf for the full adder with VHDL half adder components is the same as Figure ANS4.13.

Chapter 5
5.1 $Y = D_3D_2D_1D_0$; $Y = D_3D_2D_1D_0$
5.3 See Figure ANS5.3.
5.5  a. 32;  b. 64;  c. 256;  $m = 2^n$.

5.7  A selected signal assignment assigns an output value based on alternative input values. Each choice is independent of the others. A conditional signal assignment evaluates one input choice and assigns a value to an output if true. Otherwise, a second choice is evaluated, then a third, and so on. Low-priority choices are assigned only if higher-priority alternatives are false. This linked conditional structure tends to generate a more “serial” hardware, as opposed to the more “parallel” structure of the selected signal assignment. The selected signal assignment is preferable because it is generally results in a better use of chip resources and is more efficient.

5.9  See Figures ANS5.9a and ANS5.9c.
5.11 See Figure ANS5.11.

5.13

\[ a = \overline{D_3}D_2\overline{D_1}D_0 + \overline{D_3}D_2D_1\overline{D_0} + D_3D_2D_1D_0 + D_3D_2D_1D_0 \]
\[ b = \overline{D_3}D_2\overline{D_1}D_0 + D_3D_2D_1 + D_3D_2\overline{D_0} + D_3D_2D_0 + D_2D_1D_0 \]
\[ c = D_3D_2D_1D_0 + D_3D_2D_1D_0 + D_3D_2D_1D_0 \]
\[ d = D_3D_2D_1D_0 + D_3D_2D_1D_0 + D_3D_2D_1D_0 + D_3D_2D_1D_0 \]
\[ e = D_3D_0 + D_3D_2D_1 + D_3D_2D_0 \]
\[ f = \overline{D_3}D_2D_1 + D_3\overline{D_2}D_1 + D_3D_1D_0 + D_3D_2\overline{D_1}D_0 \]
\[ g = D_3D_2D_1 + D_3D_2D_1D_0 + D_3D_2D_1D_0 \]

5.17

a. 1000;  
   b. 1001;  
   c. 1001

5.19 See Figures ANS5.19a and b.

5.21 See Figure ANS5.21.
FIGURE ANS5.19A
FIGURE ANS5.19B

FIGURE ANS5.21
5.23 See Figure ANS5.23
See page 787 for Truth Tables.

5.25 \[ Y = \overline{S_2}S_1S_0D_0 + \overline{S_2}S_1S_0D_1 + \overline{S_2}S_1S_0D_2 + \overline{S_2}S_1S_0D_3 + S_2S_1S_0D_4 + S_2S_1S_0D_5 + S_2S_1S_0D_6 + S_2S_1S_0D_7 \]
\[ = \overline{1} \cdot \overline{0} \cdot \overline{1} \cdot D_0 + \overline{1} \cdot \overline{0} \cdot 1 \cdot D_1 + 1 \cdot \overline{0} \cdot \overline{1} \cdot D_2 + 1 \cdot \overline{0} \cdot 1 \cdot D_3 + 1 \cdot \overline{1} \cdot \overline{1} \cdot D_4 + 1 \cdot 0 \cdot \overline{1} \cdot D_5 + 1 \cdot \overline{1} \cdot \overline{1} \cdot D_6 + 1 \cdot 0 \cdot 1 \cdot D_7 \]
\[ = 0 \cdot D_0 + 0 \cdot D_1 + 0 \cdot D_2 + 1 \cdot D_3 + 1 \cdot D_4 + 1 \cdot D_5 + 1 \cdot D_6 + 0 \cdot D_7 \]
\[ = D_5 \]

FIGURE ANS5.23

5.27

-- quad8to1.vhd
-- Eight-channel 4-bit multiplexer
-- One of eight sets four inputs
-- (d03..d00), (d13..d10), (d23..d20), (d33..d30),
-- (d43..d40), (d53..d50), (d63..d60), (d73..d70)
-- is directed to an output (y), based on the status of three
-- select inputs (s2, s1, s0).

ENTITY quad8to1 IS
PORT(
    s  : IN  INTEGER RANGE 0 to 7;
    d0 : IN  BIT_VECTOR (3 downto 0);
    d1 : IN  BIT_VECTOR (3 downto 0);
    d2 : IN  BIT_VECTOR (3 downto 0);
    d3 : IN  BIT_VECTOR (3 downto 0);
    d4 : IN  BIT_VECTOR (3 downto 0);
    d5 : IN  BIT_VECTOR (3 downto 0);
    d6 : IN  BIT_VECTOR (3 downto 0);
    d7 : IN  BIT_VECTOR (3 downto 0);
    y  : OUT BIT_VECTOR (3 downto 0));
END quad8to1;
ARCHITECTURE mux8 OF quad8to1 IS
BEGIN
  -- Selected Signal Assignment
  MUX4: WITH s SELECT
         y <=
          d0 WHEN 0,
          d1 WHEN 1,
          d2 WHEN 2,
          d3 WHEN 3,
          d4 WHEN 4,
          d5 WHEN 5,
          d6 WHEN 6,
          d7 WHEN 7;
END mux8;

FIGURE ANS5.27

The simulation of this circuit is shown in Figure ANS5.27.

5.29 -- oct4to1.vhd
-- Four-channel 8-bit multiplexer
-- One of four sets eight inputs
-- (d07..d00), (d17..d10), (d27..d20), or (d37..d30)
-- is directed to an output (y), based on the status of two
-- select inputs (s1, s0).

ENTITY oct4to1 IS
  PORT(
      s : IN INTEGER RANGE 0 to 3;
      d0 : IN BIT_VECTOR (7 downto 0);
      d1 : IN BIT_VECTOR (7 downto 0);
      d2 : IN BIT_VECTOR (7 downto 0);
      d3 : IN BIT_VECTOR (7 downto 0);
      y : OUT BIT_VECTOR (7 downto 0));
END oct4to1;
END oct4tol;

ARCHITECTURE mux4 OF oct4tol IS
BEGIN
  -- Selected Signal Assignment
  MUX8: WITH s SELECT
    y <= d0 WHEN 0,
        d1 WHEN 1,
        d2 WHEN 2,
        d3 WHEN 3;
END mux4;

FIGURE ANS5.29

The simulation is shown in Figure ANS5.29.

5.31 ENTITY mux_8ch IS
PORT(
  sel : IN BIT_VECTOR (2 downto 0);
  d   : IN BIT_VECTOR (7 downto 0);
  y   : OUT BIT);
END mux_8ch;

ARCHITECTURE a OF mux_8ch IS
BEGIN
  -- Selected Signal Assignment
  MUX8: WITH sel SELECT
    y <= d(0) WHEN "000",
        d(1) WHEN "001",
        d(2) WHEN "010",
        d(3) WHEN "011",
        d(4) WHEN "100",
        d(5) WHEN "101",
        d(6) WHEN "110",
        d(7) WHEN "111";
END a;

An 8-to-1 MUX can be easily extended to a 16-bit device by adding one select input, eight data inputs, and eight
FIGURE AN5.37A
5.35 00110011; 00001111
5.37 See Figure ANS5.37.

5.39 An analog switch can transmit a range of positive and negative voltages, not just 0V and 5V.
5.41 See Figure ANS5.41
5.43 See Figure ANS5.43. The glitch in the simulation is caused by propagation delay.
5.45 \[ \text{AEQB} = (A_5 \oplus B_5)(A_4 \oplus B_4)(A_3 \oplus B_3)(A_2 \oplus B_2) \\
(A_1 \oplus B_1)(A_0 \oplus B_0) \]

\[ \text{AGTB} = A_3B_5 + A_4B_4(A_5 \oplus B_5) + \\
A_3B_3(A_5 \oplus B_5)(A_4 \oplus B_4) + \\
A_2B_4(A_5 \oplus B_5)(A_3 \oplus B_3) + \\
(A_2 \oplus B_2)(A_1 \oplus B_1) \]

\[ \text{ALTB} = A_9B_5 + A_8B_4(A_9 \oplus B_9) + \\
A_9B_3(A_9 \oplus B_9)(A_8 \oplus B_8) + \\
A_2B_8(A_9 \oplus B_9)(A_7 \oplus B_7) + \\
A_1B_7(A_9 \oplus B_9)(A_6 \oplus B_6) \]
(A_2 \oplus B_2)
+ \overline{A_0B_0}(A_1 \oplus B_1)(A_2 \oplus B_2)(A_3 \oplus B_3)
\overline{(A_1 \oplus B_1)(A_2 \oplus B_2)}

5.47 -- cmp4x6.vhd

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY cmp4x6 IS
    PORT(
a, b : IN INTEGER RANGE 0 TO 15;
altb, aleb, aeqb, aneb, ageb, agtb : OUT STD_LOGIC);
END cmp4x6;

ARCHITECTURE a OF cmp4x6 IS
SIGNAL compare : STD_LOGIC_VECTOR (5 downto 0);
BEGIN
PROCESS (a,b)
BEGIN
IF a<b THEN
    compare <= "001011";
ELSIF a=b THEN
    compare <= "100101";
ELSIF a>b THEN
    compare <= "111000";
ELSE
    compare <= "111111";
END IF;
altb  <=    compare (5); —— a is less than b
aleb  <=    compare (4); —— a is less than or equal to b
aeqb  <=    compare (3); —— a equals b
aneb  <=    compare (2); —— a is not equal to b
ageb  <=    compare (1); —— a is greater than or equal to b
agtb  <=    compare (0); —— a is greater than b
END PROCESS;
END a;

5.49 a. 1111100; five 1s: PE
b. 1010110; four 1s; P_E = 0; P_O = 1;
c. 0001101; three 1s; P_E = 1; P_O = 0

5.51 a. ABCDEFGHP = 110101100; P' = 1; Error in bit D.
b. ABCDEFGHP = 110011101; P' = 1; Error in parity bit.
c. ABCDEFGHP = 110001100; P' = 0; Data received
Answers to Selected Odd-Numbered Problems

5.53 See Figure ANS5.53.

Chapter 6

6.1 a. 11111; b. 100000; c. 11110; d. 101010; e. 101100; f. 1100100

6.3

<table>
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<tr>
<th>Decimal</th>
<th>True Magnitude</th>
<th>1’s Complement</th>
<th>2’s Complement</th>
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<td>11101110</td>
<td>10010001</td>
<td>10010010</td>
</tr>
<tr>
<td>b. -67</td>
<td>01000011</td>
<td>01000011</td>
<td>01000011</td>
</tr>
<tr>
<td>c. -54</td>
<td>10110110</td>
<td>11001001</td>
<td>11001010</td>
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<tr>
<td>d. -93</td>
<td>11011101</td>
<td>10100010</td>
<td>10100011</td>
</tr>
<tr>
<td>e. 0</td>
<td>00000000</td>
<td>00000000</td>
<td>00000000</td>
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<tr>
<td>f. -1</td>
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<td>11111111</td>
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<td>g. 127</td>
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<td>01111111</td>
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<tr>
<td>h. -127</td>
<td>11111111</td>
<td>10000000</td>
<td>10000001</td>
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</table>

6.5 Largest: 01111111₂ = +127₁₀; smallest: 10000000₂ = -128₁₀

6.7 Overflow in an 8-bit signed addition results if the sum is outside the range -128 ≤ sum ≤ 127. The sums in parts a and f do not generate an overflow. The sums in parts b, c, d, and e do.

6.9 a. 8D:

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<td>11110000010</td>
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<td>7730</td>
<td>11110001000</td>
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8421 BCD | Excess-3 |
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<td>0100 1011 1011 1100</td>
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<tr>
<td>0010 0011 1000 0101</td>
<td>1010 0110 1000 1000</td>
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<tr>
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<td>0000 1000 0000 0000</td>
</tr>
<tr>
<td>0001 1010 1101 0001</td>
<td>0001 1000 0101 1001</td>
</tr>
<tr>
<td>0011 0111 0011 0000</td>
<td>1010 1010 0110 0011</td>
</tr>
</tbody>
</table>

57 41 52 4E 49 4E 47 21 20 54 68 69 73 20
WARNING! SP ThisSP commandSP erases SP 20 36 34 30 4D 20 6F 66 20 6D 65 6D 6F 72 79 2E SP 6 4 0 M SP of SP memory.

FIGURE ANS6.27

b. 120;

6.11

6.15 The sequence of codes yields the following text:

6.21 A fast carry circuit is “flatter”, but “wider” than a ripple carry circuit. There are more gate levels for an input change to propagate through in a ripple carry circuit. The ripple carry is thus slower. The limitation on a fast carry circuit is its width, both in the number of gates and on the number of inputs on the gates. Both factors increase with adder bit size.

6.23 A carry is generated if the MSB of either A or B is HIGH AND the second bit of either A or B is HIGH AND the third bits of both A and B are HIGH.

6.25 To generate all possible combinations of input for an 8-bit adder requires $2^{16} = 65,536$ combinations. (A simulation with one change every 40 ns would have an end time of 2.62144 ms.)

6.27 See Figure ANS6.27

The transition from the sum FFF+000=FFF to FFF+001=000 (plus a carry) is given in the following table:

<table>
<thead>
<tr>
<th>Time</th>
<th>Sum (Hex)</th>
<th>Sum (Binary)</th>
<th>From a1 to:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>FFF</td>
<td>1111 1111 1111</td>
<td></td>
</tr>
</tbody>
</table>
The 4-bit parallel adder/subtractor is shown in Figure ANS6.29a. The component \textbf{add4}, a parallel binary adder, is shown in Figure ANS6.29b.

\textbf{SUB} = 1: Input carry is forced HIGH, automatically
adding 1 to the output sum; the XOR gates act as inverters, making the inputs to the adder equal to the one’s complement of B; the output is $A + (\text{one’s complement of } B) + 1 = A - B$

SUB = 0: Input carry is forced LOW, adding 0 to the output sum; the XOR gates act as noninverting buffers, making the inputs to the adder equal the true binary value of B; the output is $A + B + 0 = A + B$.

6.31 See Figure ANS6.31.

6.33 -- addsubv1.vhd
-- 4-bit parallel adder with overflow detection,
-- using a generate statement and components
-- overflow: SOP network

ENTITY addsubv1 IS
    PORT(
        sub : IN BIT;
        a, b : IN BIT_VECTOR(4 downto 1);
        c4, v : OUT BIT;
        sum : BUFFER BIT_VECTOR(4 downto 1));
END addsubv1;

ARCHITECTURE adder OF addsubv1 IS
    -- Component declaration
    COMPONENT full_add
        PORT(
            a, b, c_in : IN BIT;
            c_out, sum : OUT BIT);
END COMPONENT;

END COMPONENT;
-- Define a signal for internal carry bits
SIGNAL c         : BIT_VECTOR (4 downto 0);
SIGNAL b_comp    : BIT_VECTOR (4 downto 1);
BEGIN
-- Carry input depends on add or subtract (sub=1 for subtract)
c(0)  <=   sub;
adders:
FOR I IN 1 to 4 GENERATE
-- invert b for subtract function (b(i) xor 1)
-- do not invert b for add function (b(i) xor 0)
b_comp(i) <= b(i) xor sub;
adder: full_add PORT MAP (a(i), b_comp(i), c(i-1), c(i), sum(i));
END GENERATE;
c4    <=     c(4);
v <= (a(4) and b(4) and (not sum(4)))
or ((not a(4)) and (not b(4)) and sum(4));
END adder;

-- addsubv2.vhd
-- 4-bit parallel adder with overflow detection,
-- using a generate statement and components
-- overflow: xor gate

ENTITY addsubv2 IS
   PORT(
      sub         : IN  BIT;
      a, b        : IN  BIT_VECTOR(4 downto 1);
      c4, v       : OUT BIT;
      sum         : OUT BIT_VECTOR(4 downto 1));
END addsubv2;

ARCHITECTURE adder OF addsubv2 IS
-- Component declaration
COMPONENT full_add
   PORT(
      a, b, c_in  : IN  BIT;
      c_out, sum   : OUT BIT);
END COMPONENT;
-- Define a signal for internal carry bits
SIGNAL c         : BIT_VECTOR (4 downto 0);
SIGNAL b_comp    : BIT_VECTOR (4 downto 1);
BEGIN
-- Carry input depends on add or subtract (sub=1 for subtract)
c(0)  <=   sub;
adders:
FOR i IN 1 to 4 GENERATE
-- invert b for subtract function (b(i) xor 1)
-- do not invert b for add function (b(i) xor 0)
b_comp(i) <= b(i) xor sub;
adder: full_add PORT MAP (a(i), b_comp(i), c(i-1), c(i), sum(i));
END GENERATE;
c4    <=     c(4);
v <= c(4) xor c(3);
END adder;
6.35 1999; 3½ digits
6.37 1 followed by \(n\) 9s.
6.39 See Figure 6.26 in text.
6.41
--- add4bcd.vhd
--- 4-bit bcd adder
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY add4bcd IS
  PORT(
    c0                : IN  STD_LOGIC;
    a_bcd, b_bcd      : IN  STD_LOGIC_VECTOR(4 down to 1);
    sum_bcd           : OUT STD_LOGIC_VECTOR(5 downto 1));
END add4bcd;
ARCHITECTURE adder OF add4bcd IS
  -- Component declaration
  COMPONENT add4gen
    PORT(
      c0          : IN  STD_LOGIC;
      a, b        : IN  STD_LOGIC_VECTOR(4 downto 1);
      c4          : OUT STD_LOGIC;
      sum         : OUT STD_LOGIC_VECTOR(4 downto 1));
  END COMPONENT;
  COMPONENT bin2bcd
    PORT(
      bin : IN STD_LOGIC_VECTOR(5 downto 1);
      bcd : OUT STD_LOGIC_VECTOR(5 downto 1));
  END COMPONENT;
  -- Define a signal for internal carry bits
  SIGNAL connect : STD_LOGIC_VECTOR (5 downto 1);
BEGIN
  adder: add4gen PORT MAP (c0, a_bcd, b_bcd, connect(5),
    connect (4 downto 1));
  converter: bin2bcd PORT MAP (bin, bcd);

FIGURE ANS7.1
FIGURE ANS7.3
MAP (connect, sum_bcd);

\[ \begin{array}{c}
\text{S} & \quad \text{Q} \\
\text{R} & \quad \text{Q}
\end{array} \]

**FIGURE ANS7.5**

END adder;

6.43 The circuit will be like Figure 6.27 in the text, minus the thousands digit. It will generate a 3½ digit output.

Chapter 7

7.1 See Figure ANS7.1.

7.3 See Figure ANS7.3.

7.5 See Figure ANS7.5.

\[ \begin{array}{c|c}
S & R \\
\hline
0 & 0 \\
0 & 1 \\
1 & 0 \\
1 & 1
\end{array} \]

- Latch tries to set and reset at the same time. Forbidden state.
- Set input active. Q = 1.
- Reset input active. Q = 0.
- Neither set nor reset active. No change.

7.7 See Figure ANS7.7.

7.9 See Figure ANS7.9.

**FIGURE ANS7.7**

**FIGURE ANS7.9**

NAND waveforms

NOR waveforms
Answers to Selected Odd-Numbered Problems

i.

ii.

iii.

FIGURE ANS7.11

FIGURE ANS7.13
7.11  a. See Figure ANS7.11.
    b.  i. R is last input active. Latch resets;  ii. S is last in

FIGURE ANS7.15

FIGURE ANS7.17
7.19  -- ltch8prm.vhd
-- D latch with active-HIGH level-sensitive enable

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;

ENTITY ltch8prm IS
    PORT(d_in    : IN  STD_LOGIC_VECTOR(7 downto 0);
        enable : IN STD_LOGIC;
        q_out  : OUT STD_LOGIC_VECTOR(7 downto 0));
END ltch8prm;

ARCHITECTURE a OF ltch8prm IS
BEGIN
    -- Instantiate a latch from a MAX+PLUS II primitive
    latch8:
        FOR i IN 7 downto 0 GENERATE
            latch_primitive: latch
                PORT MAP (d => d_in(i), ena => enable, q => q_out(i));
        END GENERATE;
END a;

See Figure ANS7.19.

7.21  See Figure ANS7.21.
See Figure ANS7.23.

See Figure ANS7.25.

---

7.27

-- dff12lpm.vhd
-- 12-BIT D flip-flop
-- Uses a flip-flop component from the Library of Parameterized Modules (LPM)

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;
ENTITY dff12lpm IS
  PORT(d_in : IN STD_LOGIC_VECTOR(11 downto 0);
       clk  : IN STD_LOGIC;
       q_out : OUT STD_LOGIC_VECTOR(11 downto 0));
END dff12lpm;
ARCHITECTURE a OF dff12lpm IS
BEGIN
  -- Instantiate flip-flop from an LPM component
dff12: lpm_ff
    GENERIC MAP (LPM_WIDTH => 12)
    PORT MAP (data => d_in,
               clock => clk,
               q    => q_out);
END a;
7.29 See Figure ANS7.29.

7.31 See Figure ANS7.31. The circuit generates the following repeating pattern: 111, 110, 101, 100, 011, 010, 001, 000. This is a 3-bit binary down-count sequence.

7.33 The circuit generates a 4-bit binary sequence from 0000 to 1111, then repeats indefinitely.

7.35 See Figure 7.35.
7.37  See Figure 7.37

7.39  Similarity: an asynchronous circuit and an asynchronous input cause outputs to change out of synchronization with a system clock. Difference: an asynchronous circuit may be clocked, but at different times throughout the circuit; an asynchronous input is independent of the clock function altogether.

FIGURE 7.37

7.41  -- d12lpmcl.vhd
     -- 4-BIT D latch with active-HIGH level-sensitive enable
     -- Uses a latch component from the Library of Parameterized Modules

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY d12lpmcl IS
    PORT(d_in               : IN  STD_LOGIC_VECTOR(11 downto 0);
         clk, set, reset   : IN  STD_LOGIC;
         q_out             : OUT STD_LOGIC_VECTOR(11 downto 0));
END d12lpmcl;

ARCHITECTURE a OF d12lpmcl IS
    SIGNAL clrn  : STD_LOGIC;
    SIGNAL prn  : STD_LOGIC;
BEGIN

-- Instantiate flip-flop from an LPM component
dff12: lpm_ff
GENERIC MAP (LPM_WIDTH => 12)
PORT MAP (  data  => d_in,
clock => clk,
aclr  => clrn,
aset  => prn,
q     => q_out);

-- Make set and reset active-LOW
clrn <= not reset;
prn <= not set;
END a;

FIGURE ANS7.41

7.43 See Figure ANS7.43.

FIGURE ANS7.43

7.45 -- syn4tprm.vhd
-- 4-bit sync counter (TFF primitives)

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY altera;
USE altera.maxplus2.ALL;

ENTITY syn4tprm IS
  PORT (clock, reset : IN STD_LOGIC;
        q : OUT STD_LOGIC_VECTOR(3 downto 0));
END syn4tprm;
ARCHITECTURE a OF syn4tprm IS
   -- Declare component only with ports actually used
   COMPONENT TFF
      PORT ( t : IN STD_LOGIC;
             clk : IN STD_LOGIC;
             clrn: IN STD_LOGIC;
             q : OUT STD_LOGIC);
   END COMPONENT;
   SIGNAL q_int : STD_LOGIC_VECTOR(2 downto 0);
   SIGNAL t_int : STD_LOGIC_VECTOR(3 downto 0);
BEGIN
   -- Instantiate 4 T flip-flops.
   ff0: tff
      PORT MAP (t_int(0), clock, reset, q_int(0));
   ff1: tff
      PORT MAP (t_int(1), clock, reset, q_int(1));
   ff2: tff
      PORT MAP (t_int(2), clock, reset, q_int(2));
   ff3: tff
      PORT MAP (t_int(3), clock, reset, q(3));
   -- Connect flip-flops internally
   t_int(0) <= '1';
   t_int(1) <= q_int(0);
   t_int(2) <= q_int(0) and q_int(1);
   t_int(3) <= q_int(0) and q_int(1) and q_int(2);
   q(0) <= q_int(0);
   q(1) <= q_int(1);
   q(2) <= q_int(2);
END a;

7.47 tsu = 20 ns, tb = 0
7.49 clock pulse width: tc = 12 ns; setup time: tsu = 10 ns; hold time: tb = 5 ns

Chapter 8
8.1 See Figure 8.2.
8.7 a. 4;  b. 6;  c. 8
8.9 A global architecture cell configures all macrocells in the PLD. A local architecture cell works only on the macrocell of which it is a part.
8.11 Registered/active LOW; registered/active HIGH; combinatorial/active LOW; combinatorial/active HIGH
8.13 No. Global clock only.
8.15 Global. These functions operate simultaneously on all macrocells.
8.17 a. 32;
     b. 64;
     c. 128;
     d. 160
8.19 n/16 Logic Array Blocks for n macrocells. (e.g. 128/16 = 8 LABs for an EPM7128S)
8.21 Macrocells without pin connections can be used for internal logic.
8.23 A MAX7000S macrocell can be reset from a global clear pin (GCLRn) or locally from a product term.
8.25 5 dedicated product terms; by using terms from shared logic expanders and parallel logic expanders; 5 dedicated, up to 15 from parallel logic expanders; up to 16 from shared logic expanders.
8.27 A sum-of-products network constructs Boolean expressions by switching signals into an OR-gate output via a programmable matrix of AND gates. A look-up table network stores the output values of the network in a small memory whose storage locations are selected by combinations of the input signals.
8.29 A carry chain allows for efficient fast-carry implementation of adders, comparators, and other circuits whose inputs become wider with higher-order bits.
8.31 2048
9.1 See Figure ANS9.1. The 12-bit counter recycles to 0 after 4096 cars have entered the parking lot. The last car causes all bits to go LOW. The negative edge on the MSB clocks a flip flop whose output enables the LOT FULL sign. Every car out of the gate resets the flip-flop and turns off the sign.

A better circuit would have the exit gate make the counter output decrease by 1 with every vehicle exiting.

9.3 See Figure ANS9.3.

9.5 a. See Figure ANS9.5
   b. i. 0100;
        ii. 0110;
        iii. 0011

FIGURE ANS9.1

FIGURE ANS9.3

FIGURE ANS9.5
9.7 Figure ANS9.7 shows the timing diagram of a mod-10 counter.

<table>
<thead>
<tr>
<th>$Q_3$</th>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<tr>
<td>0</td>
<td>0</td>
<td>1</td>
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<td>0</td>
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<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

9.9 $Q_0$: 24 kHz; $Q_1$: 12 kHz; $Q_2$: 6 kHz; $Q_3$: 3 kHz

9.11 See Figure ANS9.11

9.13 $J_0 = K_0 = 1$
$J_1 = K_1 = Q_0$
$J_2 = K_2 = Q_1Q_0$
$J_3 = K_3 = Q_2Q_1Q_0$
$J_4 = K_4 = Q_3Q_2Q_1Q_0$
$J_5 = K_5 = Q_4Q_3Q_2Q_1Q_0$
$J_6 = K_6 = Q_5Q_4Q_3Q_2Q_1Q_0$
$J_7 = K_7 = Q_6Q_5Q_4Q_3Q_2Q_1Q_0$

9.15 a. $J_3 = Q_2Q_1Q_0$
$K_3 = Q_3Q_0$
$J_2 = Q_2Q_1Q_0$
$K_2 = Q_1Q_0$
$J_1 = Q_0$
$K_1 = Q_0$
$J_0 = 1$
$K_0 = 1$

b. 1011, 0000, 0001

9.19 See Figure ANS9.19

FIGURE ANS9.7

FIGURE ANS9.11
9.21 Boolean equations:
\[
\begin{align*}
D_3 &= Q_3 Q_2 + Q_3 Q_2 \\
D_2 &= Q_1 Q_0 \\
D_1 &= Q_1 Q_0 + Q_1 Q_0 \\
D_0 &= Q_2 Q_0
\end{align*}
\]

9.23 See Figure ANS9.23a for a simulation of the clear function and Figure ANS9.23b for the recycle point of the counter.

9.25 See Figure 9.22 in the text. Asynchronous load transfers data directly to the flip-flops of a counter as soon as the load input is asserted; it does not wait for a clock edge. Synchronous load waits for an active clock edge to load a value into the counter flip-flops.

9.27 Figure ANS9.27 shows the part of the simulation where the value 1AH is synchronously loaded into the counter.

9.29 See Figure ANS9.29

9.31 \[
\begin{align*}
D_0 &= Q_0 \\
D_1 &= Q_0 \overline{Q_0} + Q_0 \overline{Q_1} \\
D_2 &= Q_1 Q_0 \overline{Q_0} + Q_1 Q_0 \overline{Q_1} \\
D_3 &= Q_2 Q_1 Q_0 \overline{Q_0} + Q_2 Q_1 Q_0 \overline{Q_1}
\end{align*}
\]
The right-hand product term of each equation represents the down-count logic, which is enabled whenever \( \overline{DIR} = 0 \). The left-hand product term is the up-count logic, enabled when \( DIR = 1 \). \( D_0 \) is always the opposite of \( Q_0 \) regardless of whether the count is up or down.
FIGURE ANS 9.27

FIGURE ANS 9.29
9.33 The circuit is shown in Figure ANS9.33a. The counter module `sl_count` is shown in Figure 9.25 in the text. The simulation is shown in Figure ANS9.33b.

**FIGURE ANS9.33A**
9.35  -- ct_mod24
      -- Presettable counter with synchronous clear and load
      -- and a modulus of 24

ENTITY ct_mod24 IS
  PORT(
    clk                   : IN    BIT;
    clear, direction     : IN    BIT;
    q                     : OUT INTEGER RANGE 0 TO 23);
END ct_mod24;

ARCHITECTURE a OF ct_mod24 IS
BEGIN
  PROCESS (clk)
  VARIABLE    cnt       : INTEGER RANGE 0 TO 23;
  BEGIN
    IF (clk'EVENT AND clk = '1') THEN
      IF (clear = '0') THEN  —— Synchronous clear
        cnt   :=    0;
      ELSIF (direction = '0') THEN
        IF cnt = 0 THEN
          cnt := 23;
        ELSE
          cnt := cnt - 1;
        END IF;
      ELSIF (direction = '1') THEN
        IF cnt = 23 THEN
          cnt := 0;
        ELSE
          cnt := cnt + 1;
        END IF;
      END IF;
      q     <=    cnt;
    END PROCESS;
  END a;

See Figure ANS9.35 for simulation.
9.37

-- sst1_lpm.vhd
-- 12-bit LPM counter with sst1 and aclr (Chapter problem)

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY sst1_lpm IS
  PORT(
    clk             : IN   STD_LOGIC;
    clear, set      : IN STD_LOGIC;
    q               : OUT  STD_LOGIC_VECTOR (11 downto 0));
END sst1_lpm;

ARCHITECTURE a OF sst1_lpm IS
BEGIN
  counter1: lpm_counter
    GENERIC MAP (LPM_WIDTH => 12)
    PORT MAP (  clock => clk,
                sset => set,
                aclr => clear,
                q    => q);
END a;

The counter in this problem sets to all 1s (1111 1111 1111 = FFFH), rather than 0111 1111 1111 (= 7FFH). See Figure ANS9.37 for the simulation of the counter in problem 9.37.
9.39  -- lpm8term
-- 8-bit presettable counter with synchronous clear and load,
-- count enable, a directional control port,
-- and terminal count decoding

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY lpm8term IS
  PORT(
    clk, count_ena             : IN     STD_LOGIC;
    clear, load, direction     : IN     STD_LOGIC;
    p                          : IN STD_LOGIC_VECTOR(7 downto 0);
    max_min                    : OUT STD_LOGIC;
    q                                  : OUT   STD_LOGIC_VECTOR(7 downto 0));
END lpm8term;

ARCHITECTURE a OF lpm8term IS
  SIGNAL cnt : STD_LOGIC_VECTOR(7 downto 0);
BEGIN
  counter1: lpm_counter
    GENERIC MAP (LPM_WIDTH => 8)
    PORT MAP (  clock  => clk,
                updown => direction,
                cnt_en => count_ena,
                data   => p,
                sload  => load,
                sclr   => clear,
                q      => cnt);

  q <= cnt;
  PROCESS (clk, cnt)
  BEGIN
    -- Terminal count decoder
    IF (cnt = "00000000" and direction = '0') THEN
      max_min <= '1';
    ELSIF (cnt = "11111111" and direction = '1') THEN
      max_min <= '1';
    ELSE
      max_min <= '0';
    END IF;
  END PROCESS;
END a;
9.41 See Figure ANS9.41.
9.43 001111, 000000, 000000, 110000
9.45 See Figure ANS9.45. The serial output is the same as the serial input, only delayed by eight clock pulses and synchronized to the positive edge of the clock.
9.47 See Figure ANS9.47.
FIGURE ANS9.47
9.51 — Left-shift register of generic width

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY slt_bhv IS
    GENERIC (width : POSITIVE);
    PORT(
        serial_in, clk : IN        STD_LOGIC;
        q                 : BUFFER    STD_LOGIC_VECTOR(width-1 downto 0));
END slt_bhv;

ARCHITECTURE left_shift of slt_bhv IS
BEGIN
    PROCESS (clk)
    BEGIN
        IF (clk'EVENT and clk = '1') THEN
            q(width-1 downto 0) <= q(width-2 downto 0) & serial_in;
        END IF;
    END PROCESS;
END left_shift;

-- 32-bit left-shift register
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY slt32_bhv IS
    PORT(
        data_in, clock : IN        STD_LOGIC;
        qo                : BUFFER    STD_LOGIC_VECTOR(31 downto 0));
END slt32_bhv;

ARCHITECTURE left_shift of slt32_bhv IS
COMPONENT slt_bhv
    GENERIC (width : POSITIVE);
    PORT(
        serial_in, clk : IN  STD_LOGIC;
        q                 : OUT STD_LOGIC_VECTOR(31 downto 0));
END COMPONENT;
BEGIN
    Shift_left_32: slt_bhv
        GENERIC MAP (width=> 32)
        PORT MAP (serial_in => data_in,
                        clk       => clock,
                        q         => qo);
END left_shift;

Figure ANS9.51 shows a partial simulation of the shift register.
9.53 The generic component has a default width of 8 bits. The instantiated component has an assigned width of 16 bits. The generic map in the instantiated component overrides the default parameter.

9.55

-- srg10lpm.vhd

-- 10-bit serial shift register (shift right)
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
LIBRARY lpm;
USE lpm.lpm_components.ALL;

ENTITY srg10lpm IS
    PORT(
        clk         : IN  STD_LOGIC;
        serial_in   : IN  STD_LOGIC;
        sync_set    : IN  STD_LOGIC;
        serial_out  : OUT STD_LOGIC);
END srg10lpm;

ARCHITECTURE lpm-shift of srg10lpm IS
    COMPONENT lpm_shiftreg
        GENERIC(LPM_WIDTH: POSITIVE; LPM_SVALUE: STRING);
        PORT(
            clock, shiftin    : IN  STD_LOGIC;
            sset              : IN  STD_LOGIC;
            shiftout          : OUT STD_LOGIC);
    END COMPONENT;
BEGIN
    Shift_10: lpm_shiftreg
        GENERIC MAP (LPM_WIDTH=> 10, LPM_SVALUE => "960")
        PORT MAP (clk, serial_in, sync_set, serial_out);
END lpm_shift;
The parameter LPM_SVALUE is set to 960, the decimal equivalent of $16^2$. Figure ANS9.55 shows the simulation of the shift register.

FIGURE ANS9.55

<table>
<thead>
<tr>
<th>$Q_4$</th>
<th>$Q_3$</th>
<th>$Q_2$</th>
<th>$Q_1$</th>
<th>$Q_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

All gates used in the decoder of Figure 9.84 remain unchanged except those decoding the MSB/LSB pairs ($Q_3 Q_0$ and $Q_2 Q_0$). Change these to decode $Q_4 Q_0$ and $Q_4 Q_3$. Add two new gates to decode $Q_4 Q_3$ (2nd state) and $Q_4 Q_1$ (7th state).

9.59  
See Figure ANS9.59

FIGURE ANS9.59

Chapter 10

10.1  Mealy machine. The output is fed by combinational, as well as sequential, logic.

$D_3 = \bar{Q}_4 \bar{Q}_3 Q_0 + Q_3 Q_1 + Q_4 Q_0$

$D_2 = \bar{Q}_4 Q_3 Q_0 + \bar{Q}_3 Q_1 + Q_2 Q_0$

$D_1 = \bar{Q}_4 Q_3 Q_0 + \bar{Q}_3 Q_2 Q_0 + Q_4 \bar{Q}_0$

$D_0 = \bar{Q}_4 Q_3 Q_1 + \bar{Q}_3 Q_2 Q_1 + \bar{Q}_4 Q_2 Q_1 + Q_4 Q_0 Q_1$

See Figure ANS10.3.
10.5  \[ J_2 = Q_1\bar{Q}_0 \]
     \[ K_2 = Q_1Q_0 \]
     \[ J_1 = \bar{Q}_2Q_0 \]
     \[ K_1 = Q_2Q_0 \]
     \[ J_0 = \bar{Q}_2Q_1 + Q_2Q_1 \]
     \[ K_0 = Q_2Q_1 + Q_2\bar{Q}_1 \]

See Figure ANS10.5

10.7  \[ D_1 = \bar{Q}_1Q_0\text{in1} \]
     \[ D_0 = Q_0\text{in1} \]
     \[ \text{out1} = \bar{Q}_1Q_0\text{in1} \]
     \[ \text{out2} = Q_1\text{in1} \]

See Figure ANS10.7. The circuit generates a HIGH pulse on \text{out1} when \text{in1} goes LOW and a HIGH pulse on \text{out2} when the input goes back HIGH.
FIGURE ANS10.7
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY prob10_9 IS
  PORT(
    clk, in1 : IN STD_LOGIC;
    out1, out2 : OUT STD_LOGIC);
END prob10_9;

ARCHITECTURE a OF prob10_9 IS
  TYPE PULSER IS (s0, s1, s2, s3);
  SIGNAL sequence: PULSER;
  BEGIN
    PROCESS (clk)
    BEGIN
      IF clk'EVENT AND clk = '1' THEN
        CASE sequence IS
        WHEN s0 =>
          IF in1 = '1' THEN
            sequence <= s0;  -- no change if in1 = 1
            out1 <= '0';
            out2 <= '0';
          ELSE
            sequence <= s1;  -- proceed if in1 = 0
            out1 <= '1';      -- pulse on out1
            out2 <= '0';
          END IF;
        WHEN s1 =>
          IF in1 = '0' THEN
            sequence <= s1;  -- outputs LOW
            out1 <= '0';
            out2 <= '0';
          ELSE
            sequence <= s2;
            out1 <= '0';
            out2 <= '1';      -- pulse on out2
          END IF;
        WHEN s2 =>
          sequence <= s0;
          out1 <= '0';
          out2 <= '0';
        WHEN others =>
          sequence <= s0;
          out1 <= '0';
          out2 <= '0';
        END CASE;
      END IF;
    END PROCESS;
  END a;

See Figure ANS10.9.
10.11 LIBRARY ieee;
USE ieee.std_logic_1164.ALL;
ENTITY prob10_11 IS
PORT(
  clk, go, reset, eoc : IN STD_LOGIC;
  sc, oe : OUT STD_LOGIC);
END prob10_11;
ARCHITECTURE a OF prob10_11 IS
TYPE ADC IS (idle, start, waiting, read);
SIGNAL state: ADC;
SIGNAL outputs: STD_LOGIC_VECTOR(1 downto 0);
BEGIN
  sc <= outputs(1);
  oe <= outputs(0);
  PROCESS (clk)
  BEGIN
    IF clk'EVENT AND clk = '1' THEN
      IF reset = '0' THEN
        state <= idle;
        outputs <= "01";
      ELSE
        CASE state IS
          WHEN idle =>
            IF go = '0' THEN
              state <= idle;
              outputs <= "01";
            ELSIF go = '1' THEN
              state <= start;
              outputs <= "11";
            END IF;
          WHEN start =>
            state <= waiting;
            outputs <= "01";
          WHEN waiting =>
            IF eoc = '0' THEN
              state <= waiting;
              outputs <= "01";
            ELSIF eoc = '1' THEN
              state <= read;
              outputs <= "00";
            END IF;
          END CASE;
      END IF;
    END PROCESS;
  END IF;
END a;
WHEN read =>
    state <= idle;
    outputs <= "01";
END CASE;
END IF;
END IF;
END PROCESS;
END a;

See Figure ANS 10.11.

10.13 A NAND latch can only debounce a switch with a normally open and a normally closed contact: one to set and the other to reset the latch. The pushbutton on the Altera UP-1 board has only a normally open contact.

10.15 8.33 ms
10.17 Four clock periods. 8.33 ms
10.19 -- prob10_19.vhd
LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY prob10_19 IS
PORT(
    clk, in1, in2 : IN STD_LOGIC;
    out1 : OUT STD_LOGIC);
END prob10_19;

ARCHITECTURE a OF prob10_19 IS
    TYPE STATE_TYPE IS (s0, s1, s2, s3, s4);
    SIGNAL state: STATE_TYPE;
BEGIN
PROCESS (clk)
BEGIN
IF clk'EVENT AND clk = '1' THEN
CASE state IS
WHEN s0 =>
    state <= s1;
    out1 <= '0';
WHEN s1 =>
    IF in1 = '1' THEN
        state <= s1;
        out1 <= '0';
    ELSEIF in1 = '0' THEN
        state <= s2;
        out1 <= '1';
    END IF;
WHEN s2 =>
    state <= s3;
    out1 <= '0';
WHEN s3 =>
    state <= s4;
    out1 <= '0';
WHEN s4 =>
    IF in2 = '1' THEN
        state <= s4;
        out1 <= '0';
    ELSEIF in2 = '0' THEN
        state <= s0;
        out1 <= '1';
    END IF;
END CASE;
END IF;
END PROCESS;
END a;

See Figure ANS10.19
10.23 See Figure ANS10.23.

FIGURE ANS10.23

Chapter 11

11.1 TTL: advantages—relatively high speed, high current driving capability; disadvantages—rigid power supply requirements. CMOS: advantages—low power consumption, high noise immunity, flexible power supply requirements; disadvantages—low output current ECL: advantages—high speed; disadvantages—high susceptibility to noise, high power consumption.

11.3 \( t_{	ext{PHL}} = 12 \text{ ns} \), \( t_{	ext{PLH}} = 10 \text{ ns} \)

11.5 Transition from state 1 to state 2: \( t_0 = t_{	ext{PHLH0}} + t_{	ext{PHL00}} = 16 \text{ ns} + 15 \text{ ns} = 31 \text{ ns} \). Transition from state 2 to state 3: \( t_0 = t_{	ext{PLH00}} = 15 \text{ ns} \) (Assume \( V_{	ext{CC}} = 4.5 \text{ volts}; T = 25^\circ \text{C} \) to \(-55^\circ \text{C}\))

11.7 Driving gate (74LS00): \( I_{	ext{OL}} = 8 \text{ mA} \); \( I_{	ext{OH}} = -0.4 \text{ mA} \)
Load gate (74S32): \( I_{	ext{IL}} = -2 \text{ mA} \); \( I_{	ext{IH}} = 20 \mu\text{A} \)
\( n_\text{L} = I_{	ext{OL}}/I_{	ext{IL}} = 8 \text{ mA} / 2 \mu\text{A} = 4 \)
\( n_\text{H} = I_{	ext{OH}}/I_{	ext{IH}} = 0.4 \text{ mA} / 0.02 \text{ mA} = 20 \)
\( n = n_{\text{L}} = 4 \)

11.9 Source: \( I_{	ext{OH}} = -0.14 \text{ mA} \); sink: \( I_{	ext{OL}} = 2.8 \text{ mA} \)

11.11 a. 44 mW;
   b. 39 mW;
   c. 29 mW;
   d. 20 mW

11.13 a. 4.5 \mu\text{W};
   b. 28.3 \mu\text{W};
   c. 4.46 mW

11.15 a. 550 \mu\text{W};
   b. \(-56.4\%\)

11.17 The outputs of a 74LS00 gates are guaranteed to produce output voltages of \( V_{	ext{OH}} \geq 2.7 \text{ V} \) and \( V_{	ext{OL}} \leq 0.8 \text{ V} \). The inputs of a 74HCT series gate are voltage compatible with LSTTL outputs since \( V_{	ext{IH}} \geq 2 \text{ V} \) and \( V_{	ext{IL}} \leq 0.5 \text{ V} \). This is not the case for 74HC series gates, where \( V_{	ext{IH}} \leq 3.15 \text{ V} \) and \( V_{	ext{IL}} \leq 1.35 \text{ V} \). The 74LS gate is not guaranteed to drive the 74HC gate in the HIGH state.

11.19 10 loads, since the 74HC output voltages are defined for an output current of \( \pm 4 \text{ mA} \).

11.21 HIGH input: The base-emitter junction of transistor Q1 is reverse-biased. Current flows, by default, through the base-collector junction of Q1, supplying base current to Q2, saturating it. This in turn, saturates Q3, making its collector LOW.

LOW input: Current has a path to ground via the base-emitter junction of Q1. This transports charge away from the base region of Q3, making it cutoff. Since Q3 is cutoff, no base current flows in Q3. If an external pull-up resistor is connected to the collector of Q3, the output will be HIGH. Otherwise, it is floating.

11.23 See Figure 11.31 in text. \( Y = AB \cdot CD \cdot EF = AB + CD + EF \)

11.27 Yes. When the output transistor saturates (output LOW), there will be a direct connection from the output to \( V_{	ext{CC}} \). Since there is relatively little resistance in the current path, the current will likely exceed the rated output current, \( I_{	ext{OL}} \).

11.29 a. \( Q_3 \) and \( Q_4 \) are never on at the same time because the phase splitter, \( Q_2 \), keeps them in opposite states. The voltage in the circuit is divided such that when \( Q_2 \) is on, it pulls the base of \( Q_4 \) into the cutoff region for that transistor. At the same time, \( Q_1 \) is supplied with base current and thus saturated. When \( Q_2 \) is off, there is no base current in \( Q_3 \), making it cutoff. The base voltage at \( Q_3 \) is now such that it is on.

b. Switching noise originates in a totem pole output because the HIGH output transistor, \( Q_4 \), can switch on faster than the LOW output transistor, \( Q_1 \), can switch off. For a brief time, both transistors are on, causing a supply current spike. This can be counteracted by connecting a small capacitor between the supply voltage, \( V_{	ext{CC}} \), and ground.

11.31 7.58 mA, 95% of \( I_{	ext{OL}} \); 2.12 mA, 530% of \( I_{	ext{OH}} \) The first circuit is more suitable, as it can drive a higher current to the LED and still remain within the output specification of the inverter.

11.33 Store MOS devices in antistatic or conducting material. Work only on an antistatic work surface and wear a conductive wrist strap. Connect all unused device inputs to power or ground. Do not touch the pins of the MOS device.
11.35 See Figure 11.56 in text.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Q₁</th>
<th>Q₂</th>
<th>Q₃</th>
<th>Q₄</th>
<th>Q₅</th>
<th>Q₆</th>
<th>Y</th>
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<td>0</td>
<td>ON</td>
<td>ON</td>
<td>OFF</td>
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<td>ON</td>
<td>0</td>
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<tr>
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<td>1</td>
<td>OFF</td>
<td>OFF</td>
<td>ON</td>
<td>ON</td>
<td>ON</td>
<td>ON</td>
<td>1</td>
</tr>
</tbody>
</table>

11.37 The state of flip-flop output Q selects which signal is switched to the data converter/display driver by enabling one of the CMOS transmission gates. When Q=1, the wheel rotation sensor is selected. Q=0 selects the engine rotation sensor.

11.39 No. TTL power dissipation, and therefore the speed-power product, depends on the logic states of the device outputs, not on frequency.

11.41 74HCNN: pin replacement for TTL device; CMOS-compatible inputs; TTL-compatible outputs. 74HC4NNN: pin replacement for CMOS device; CMOS-compatible inputs; TTL-compatible outputs. 74HCTNN: pin replacement for TTL device; TTL-compatible inputs; TTL-compatible outputs. 74HCUNN: unbuffered CMOS outputs.

Chapter 12

12.1 Analog Voltage | Code | Analog Voltage | Code |
<table>
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<tr>
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<td>0 – 0.75</td>
<td>000</td>
<td>0.000 – 0.375</td>
<td>0000</td>
</tr>
<tr>
<td>0.75 – 2.25</td>
<td>001</td>
<td>0.375 – 1.125</td>
<td>0001</td>
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<tr>
<td>2.25 – 3.75</td>
<td>010</td>
<td>1.125 – 1.875</td>
<td>0010</td>
</tr>
<tr>
<td>3.75 – 5.25</td>
<td>011</td>
<td>1.875 – 2.625</td>
<td>0011</td>
</tr>
<tr>
<td>5.25 – 6.75</td>
<td>100</td>
<td>2.625 – 3.375</td>
<td>0100</td>
</tr>
<tr>
<td>6.75 – 8.25</td>
<td>101</td>
<td>3.375 – 4.125</td>
<td>0101</td>
</tr>
<tr>
<td>8.25 – 9.75</td>
<td>110</td>
<td>4.125 – 4.875</td>
<td>0110</td>
</tr>
<tr>
<td>9.75 – 12.00</td>
<td>111</td>
<td>4.875 – 5.625</td>
<td>0111</td>
</tr>
</tbody>
</table>

12.3 Fraction of T | Sine Voltage | Digital Code
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<tbody>
<tr>
<td>0</td>
<td>0 V</td>
<td>0000</td>
</tr>
<tr>
<td>T/8</td>
<td>4.59 V</td>
<td>0110</td>
</tr>
<tr>
<td>T/4</td>
<td>8.48 V</td>
<td>1011</td>
</tr>
<tr>
<td>3T/8</td>
<td>11.09 V</td>
<td>1111</td>
</tr>
<tr>
<td>T/2</td>
<td>12.00 V</td>
<td>1111</td>
</tr>
<tr>
<td>5T/8</td>
<td>11.09 V</td>
<td>1111</td>
</tr>
<tr>
<td>3T/4</td>
<td>8.48 V</td>
<td>1011</td>
</tr>
<tr>
<td>7T/8</td>
<td>4.59 V</td>
<td>0110</td>
</tr>
<tr>
<td>T</td>
<td>0 V</td>
<td>0000</td>
</tr>
</tbody>
</table>

12.5 6 bits, since $64 = 2^6$. Resolution = 500 mV/64 = 7.8125 mV.

12.7 $n+1$ (One extra bit for each doubling of the number of codes.)

12.9 a. $V_a = (\text{code/2}) \times V_{\text{ref}} = (12/16) \times 0.75 \times V_{\text{ref}}$; b. $V_a = (\text{code/2^2}) \times V_{\text{ref}} = (200/256) \times 0.78125 \times V_{\text{ref}}$; c. A 4-bit and 8-bit quantization of the same analog voltage are the same in the first four bits. The additional bit in the lower 4 bits adds an extra voltage to the analog output.

12.11 From most to least significant bits: 1 kΩ, 2 kΩ, 4 kΩ, 8 kΩ, 16 kΩ, 32 kΩ, 64 kΩ, 128 kΩ, 256 kΩ, 512 kΩ, 1024 kΩ, 2048 kΩ, 4096 kΩ, 8192 kΩ, 16,384 kΩ, 32,768 kΩ. All resistors greater than 64 kΩ are specified to three or more significant figures. These values, which are necessary to maintain conversion accuracy, are not available as commercial components.

12.13 a. $V_a = (15/16) \times 12 \text{ V} = 11.25 \text{ V}$; b. $V_a = (11/16) \times 12 \text{ V} = 8.25 \text{ V}$; c. $V_a = (6/16) \times 12 \text{ V} = 4.8 \text{ V}$; d. $V_a = (3/16) \times 12 \text{ V} = 2.25 \text{ V}$.

12.15 Resolution = (1/256)(2.2 kΩ × 12 V)/6.8 kΩ = 15.16 mV

12.19 There are only 16 steps in the waveform and they reach to 15/16 of the reference value. Therefore, the four least significant bits are stuck at logic LOW.

12.21 Offset error (OE) = 0.5 V; OE = 0.333 LSB; OE = 4.167% FS. Gain error = 0; Linearity error = 0

12.23 Linearity error (LE) = 0.175 V; LE = 0.35 LSB; LE = 4.375%. Gain error = 0; Offset error = 0

12.25 The priority encoder converts the highest active comparator voltage to a digital code. The enable input of the latch can be pulsed with a waveform having the same frequency as the sampling frequency.
12.27

<table>
<thead>
<tr>
<th>Bit</th>
<th>New Digital Value</th>
<th>Analog Equivalent from DAC</th>
<th>$V_{\text{analog}} \geq V_{\text{DAC}}$</th>
<th>Comparator Output</th>
<th>Accumulated Digital Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q₇</td>
<td>10000000</td>
<td>8 V</td>
<td>No</td>
<td>0</td>
<td>00000000</td>
</tr>
<tr>
<td>Q₆</td>
<td>01000000</td>
<td>4 V</td>
<td>Yes</td>
<td>1</td>
<td>01000000</td>
</tr>
<tr>
<td>Q₅</td>
<td>01100000</td>
<td>6 V</td>
<td>No</td>
<td>0</td>
<td>01000000</td>
</tr>
<tr>
<td>Q₄</td>
<td>01010000</td>
<td>5 V</td>
<td>No</td>
<td>0</td>
<td>01000000</td>
</tr>
<tr>
<td>Q₃</td>
<td>01001000</td>
<td>4.5 V</td>
<td>Yes</td>
<td>1</td>
<td>01001000</td>
</tr>
<tr>
<td>Q₂</td>
<td>01001100</td>
<td>4.75 V</td>
<td>Yes</td>
<td>1</td>
<td>01001100</td>
</tr>
<tr>
<td>Q₁</td>
<td>01001110</td>
<td>4.875 V</td>
<td>No</td>
<td>0</td>
<td>01001100</td>
</tr>
<tr>
<td>Q₀</td>
<td>01001101</td>
<td>4.8125 V</td>
<td>No</td>
<td>0</td>
<td>01001100</td>
</tr>
</tbody>
</table>

12.29 \((8 \text{ V}/12 \text{ V}) \times 16 = 10.667\). Since the SAR method of A/D conversion truncates a result, the new code value will be 1010. The new hex digit is A.

12.31 ±½ LSB

12.33 a. Integrating phase: The slope for a Full Scale input is given by:

\[-\frac{V_{\text{in}}}{RC} = -16 \text{ V}/(80 \text{ k}\Omega)(0.1 \text{ }\mu\text{F}) = -2 \text{ V/ms}.

Since the slope is proportional to the input voltage, the slope for a 3 V input is:

\((3/16) \times (-2 \text{ V/ms}) = -0.375 \text{ V/ms}\)

b. Rezeroing phase: At +2 V/ms, the integrator would take 8 seconds to rezero from Full Scale. This is always the slope when the circuit rezeros.

c. It would take \((3/16) \times (8 \text{ s}) = 1.5 \text{ s}\) to rezero for an input of 3 V.

d. The integrator waveform is similar to that for the input of 1/4 Full Scale shown in Figure 12.28 in the text.

e. Code = \((3/16) \times 256 = 48_{10} = 00110000_2\).

12.35 See Figure 12.29 in text.

12.37 200 kHz

12.39 The sampling frequency is 13/12 times the period of the sampled analog waveform. This is 1-1/12 periods, or 30 degrees greater than the sampled waveform. Thus one full cycle of the alias frequency is \(5.2 \mu\text{s} \times 12 = 62.4 \mu\text{s}\). The alias frequency is approximately 16 kHz.

12.41 --- adc_cont.vhd

--- State machine interface to ADC0808
--- Continuous conversion, single latch,
--- analog channel selected externally

LIBRARY ieee;
USE ieee.std_logic_1164.ALL;

ENTITY adc_cont IS
PORT(
clock, reset, eoc : IN STD_LOGIC;
sc, oe, en : OUT STD_LOGIC)
END adc_cont;
ARCHITECTURE adc OF adc_cont IS
TYPE state_type IS (start, wait1, wait2, read, store);
SIGNAL state: state_type;
SIGNAL outputs: STD_LOGIC_VECTOR (1 to 3);
BEGIN
PROCESS (clock, reset)
BEGIN
IF (reset = '0') THEN
    state <= start;
    outputs <= "000";
ELSIF (clock'EVENT and clock = '1') THEN
    CASE state IS
    WHEN start =>
        state <= wait1;
        outputs <= "100";
    WHEN wait1 =>
        IF (eoc = '1') THEN
            state <= wait1;
            outputs <= "000";
        ELSIF (eoc = '0') THEN
            state <= wait2;
            outputs <= "000";
        END IF;
    WHEN wait2 =>
        IF (eoc = '0') THEN
            state <= wait2;
            outputs <= "000";
        ELSIF (eoc = '1') THEN
            state <= read;
            outputs <= "011";
        END IF;
    WHEN read =>
        state <= store;
        outputs <= "010";
    WHEN store =>
        state <= start;
        outputs <= "000";
    END CASE;
END IF;
sc <= outputs(1);
oe <= outputs(2);
en <= outputs(3);
END PROCESS;
END adc;
See Figure ANS12.41.

12.43  23.5 kHz

FIGURE ANS12.41
To change the cell contents to a 0, we make the BIT line LOW and the ROW SELECT line HIGH. The ROW SELECT line gives access to the cell by turning on Q₄ and Q₆, completing the conduction path between the BIT lines and the flip flop inputs. The LOW on the BIT line pulls the gate of Q₃ LOW, turning it OFF. This breaks the conduction path from Q₃ drain to source and makes V_{DS₄} = V_{DD}, a logic HIGH. This HIGH is applied to the gate of Q₅, turning it ON. A conduction path is established between Q₅ drain and source, pulling the drain of Q₅ LOW. The cell now stores a logic 0.

A selected RAM cell is at the junction of an active ROW line and an active COLUMN line in a rectangular matrix of cells.

The DRAM in Problem 13.10 has 10 multiplexed ROW/COLUMN address lines. Adding one more line makes 11 lines, each of which are used for a ROW address and also a column address. This make a total of 22 lines, giving an address capacity of 2^{22} = 4M locations. Adding another address line gives 12 multiplexed lines, each used for ROW and COLUMN, giving a total of 2^{24} = 16M locations.

The primary difference between the different types of ROM is how easy each type is to program and erase.

Mask-programmed ROM has the data manufactured into the device, making it difficult to program and impossible to erase. It is relatively cheap to mass-produce and is useful for storing unchanging data that must always be retained, including after power failure. An example is the “boot ROM” in a personal computer that contains data for minimal start-up instructions.

UV-erasable EPROM is fairly expensive because of the specialized packaging it requires. It is user-programmable and can be easily erased by exposure to ultraviolet light when removed from the circuit. It is useful for unfinished designs, since stored data can be changed as development of a product proceeds.

EEPROM can be used for applications which require data to be stored after power is removed from a device, but which require periodic in-circuit changes of data. One example might be an EEPROM which stores the numbers of several local channels in a digitally-programmed car radio.

Unlike EEPROM, flash memory is organized into sectors that can be erased all at one time. One sector, called the boot block, can be protected against unauthorized erasure or modification, thus adding a level of security to the memory.

EEPROM has slower access time and smaller bit capacity than RAM. It also has a finite number of program/erase cycles.

FIFO: buffer for serial data transmission; LIFO: memory stack in a microcomputer

\[ 4K = 2^{12}. \text{ Range = 0000 0000 0000 to 1111 1111 1111 (000H to FFFH); End address = Start + Maximum = 2000H + FFFH = 2FFFH.} \]

\[ 8K = 2^{13}. \text{ Range = 0000 0000 0000 to 1 1111 1111 1111 (000H to FFFH); End = Start + Maximum = 6000H + 1FFFH = 7FFFH} \]

See Figure ANS13.21.

See Figure ANS13.23

FIGURE ANS13.21
13.25

<table>
<thead>
<tr>
<th>Device</th>
<th>Start Address</th>
<th>End Address</th>
<th>Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPROM</td>
<td>0000H</td>
<td>3FFFH</td>
<td>16K</td>
</tr>
<tr>
<td>SRAM₁</td>
<td>4000H</td>
<td>7FFFH</td>
<td>16K</td>
</tr>
<tr>
<td>SRAM₂</td>
<td>8000H</td>
<td>BFFFH</td>
<td>16K</td>
</tr>
<tr>
<td>SRAM₃</td>
<td>E000H</td>
<td>FFFFH</td>
<td>8K</td>
</tr>
</tbody>
</table>

13.27

<table>
<thead>
<tr>
<th>Device</th>
<th>Start Address</th>
<th>End Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000000H</td>
<td>0FFFFFFH</td>
</tr>
<tr>
<td>1</td>
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<td>1FFFFFFH</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>8000000H</td>
<td>8FFFFFFH</td>
</tr>
<tr>
<td>9</td>
<td>9000000H</td>
<td>9FFFFFFH</td>
</tr>
<tr>
<td>10</td>
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<td>AFFFFFFH</td>
</tr>
<tr>
<td>11</td>
<td>B000000H</td>
<td>BFFFFFFH</td>
</tr>
<tr>
<td>12</td>
<td>C000000H</td>
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<tr>
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<td>D000000H</td>
<td>DFFFFFFH</td>
</tr>
<tr>
<td>14</td>
<td>E000000H</td>
<td>EFFFFFFH</td>
</tr>
<tr>
<td>15</td>
<td>F000000H</td>
<td>FFFFFFFH</td>
</tr>
</tbody>
</table>