Mathematics

Teacher’s Guide
Unit 3

This book was collaboratively developed and reviewed by educators from public and private schools, colleges, and/or universities. We encourage teachers and other education stakeholders to email their feedback, comments, and recommendations to the Department of Education at action@deped.gov.ph.

We value your feedback and recommendations.

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Introduction

This Teacher's Guide has been prepared to provide teachers of Grade 10 Mathematics with guidelines on how to effectively use the Learner's Material to ensure that learners will attain the expected content and performance standards.

This book consists of four units subdivided into modules which are further subdivided into lessons. Each module contains the content and performance standards and the learning competencies that must be attained and developed by the learners which they could manifest through their products and performances.

The special features of this Teacher's Guide are:

A. **Learning Outcomes.** Each module contains the content and performance standards and the products and/or performances expected from the learners as a manifestation of their understanding.

B. **Planning for Assessment.** The assessment map indicates the type of assessment and categorized the objectives to be assessed into knowledge, process/skills, understanding, and performance.

C. **Planning for Teaching-Learning.** Each lesson has Learning Goals and Targets, a Pre-Assessment, Activities with answers, What to Know, What to Reflect on and Understand, What to Transfer, and Summary / Synthesis / Generalization.

D. **Summative Test.** After each module, answers to the summative test are provided to help the teachers evaluate how much the learners have learned.

E. **Glossary of Terms.** Important terms in the module are defined or clearly described.

F. **References and Other Materials.** This provides the teachers with the list of reference materials used, both print and digital.

We hope that this Teacher's Guide will provide the teachers with the necessary guide and information to be able to teach the lessons in a more creative, engaging, interactive, and effective manner.
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Figure 1. The Conceptual Framework of Mathematics Education
K to 12 BASIC EDUCATION CURRICULUM

CONCEPTUAL FRAMEWORK

Mathematics is one subject that pervades life at any age and in any circumstance. Thus, its value goes beyond the classroom and the school. Mathematics as a school subject, therefore, must be learned comprehensively and with much depth.

The twin goals of mathematics in the basic education levels, K-10, are the development of Critical Thinking and Problem Solving skills.

Critical thinking, according to Scriven and Paul (1987) is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action.

On the other hand, according to Polya (1945 & 1962), mathematical problem solving is finding a way around a difficulty, around an obstacle, and finding a solution to a problem that is unknown.

These two goals are to be achieved with an organized and rigorous curriculum content, a well-defined set of high-level skills and processes, desirable values and attitudes, and appropriate tools, taking into account the different contexts of Filipino learners.

There are five content areas in the curriculum, as adopted from the framework prepared by MATHTED and SEI (2010): Numbers and Number Sense, Measurement, Geometry, Patterns and Algebra, and Probability and Statistics.

The specific skills and processes to be developed are: knowing and understanding; estimating, computing and solving; visualizing and modelling; representing and communicating; conjecturing, reasoning, proving and decision-making; and applying and connecting.

The following values and attitudes are to be honed as well: accuracy, creativity, objectivity, perseverance, and productivity.

We recognize that the use of appropriate tools is necessary in teaching mathematics. These include: manipulative objects, measuring devices, calculators and computers, smart phones and tablet PCs, and the Internet.

We define context as a locale, situation, or set of conditions of Filipino learners that may influence their study and use of mathematics to develop critical thinking and problem solving skills. Contexts refer to beliefs, environment, language and culture that include traditions and practices, as well as the learner’s prior knowledge and experiences.
The framework is supported by the following underlying learning principles and theories: Experiential and Situated Learning, Reflective Learning, Constructivism, Cooperative Learning and Discovery and Inquiry-based Learning. The mathematics curriculum is grounded in these theories.

Experiential Learning as advocated by David Kolb is learning that occurs by making sense of direct everyday experiences. Experiential Learning theory defines learning as "the process whereby knowledge is created through the transformation of experience. Knowledge results from the combination of grasping and transforming experience" (Kolb, 1984, p. 41). Situated Learning, theorized by Lave and Wenger, is learning in the same context in which concepts and theories are applied.

Reflective Learning refers to learning that is facilitated by reflective thinking. It is not enough that learners encounter real-life situations. Deeper learning occurs when learners are able to think about their experiences and process these, allowing them the opportunity to make sense of and derive meaning from their experiences.

Constructivism is the theory that argues that knowledge is constructed when the learner is able to draw ideas from his/her own experiences and connect them to new ideas.

Cooperative Learning puts premium on active learning achieved by working with fellow learners as they all engage in a shared task. The mathematics curriculum allows for students to learn by asking relevant questions and discovering new ideas. Discovery Learning and Inquiry-based Learning (Bruner, 1961) support the idea that students learn when they make use of personal experiences to discover facts, relationships, and concepts.
K to 12 BASIC EDUCATION CURRICULUM

BRIEF COURSE DESCRIPTION

Mathematics from K-10 is a skills subject. As such, it is all about quantities, shapes and figures, functions, logic, and reasoning. Mathematics is also a tool of science and a language complete with its own notations and symbols and “grammar” rules, with which concepts and ideas are effectively expressed.

The contents of mathematics include Numbers and Number Sense, Measurement, Geometry, Patterns & Algebra and Statistics and Probability.

Numbers and Number Sense as a strand include concepts of numbers, properties, operations, estimations, and their applications.

Measurement as a strand includes the use of numbers and measures to describe, understand, and compare mathematical and concrete objects. It focuses on attributes such as length, mass and weight, capacity, time, money, and temperature, as well as applications involving perimeter, area, surface area, volume, and angle measure.

Geometry as a strand includes properties of two- and three-dimensional figures and their relationships, spatial visualization, reasoning, and geometric modelling and proofs.

Patterns and Algebra as a strand studies patterns, relationships, and changes among shapes and quantities. It includes the use of algebraic notations and symbols, equations, and most importantly, functions, to represent and analyze relationships.

Statistics and Probability as a strand is all about developing skills in collecting and organizing data using charts, tables, and graphs; understanding, analyzing and interpreting data; dealing with uncertainty; and making predictions about outcomes.

The K to 10 Mathematics Curriculum provides a solid foundation for Mathematics at Grades 11 to 12. More importantly, it provides necessary concepts and life skills needed by Filipino learners as they proceed to the next stage in their life as learners and as citizens of the Philippines.
K to 12 BASIC EDUCATION CURRICULUM

**LEARNING AREA STANDARD:** The learner demonstrates understanding and appreciation of key concepts and principles of mathematics as applied - using appropriate technology - in problem solving, critical thinking, communicating, reasoning, making connections, representations, and decisions in real life.

**KEY STAGE STANDARDS:**

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<thead>
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<th>K – 3</th>
<th>4 – 6</th>
<th>7 – 10</th>
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<tbody>
<tr>
<td>At the end of Grade 3, the learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to 10,000 and the four fundamental operations including money, ordinal numbers up to 100th, basic concepts of fractions); measurement (time, length, mass, capacity, area of square and rectangle); geometry (2-dimensional and 3-dimensional objects, lines, symmetry, and tessellation); patterns and algebra (continuous and repeating patterns and number sentences); statistics and probability (data collection and representation in tables, pictographs and bar graphs and outcomes); as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
<td>At the end of Grade 6, the learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers, number theory, fractions, decimals, ratio and proportion, percent, and integers); measurement (time, speed, perimeter, circumference and area of plane figures, volume and surface area of solid/space figures, temperature and meter reading); geometry (parallel and perpendicular lines, angles, triangles, quadrilaterals, polygons, circles, and solid figures); patterns and algebra (continuous and repeating patterns, number sentences, sequences, and simple equations); statistics and probability (bar graphs, line graphs and pie graphs, simple experiment, and experimental probability) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
<td>At the end of grade 10, the learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (sets and real numbers); measurement (conversion of units); patterns and algebra (linear equations and inequalities in one and two variables, linear functions, systems of linear equations, and inequalities in two variables, exponents and radicals, quadratic equations, inequalities, functions, polynomials, and polynomial equations and functions); geometry (polygons, axiomatic structure of geometry, triangle congruence, inequality and similarity, and basic trigonometry); statistics and probability (measures of central tendency, variability and position; combinatorics and probability) as applied - using appropriate technology - in critical thinking, problem solving, communicating, reasoning, making connections, representations, and decisions in real life.</td>
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K to 12 BASIC EDUCATION CURRICULUM

**GRADE LEVEL STANDARDS:**

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<th>GRADE LEVEL</th>
<th>GRADE LEVEL STANDARDS</th>
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<tr>
<td>K</td>
<td>The learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to 20, basic concepts on addition and subtraction); geometry (basic attributes of objects), patterns and algebra (basic concept of sequence and number pairs); measurement (time, location, non-standard measures of length, mass and capacity); and statistics and probability (data collection and tables) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations and decisions in real life.</td>
</tr>
<tr>
<td>GRADE 1</td>
<td>The learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to 100, ordinal numbers up to 10\textsuperscript{th}, money up to PhP100, addition and subtraction of whole numbers, and fractions $\frac{1}{2}$ and $\frac{1}{4}$); geometry (2- and 3-dimensional objects); patterns and algebra (continuous and repeating patterns and number sentences); measurement (time, non-standard measures of length, mass, and capacity); and statistics and probability (tables, pictographs, and outcomes) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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<td>GRADE 2</td>
<td>The learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to PhP1000, ordinal numbers up to 20\textsuperscript{th}, money up to PhP100, the four fundamental operations of whole numbers, and unit fractions); geometry (basic shapes, symmetry, and tessellations); patterns and algebra (continuous and repeating patterns and number sentences); measurement (time, length, mass, and capacity); and statistics and probability (tables, pictographs, and outcomes) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
</tr>
<tr>
<td>GRADE 3</td>
<td>The learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to 10 000; ordinal numbers up to 100\textsuperscript{th}; money up to PhP1 000; the four fundamental operations of whole numbers; proper and improper fractions; and similar, dissimilar, and equivalent fractions); geometry (lines, symmetry, and tessellations); patterns and algebra (continuous and repeating patterns and number sentences); measurement (conversion of time, length, mass and capacity, area of square and rectangle); and statistics and probability (tables, bar graphs, and outcomes) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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<td>GRADE 4</td>
<td>The learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to 100 000, multiplication and division of whole numbers, order of operations, factors and multiples, addition and subtraction of fractions, and basic concepts of decimals including money); geometry (lines, angles, triangles, and quadrilaterals); patterns and algebra (continuous and repeating patterns and number sentences); measurement (time, perimeter, area, and volume); and statistics and probability (tables, bar graphs, and simple experiments) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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<td>GRADE 5</td>
<td>The learner demonstrates understanding and appreciation of key concepts and skills involving numbers and number sense (whole numbers up to 10 000 000, order of operations, factors and multiples, fractions and decimals including money, ratio and proportion, percent); geometry (polygons, circles, solid figures); patterns and algebra (sequence and number sentences); measurement (time, circumference, area, volume, and temperature); and statistics and probability (tables, line graphs and experimental probability) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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<td>GRADE 6</td>
<td>The learner demonstrates understanding of key concepts and skills involving numbers and number sense (divisibility, order of operations, fractions and decimals including money, ratio and proportion, percent, integers); geometry (plane and solid figures); patterns and algebra (sequence, expression, and equation); measurement (rate, speed, area, surface area, volume, and meter reading); and statistics and probability (tables, pie graphs, and experimental and theoretical probability) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
</tr>
<tr>
<td>GRADE 7</td>
<td>The learner demonstrates understanding of key concepts and principles of numbers and number sense (sets and real number system); measurement (conversion of units of measurement); patterns and algebra (algebraic expressions and properties of real numbers as applied in linear equations and inequalities in one variable); geometry (sides and angles of polygons); and statistics and probability (data collection and presentation, and measures of central tendency and variability) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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K to 12 BASIC EDUCATION CURRICULUM

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<tr>
<td>GRADE 8</td>
<td>The learner demonstrates understanding of key concepts and principles of patterns and algebra (factors of polynomials, rational algebraic expressions, linear equations and inequalities in two variables, systems of linear equations and inequalities in two variables); geometry (axiomatic structure of geometry, triangle congruence, inequalities in a triangle, and parallel and perpendicular lines); and statistics and probability (probability of simple events) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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<tr>
<td>GRADE 9</td>
<td>The learner demonstrates understanding of key concepts and principles of patterns and algebra (quadratic equations and inequalities, quadratic functions, rational algebraic equations, variations, and radicals) and geometry (parallelograms and triangle similarities and basic concepts of trigonometry) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
</tr>
<tr>
<td>GRADE 10</td>
<td>The learner demonstrates understanding of key concepts and principles of patterns and algebra (sequences, series, polynomials, polynomial equations, and polynomial functions); geometry (circles and coordinate geometry); and statistics and probability (combinatorics and probability, and measures of position) as applied - using appropriate technology - in critical thinking, problem solving, reasoning, communicating, making connections, representations, and decisions in real life.</td>
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<th>PERFORMANCE STANDARDS</th>
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<tr>
<td>Patterns and Algebra</td>
<td>demonstrates understanding of key concepts of sequences, polynomials and polynomial equations.</td>
<td>is able to formulate and solve problems involving sequences, polynomials and polynomial equations in different disciplines through appropriate and accurate representations.</td>
<td>1. generates patterns.***</td>
<td>M10AL-la-1</td>
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<td>2. illustrates an arithmetic sequence</td>
<td>M10AL-lb-1</td>
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<td>3. determines arithmetic means and nth term of an arithmetic sequence.***</td>
<td>M10AL-lb-c-1</td>
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<td>4. finds the sum of the terms of a given arithmetic sequence.***</td>
<td>M10AL-ic-2</td>
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<td>5. illustrates a geometric sequence.</td>
<td>M10AL-id-1</td>
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<td>6. differentiates a geometric sequence from an arithmetic sequence.</td>
<td>M10AL-id-2</td>
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<td>7. differentiates a finite geometric sequence from an infinite geometric sequence.</td>
<td>M10AL-id-3</td>
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<td>8. determines geometric means and nth term of a geometric sequence.***</td>
<td>M10AL-le-1</td>
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<td>9. finds the sum of the terms of a given finite or infinite geometric sequence.***</td>
<td>M10AL-le-2</td>
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<td>10. illustrates other types of sequences (e.g., harmonic, Fibonacci).</td>
<td>M10AL-if-1</td>
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<td>11. solves problems involving sequences.</td>
<td>M10AL-if-2</td>
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<td>12. performs division of polynomials using long division and synthetic division.</td>
<td>M10AL-lg-1</td>
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<td>13. proves the Remainder Theorem and the Factor Theorem.</td>
<td>M10AL-lg-2</td>
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<td>14. factors polynomials.</td>
<td>M10AL-lh-1</td>
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<td>15. illustrates polynomial equations.</td>
<td>M10AL-li-1</td>
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<td>CONTENT</td>
<td>CONTENT STANDARDS</td>
<td>PERFORMANCE STANDARDS</td>
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</tr>
<tr>
<td>Geometry</td>
<td>demonstrates understanding of key concepts of circles and coordinate geometry.</td>
<td>is able to formulate and find solutions to challenging situations involving circles and other related terms in different disciplines through appropriate and accurate representations.</td>
<td>1. derives inductively the relations among chords, arcs, central angles, and inscribed angles.</td>
<td>M10GE-lle-c-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>22. proves theorems related to chords, arcs, central angles, and inscribed angles.</td>
<td>M10GE-lle-c-d-1</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>23. illustrates secants, tangents, segments, and sectors of a circle.</td>
<td>M10GE-lle-e-1</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>24. proves theorems on secants, tangents, and segments.</td>
<td>M10GE-lle-f-1</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>25. solves problems on circles.</td>
<td>M10GE-lle-f-2</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>26. derives the distance formula.</td>
<td>M10GE-lle-g-1</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>27. applies the distance formula to prove some geometric properties.</td>
<td>M10GE-lle-g-2</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>28. illustrates the center-radius form of the equation of a circle.</td>
<td>M10GE-lle-h-1</td>
</tr>
<tr>
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<td></td>
<td>29. determines the center and radius of a circle given its equation and vice versa.</td>
<td>M10GE-lle-h-2</td>
</tr>
<tr>
<td>Grade 10- SECOND QUARTER</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Patterns and Algebra</td>
<td>demonstrates understanding of key concepts of polynomial function.</td>
<td>is able to conduct systematically a mathematical investigation involving polynomial functions in different fields.</td>
<td>19. illustrates polynomial functions.</td>
<td>M10AL-lla-1</td>
</tr>
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<td>20. graphs polynomial functions.</td>
<td>M10AL-lla-b-1</td>
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<td>21. solves problems involving polynomial functions.</td>
<td>M10AL-llb-2</td>
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<td>CONTENT</td>
<td>CONTENT STANDARDS</td>
<td>PERFORMANCE STANDARDS</td>
<td>LEARNING COMPETENCY</td>
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<td>The learner...</td>
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<tr>
<td>Grade 10- THIRD QUARTER</td>
<td></td>
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<tr>
<td>Statistics and Probability</td>
<td>demonstrates understanding of key concepts of combinatorics and probability.</td>
<td>is able to use precise counting technique and probability in formulating conclusions and making decisions.</td>
<td>33. illustrates the permutation of objects.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>34. derives the formula for finding the number of permutations of $n$ objects taken $r$ at a time.</td>
<td></td>
<td>34. illustrates the permutation of objects.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>35. solves problems involving permutations.</td>
<td></td>
<td>35. solves problems involving permutations.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>36. illustrates the combination of objects.</td>
<td></td>
<td>36. illustrates the combination of objects.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>37. differentiates permutation from combination of $n$ objects taken $r$ at a time.</td>
<td></td>
<td>37. differentiates permutation from combination of $n$ objects taken $r$ at a time.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>38. derives the formula for finding the number of combinations of $n$ objects taken $r$ at a time</td>
<td></td>
<td>38. derives the formula for finding the number of combinations of $n$ objects taken $r$ at a time</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>39. solves problems involving permutations and combinations.</td>
<td></td>
<td>39. solves problems involving permutations and combinations.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>40. illustrates events, and union and intersection of events.</td>
<td></td>
<td>40. illustrates events, and union and intersection of events.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>41. illustrates the probability of a union of two events.</td>
<td></td>
<td>41. illustrates the probability of a union of two events.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td></td>
<td>42. finds the probability of $(A \cup B)$.</td>
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<td>42. finds the probability of $(A \cup B)$.</td>
<td>M10SP-illb-1</td>
</tr>
<tr>
<td>CONTENT</td>
<td>CONTENT STANDARDS</td>
<td>PERFORMANCE STANDARDS</td>
<td>LEARNING COMPETENCY</td>
<td>CODE</td>
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<td></td>
<td>The learner...</td>
<td>The learner...</td>
<td>The learner...</td>
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<tr>
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<td></td>
<td>43. illustrates mutually exclusive events.</td>
<td>M10SP-III-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>44. solves problems involving probability.</td>
<td>M10SP-III-j-1</td>
</tr>
<tr>
<td>Grade 10- FOURTH QUARTER</td>
<td>Statistics and Probability</td>
<td>demonstrates understanding of key concepts of measures of position.</td>
<td>45. illustrates the following measures of position: quartiles, deciles and percentiles.***</td>
<td>M10SP-IVa-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>is able to conduct systematically a mini-research applying the different statistical methods.</td>
<td>46. calculates a specified measure of position (e.g. 90th percentile) of a set of data.</td>
<td>M10SP-IVb-1</td>
</tr>
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<td></td>
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<td>47. interprets measures of position.</td>
<td>M10SP-IVc-1</td>
</tr>
<tr>
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<td></td>
<td>48. solves problems involving measures of position.</td>
<td>M10SP-IVd-e-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>49. formulates statistical mini-research.</td>
<td>M10SP-IVf-g-1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>50. uses appropriate measures of position and other statistical methods in analyzing and interpreting research data.</td>
<td>M10SP-IVh-j-1</td>
</tr>
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</table>

*** Suggestion for ICT enhanced lesson when available and where appropriate
<table>
<thead>
<tr>
<th>CODE</th>
<th>NS</th>
<th>GE</th>
<th>AL</th>
<th>ME</th>
<th>SP</th>
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<tbody>
<tr>
<td>DOMAIN/COMPONENT</td>
<td>Number Sense</td>
<td>Geometry</td>
<td>Patterns and Algebra</td>
<td>Measurement</td>
<td>Statistics and Probability</td>
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</tbody>
</table>

### Code Book Legend

#### Sample: M7AL-IIg-2

<table>
<thead>
<tr>
<th>LEgend</th>
<th>Sample</th>
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</thead>
<tbody>
<tr>
<td>First Entry</td>
<td>M7</td>
</tr>
<tr>
<td>Upper Case Letter/s</td>
<td>AL</td>
</tr>
<tr>
<td>Roman Numeral</td>
<td>II</td>
</tr>
<tr>
<td>Arabic Number</td>
<td>2</td>
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</table>

<table>
<thead>
<tr>
<th>Sample</th>
<th>Lower Case Letter/s</th>
<th>Quarter</th>
<th>Week</th>
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</thead>
<tbody>
<tr>
<td>M7AL-IIg-2</td>
<td>-</td>
<td>Second Quarter</td>
<td>Week seven</td>
</tr>
</tbody>
</table>

*Zero if no specific quarter.*

*Put a hyphen (-) in between letters to indicate more than a specific week.*

Solves problems involving algebraic expressions.
Module 6: Permutations and Combinations

A. Learning Outcomes

Content Standard:

The learner demonstrates understanding of key concepts of combinatorics.

Performance Standard:

The learner is able to use precise counting techniques in formulating conclusions and in making wise decisions.

Unpacking the Standards for Understanding

<table>
<thead>
<tr>
<th>Subject: Mathematics 10</th>
<th>Learning Competencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter: Third Quarter</td>
<td>1. Illustrate the permutation of objects</td>
</tr>
<tr>
<td>Topic: Permutations,</td>
<td>2. Derive the formula for finding the number of permutations of ( n ) objects taken ( r ) at a time</td>
</tr>
<tr>
<td>Combinations</td>
<td>3. Solve problems involving permutations</td>
</tr>
<tr>
<td>Lessons:</td>
<td>4. Illustrate the combination of objects</td>
</tr>
<tr>
<td>1. Permutations</td>
<td>5. Differentiate permutation from combination of ( n ) objects taken ( r ) at a time</td>
</tr>
<tr>
<td>• Illustration of the</td>
<td>6. Derive the formula for finding the number of combinations of ( n ) objects taken ( r ) at a time</td>
</tr>
<tr>
<td>• Permutations of ( n</td>
<td>7. Solve problems involving permutations and combinations</td>
</tr>
<tr>
<td>Objects Taken ( r ) at a Time</td>
<td></td>
</tr>
<tr>
<td>• Solving Problems</td>
<td></td>
</tr>
<tr>
<td>• Illustration of the</td>
<td></td>
</tr>
<tr>
<td>• Combinations of ( n</td>
<td></td>
</tr>
<tr>
<td>Objects Taken ( r ) at a Time</td>
<td></td>
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<tr>
<td>• Solving Problems</td>
<td></td>
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<tr>
<td>• Illustration of the</td>
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<tr>
<td>• Permutations and</td>
<td></td>
</tr>
<tr>
<td>• Combinations of ( n</td>
<td></td>
</tr>
<tr>
<td>Objects Taken ( r ) at a Time</td>
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<td></td>
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</tbody>
</table>

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**Essential Understanding**
Students will understand that the concepts of permutations and combinations are important tools in forming conclusions and in making wise decisions.

**Essential Question:**
How do the concepts of permutations and combinations help in forming conclusions and in making wise decisions?

**Transfer Goal:**
Students will be able to apply the key concepts of permutations and combinations in forming conclusions and in making wise decisions.

### B. Planning for Assessment

**Product/Performance**

The following are products and performances that students are expected to come up with in this module.

1. Enumerate situations in real life that illustrate permutations and combinations
2. Formulate equations involving permutations and combinations that represent real-life situations
3. Solve equations involving permutations and combinations
4. Formulate and solve problems that involve permutations and combinations
5. Role-play to demonstrate the applications of the concepts of permutations and combinations in formulating conclusions and in making wise decisions.

**Assessment Map**

<table>
<thead>
<tr>
<th>TYPE</th>
<th>KNOWLEDGE</th>
<th>PROCESS/ SKILLS</th>
<th>UNDERSTANDING</th>
<th>PERFORMANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Assessment/Diagnostic</td>
<td>Pre-Test: Part I</td>
<td>Pre-Test: Part I</td>
<td>Pre-Test: Part I</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Identifying situations that involve permutations or combinations</td>
<td>Solving equations involving permutations and combinations of $n$ objects taken $r$ at a time</td>
<td>Solving problems involving permutations and combinations</td>
<td>Differentiating between permutations and combinations</td>
</tr>
<tr>
<td>TYPE</td>
<td>KNOWLEDGE</td>
<td>PROCESS/ SKILLS</td>
<td>UNDERSTANDING</td>
<td>PERFORMANCE</td>
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</tr>
<tr>
<td>Pre-Test: Part II Situational Analysis</td>
<td>Identifying the different food choices that will be offered in a restaurant</td>
<td>Writing the mathematical expressions or equations describing the situation</td>
<td>Solving problems on permutations and combinations</td>
<td>Formulating problems related to a given situation</td>
</tr>
<tr>
<td>Pre-Test: Part II Situational Analysis</td>
<td>Determining the mathematics concepts or principles that are involved in the situation</td>
<td>Solving the equations formed</td>
<td>Explaining the bases of the sample menu for the week</td>
<td>Presenting a sample menu for the week</td>
</tr>
<tr>
<td>Formative Quiz: Lesson 1</td>
<td>Identifying situations that involve permutations</td>
<td>Solving equations involving permutations</td>
<td>Differentiating situations that involve permutations from those that do not</td>
<td>Demonstrating how knowledge of permutations can help one formulate conclusions and make wise decisions</td>
</tr>
<tr>
<td>Formative Quiz: Lesson 1</td>
<td>Citing situations that illustrate permutations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quiz: Lesson 2</td>
<td>Identifying situations that involve combinations</td>
<td>Solving equations involving combinations</td>
<td>Differentiating situations that involve combinations from those that involve permutations</td>
<td></td>
</tr>
<tr>
<td>Quiz: Lesson 2</td>
<td>Citing situations that illustrate combinations</td>
<td></td>
<td>Solving problems involving combinations</td>
<td></td>
</tr>
<tr>
<td>Quiz: Lesson 2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>TYPE</td>
<td>KNOWLEDGE</td>
<td>PROCESS/ SKILLS</td>
<td>UNDERSTANDING</td>
<td>PERFORMANCE</td>
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</tr>
<tr>
<td></td>
<td>Identifying situations that involve permutations or combinations</td>
<td>Solving equations involving permutations and combinations of ( n ) objects taken ( r ) at a time</td>
<td>Solving problems involving permutations and combinations</td>
<td>Products and performances related to or involving permutations and combinations</td>
</tr>
<tr>
<td></td>
<td>Differentiating between permutations and combinations</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-Test:</td>
<td>Post-Test: Part II Situational Analysis</td>
<td>Post-Test: Part II Situational Analysis</td>
<td>Post-Test: Part II Situational Analysis</td>
<td>Post-Test: Part II Situational Analysis</td>
</tr>
<tr>
<td>Part II</td>
<td>Identifying situations where permutations and combinations are used</td>
<td>Writing expressions and equations that describe the situation</td>
<td>Solving problems</td>
<td>Formulating problems related to the given situation</td>
</tr>
<tr>
<td>Situational</td>
<td></td>
<td></td>
<td>Explaining how the problems/situations posed an opportunity for making conclusions and wise decisions</td>
<td>Making the best groupings of the members of the class for the class activity</td>
</tr>
<tr>
<td>Analysis</td>
<td></td>
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<tr>
<td>Self –</td>
<td>Journal Writing:</td>
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<tr>
<td>Assessment</td>
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</tr>
<tr>
<td></td>
<td>Expressing understanding of permutations and combinations and their</td>
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<td></td>
<td>applications or use in real life</td>
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</table>
Assessment Matrix (Summative Test)

<table>
<thead>
<tr>
<th>Levels of Assessment</th>
<th>What will I assess?</th>
<th>How will I assess?</th>
<th>How will I score?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knowledge 15%</td>
<td>The learner demonstrates understanding of key concepts of permutations and combinations. Illustrate permutations and combinations.</td>
<td>Paper and Pencil Test</td>
<td>1 point for every correct response</td>
</tr>
<tr>
<td>Process/Skills 25%</td>
<td>Solve equations involving permutations and combinations</td>
<td>Part I items 8, 11, 12, 13, 14, 15, 16, 17, 19, 20 Part II items 6,7</td>
<td>1 point for every correct response</td>
</tr>
<tr>
<td>Understanding 30%</td>
<td>Solving problems involving permutations and combinations</td>
<td>Part I items 7, 9, 10, 18, 21, 22, 23, 24, 25, 26, 27, 28 Part II items 7</td>
<td>1 point for every correct response</td>
</tr>
<tr>
<td>Product/Performance 30%</td>
<td>Use precise counting techniques in formulating conclusions and in making wise decisions</td>
<td>Part II item 5</td>
<td>Rubric on Problem Formulated and Solved</td>
</tr>
</tbody>
</table>

C. Planning for Teaching-Learning

This module covers the key concepts of Combinatorics, namely, Permutations and Combinations. It is divided into two lessons: Lesson 1 – Permutations and Lesson 2 – Combinations.
In Lesson 1, students will identify real-life situations that involve permutations, illustrate permutations of objects, and solve problems involving permutations of \( n \) objects taken \( r \) at a time.

In Lesson 2, students will identify situations that involve combinations, differentiate them from those that involve permutations, and solve problems that involve combinations, or both permutations and combinations.

In both lessons, students are given appropriate activities to develop their knowledge, skills, and understanding of permutations and combinations, while utilizing the other mathematics concepts they have previously learned.

As an introduction to the lesson, show the students the pictures below, then ask the questions that follow.

Look at the pictures. Have you ever wondered why some locks such as the one shown have codes in them? Do you know why a shorter code is “weak,” while a longer code is a “strong” personal password in a computer account? Have you ever realized that there are several possible ways in doing most tasks or activities, like planning a seating arrangement or predicting the possible outcomes of a race? Have you ever been aware that there are numerous possible choices in selecting from a set, like deciding which combination of dishes to serve in a catering service or deciding which dishes to order from a menu? Do you know that awareness of these can help you form conclusions and make wise decisions?

Encourage students to find the answers to these questions and discover the various applications of permutations and combinations in real life through this module.
Objectives:

After the students have gone through the lessons in this module, they should be able to:
1. identify real-life situations that illustrate permutations and combinations;
2. write mathematical expressions and equations to represent situations involving permutations and combinations;
3. solve equations involving permutations and combinations; and
4. formulate and solve problems involving permutations and combinations

PRE-ASSESSMENT:

Check students’ prior knowledge, skills, and understanding of mathematics concepts related to Permutations and Combinations. Assessing these will facilitate teaching and students’ understanding of the lessons in this module.

Answer Key

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<tbody>
<tr>
<td>Part II</td>
<td>(Use the Rubric to rate students’ works/outputs.)</td>
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Solutions of some of the problems can be found at the end of this section.

LEARNING GOALS AND TARGETS:

Students are expected to demonstrate understanding of key concepts of permutations and combinations, and formulate and solve problems involving these concepts.

Lesson 1: Permutations

What to KNOW

Assess students’ knowledge of the basic counting technique called the Fundamental Counting Principle (FCP). Assessing this will facilitate students’ understanding of permutations. Remind them that as they go through the lesson,
they must keep in mind the important question: *How does the concept of permutations help in formulating conclusions and in making wise decisions?*

Let students do Activity 1 in pairs, using the “Think-Pair-Share” strategy. Since it is a review activity, this strategy is suggested to allow students to think individually about the answers first, make their own list as required, then discuss with a partner to come up with a final answer, which they will share to the class afterwards. This activity aims to make them list their answers first and then lead them to recall the FCP.

### Activity 1: Can You Show Me The Way?

<table>
<thead>
<tr>
<th>Answer Key</th>
</tr>
</thead>
</table>
| **A.** 1. blouses - stripes, with ruffles, long-sleeved, sleeveless  
  skirt - red, pink, black  
  possible outfits:  
  | blouse - skirt | blouse - skirt  
  | stripes - red | long-sleeved - red  
  | stripes - pink | long-sleeved - pink  
  | stripes - black | long-sleeved - black  
  | ruffles - red | sleeveless - red  
  | ruffles - pink | sleeveless - pink  
  | ruffles - black | sleeveless - black  
  |  
  | 2. 12 blouse-and-skirt pairs are possible  
  | 3. Another way of answering item 1 is through a tree diagram.  
  | blouse | skirt  
  | stripes | red  
  | | pink  
  | | black  
  | ruffles | red  
  | | pink  
  | | black  
  | long-sleeved | red  
  | | pink  
  | | black  
  | sleeveless | red  
  | | pink  
  | | black  
  |  
  | Students must realize/recall that the number of possible blouse-skirt pairs can be obtained by using the FCP:  
  4 choices for blouse x 3 choices for skirt = 12 possible pairs  
  |
B. 1. Possible codes containing the four digits 7, 4, 3, 1:
   (The list must be made systematically to ensure completeness.)
   
<p>| | | | |</p>
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<tbody>
<tr>
<td>1347</td>
<td>3147</td>
<td>4137</td>
<td>7134</td>
</tr>
<tr>
<td>1374</td>
<td>3174</td>
<td>4173</td>
<td>7143</td>
</tr>
<tr>
<td>1437</td>
<td>3417</td>
<td>4317</td>
<td>7314</td>
</tr>
<tr>
<td>1473</td>
<td>3471</td>
<td>4713</td>
<td>7341</td>
</tr>
<tr>
<td>1734</td>
<td>3714</td>
<td>4713</td>
<td>7413</td>
</tr>
<tr>
<td>1743</td>
<td>3741</td>
<td>4731</td>
<td>7431</td>
</tr>
</tbody>
</table>

2. There are 24 possible codes.
3. The list is quite long.

Again, using the Fundamental Counting Principle:

1st digit  2nd digit  3rd digit  4th digit
4 choices • 3 choices • 2 choices • 1 choices = 24 possible choices

Ask or point out to the students why the number of choices is decreasing.

Answers to Guide Questions:

a. We determined the different possibilities asked for by listing. We also used tree diagram as well as the Fundamental Counting Principle. Another way of finding the answers is by making a table. Making a table is especially appropriate if the problem/situation involves a pair of dice. You may make an example on the dice after this particular activity.

b. It is hard making a list when the list is long.

Having recalled the Fundamental Counting Principle, let the students do Activity 2. This activity provides them some more opportunities to use the Fundamental Counting Principle. When students present the solutions to the class, ask some questions regarding the parts of the solution, such as why are the factors being used decreasing, or how do they know how many factors will be used in the multiplication process, and other pertinent questions.

Activity 2. Count Me In!

Answer Key

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 720</td>
<td>6. 11 880</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. 216</td>
<td>7. 15 120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. 720</td>
<td>8. 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. 360</td>
<td>9. 120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. 72</td>
<td>10. 6720</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Answers to Guide Questions:

a. We found the answer to each question by multiplying the number of ways that each successive subtask can be done in order to finish the whole task. In situations which involve choosing, we multiply the number of choices for the first position by the number of choices for the second position by the number of choices for the third position, and so on.

b. Solutions:

1. Number of possible outcomes for winners is:
   
   \[
   N = 10 \text{ possible winners} \times 9 \text{ possible winners} \times 8 \text{ possible winners}
   \]
   
   \[= 720 \text{ possible outcomes for the top three in the race} \]

2. \[N = \text{number of shirts} \times \text{number of pants} \times \text{number of shoes} \]
   
   \[= (12)(6)(3) \]
   
   \[= 216 \text{ possible outfits} \]

3. \[N = (6)(5)(4)(3)(2)(1) \] because there are 6 choices for the 1st position, 5 choices left for the 2nd position, 4 choices for the 3rd, and so on.
   
   \[N = 720 \text{ possible arrangements of the plants} \]

4. \[N = (6)(5)(4)(3) \]
   
   There are 6 choices for the thousands digit, 5 choices left for the hundreds digit, 4 choices left for the tens digit, and 3 choices left for the ones digit
   
   \[N = 360 \text{ different numbers} \]

5. Town A to Town B to Town C back to Town B and then to Town A
   
   A → B → C → B → A
   
   3 roads → 4 roads → 3 roads → 2 roads
   
   \[N = (3)(4)(3)(2) \]
   
   \[N = 72 \text{ possible ways of going from town A to town C and back to town A through town B} \]

6. \[N = (12)(11)(10)(9) \]
   
   \[N = 11,880 \text{ possible ways of electing the president, vice president, secretary and treasurer} \]

7. \[N = (9)(8)(7)(6)(5) \]
   
   \[N = 15,120 \text{ possible ways of placing 9 books on a shelf if there is space enough for only 5 books} \]

8. \[N = (3)(2) \]
   
   \[N = 6 \text{ possible meals} \]
9. \( N = (5)(4)(3)(2)(1) \) or \( 5! \)
   \( N = 120 \) possible ways of arranging the 5 people in a row for picture taking

10. \( N = (8)(7)(6)(5)(4) \)
    \( N = 6720 \)

In Activity 3, students will recognize situations or tasks in which order or arrangement is considered important. Divide the class into groups of 4. All groups will answer all items but call on a particular group to share to the class their results in a specific item.

**Activity 3. Does order matter?**

<table>
<thead>
<tr>
<th>Answer Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. In situations 1, 3, 6, 7, and 9, order is important.</td>
</tr>
<tr>
<td>2. <strong>Number 1:</strong> example:</td>
</tr>
<tr>
<td>1st place – runner number 8</td>
</tr>
<tr>
<td>2nd place – runner number 5</td>
</tr>
<tr>
<td>3rd place – runner number 4</td>
</tr>
<tr>
<td><strong>Number 3:</strong> Example: She may arrange the plants according to height, or according to kind, according to appearance, or any basis she wants.</td>
</tr>
<tr>
<td><strong>Number 6:</strong> Example: 1 possible result is:</td>
</tr>
<tr>
<td>President – Mrs. Cavinta</td>
</tr>
<tr>
<td>Vice President – Ms. Ternida</td>
</tr>
<tr>
<td>Secretary – Mrs. Perez</td>
</tr>
<tr>
<td>Treasurer – Mr. Cabrera</td>
</tr>
<tr>
<td><em>This is different from other possible results, such as:</em></td>
</tr>
<tr>
<td>President – Mr. Cabrera</td>
</tr>
<tr>
<td>Vice President – Mrs. Perez</td>
</tr>
<tr>
<td>Secretary – Mrs. Cavinta</td>
</tr>
<tr>
<td>Treasurer – Mrs. Ternida</td>
</tr>
<tr>
<td><strong>Number 7:</strong> Example: Suppose the different books have titles <em>Geometry, Algebra, Trigonometry, Statistics, Biology, Physics, Chemistry, Literature,</em> and <em>Health.</em> Let us code them with letters <em>G, A, T, S, B, P, C, L, H,</em> respectively.</td>
</tr>
<tr>
<td>One possible arrangement is:</td>
</tr>
<tr>
<td>( G – A – T – S – B – P – C – L – H )</td>
</tr>
<tr>
<td>Other possible arrangements are:</td>
</tr>
<tr>
<td>( C – H – A – T – S – G – L – P – B )</td>
</tr>
<tr>
<td>( S – H – A – C – T – L – P – B – G ) and many more. All these arrangements are different from one another.</td>
</tr>
</tbody>
</table>
**Number 9:** Example: If the five people are Joyce, Valerie, Aira, Gillian, and Khyzza, one possible arrangement is:


Another possible arrangement, which is different from the first is: *Valerie – Gillian – Aira – Khyzza – Joyce.*

*There are many other different possible arrangements.*

3. Each possible arrangement is called a **permutation**.

Ask students to perform Activity 4 in small groups. This hands-on activity will emphasize why in some cases or situations, order matters.

**Activity 4. Let’s Find Out!**

Assume the four number cards are: 1 2 3 4

*(You may use other numbers.)*

Outcomes:

<table>
<thead>
<tr>
<th>Number of Pieces Used</th>
<th>Possible Arrangements</th>
<th>Number of Arrangements/Permutations</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. two number cards</strong> (e.g., 1, 2)</td>
<td>1 piece at a time: 1, 2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2 pieces at a time: 12, 21</td>
<td>2</td>
</tr>
<tr>
<td><strong>B. three number cards</strong> (e.g., 1, 2, 3)</td>
<td>1 piece at a time: 1, 2, 3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2 pieces at a time: 12, 13, 21, 23, 31, 32</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3 pieces at a time: 123, 132, 231, 213, 312, 321</td>
<td>6</td>
</tr>
<tr>
<td><strong>C. four number cards</strong> (e.g., 1, 2, 3, 4)</td>
<td>1 piece at a time: 1, 2, 3, 4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2 pieces at a time: 12, 13, 14, 21, 23, 24, 31, 32, 34, 41, 42, 43</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>3 pieces at a time: 123, 124, 132, 134, 142, 143, 213, 214, 231, 234, 241, 243, 312, 314, 321, 324</td>
<td>24</td>
</tr>
<tr>
<td>Number of Pieces Used</td>
<td>Possible Arrangements</td>
<td>Number of Arrangements/Permutations</td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------------------------------------------</td>
<td>-------------------------------------</td>
</tr>
<tr>
<td></td>
<td>341, 342, 412, 413, 421, 423, 431, 432</td>
<td></td>
</tr>
<tr>
<td>4 pieces at a time:</td>
<td>1234, 1243, 1342, 1324, 1423, 1432, 2134, 2143, 2341, 2314, 2413, 2431, 3124, 3142, 3214, 3241, 3412, 3421, 4124, 4142, 4213, 4231, 4312, 4321</td>
<td>24</td>
</tr>
</tbody>
</table>

Results: (Summary)

<table>
<thead>
<tr>
<th>Number of Objects ((n))</th>
<th>Number of Objects Taken at a Time ((r))</th>
<th>Number of Possible Arrangements/Permutations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>24</td>
</tr>
</tbody>
</table>

Answers to Guide Questions:
1. Each arrangement is called a permutation.
2. The pattern that can be seen is in the fourth column below:

<table>
<thead>
<tr>
<th>Number of Objects ((n))</th>
<th>Number of Objects Taken at a Time ((r))</th>
<th>Number of Possible Arrangements/Permutations</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>(2 = 2)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>((2)(1) = 2)</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>(3 = 3)</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>6</td>
<td>((3)(2) = 6)</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>6</td>
<td>((3)(2)(1) = 6)</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>(4 = 4)</td>
</tr>
</tbody>
</table>
Notice that the first number in the multiplication in the fourth column is equal to the value of \( n \) in the first column, and that the number of factors is equal to the value of \( r \) in the second column.

Let students summarize what they have learned so far about permutations. Allow them also to make a connection between the Fundamental Counting Principle and permutations based on what they know so far. Then, let them study the given illustrative examples found in the Learner’s Module.

**What to PROCESS**

Having learned the key concepts about permutations, let the students apply them in the succeeding activities. Activity 5 provides them the chance to practice their computational skills as they solve for the value of \( P, n, \) or \( r \) in equations involving permutations.

**Activity 5. Warm That Mind Up!**

<table>
<thead>
<tr>
<th>Answer Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 720</td>
</tr>
<tr>
<td>2. 4</td>
</tr>
<tr>
<td>3. 5</td>
</tr>
<tr>
<td>4. 9</td>
</tr>
<tr>
<td>5. 30 240</td>
</tr>
</tbody>
</table>

**Answers to Guide Questions:**

a. We calculated the different permutations by applying the formula

\[
P(n, r) = \frac{n!}{(n-r)!}.
\]

b. Aside from the concept of permutations, we applied the meaning of the term **factorial**, which says that the factorial of a number is equal to the product of the number and all the positive integers less than it.

c. There is some difficulty when the figures are large. Solving can then be facilitated by using a calculator and/or by applying cancellation in multiplication and division.

Let students do Activity 6. This activity requires students to apply their knowledge and skills to solve simple real-life problems that involve permutations. You may now ask them to work individually.
Activity 6. Mission Possible

Answer Key

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>24</td>
</tr>
<tr>
<td>2.</td>
<td>1,685,040</td>
</tr>
<tr>
<td>3.</td>
<td>2,520</td>
</tr>
<tr>
<td>4.</td>
<td>5040</td>
</tr>
<tr>
<td>5.</td>
<td>1,680</td>
</tr>
<tr>
<td>6.</td>
<td>479,001,600</td>
</tr>
<tr>
<td>7.</td>
<td>120</td>
</tr>
<tr>
<td>8.</td>
<td>360</td>
</tr>
<tr>
<td>9.</td>
<td>151,200</td>
</tr>
<tr>
<td>10.</td>
<td>60</td>
</tr>
</tbody>
</table>

To further develop the students' comprehension and understanding of permutations, let them do Activity 7. This activity, consisting of more problems on permutations, with some restrictions or conditions, expects students to think critically, instead of using the formula right away.

Activity 7. Decisions from Permutations

Answer Key

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>40,320</td>
</tr>
<tr>
<td>2. a.</td>
<td>362,880</td>
</tr>
<tr>
<td>b.</td>
<td>5760</td>
</tr>
<tr>
<td>c.</td>
<td>2880</td>
</tr>
<tr>
<td>3. a.</td>
<td>240</td>
</tr>
<tr>
<td>b.</td>
<td>3,628,800</td>
</tr>
<tr>
<td>4. a.</td>
<td>39,916,800</td>
</tr>
<tr>
<td>b.</td>
<td>2,177,280</td>
</tr>
<tr>
<td>c.</td>
<td>32,659,200</td>
</tr>
<tr>
<td>5.</td>
<td>6</td>
</tr>
</tbody>
</table>

Solutions:

1. \( P(8, 8) = 8! \)
   \[ = 40,320 \]

2. a. \( P(9, 9) = 9! \)
   \[ = 362,880 \]
   b. \( P = (4! \cdot 5!) \cdot 2! \)
   \[ = 5760 \]
   c. \( P = 5 \cdot 4 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \)
   \[ = 2880 \]

3. a. \( P = 5! \cdot 2! \)
   \[ = 240 \text{ ways} \]
   b. \( P = 10! \)
   \[ = 3,628,800 \]

4. a. \( P = (n - 1)! \)
   \[ = (12 - 1)! \]
   \[ = 11! \]
   \[ = 39,916,800 \]

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b. When three people insist on sitting beside each other, we treat these three persons “as one.” It is as if there are only 10 people.

\[ P = 9! \times 3! \]

3! is the number of permutations of the 3 people.

\[ P = 2\,177\,280 \]

c. Consider first the case that the two said persons always sit beside each other. Like in (b), it is as if there are only 11 people. The number of ways that they all can be seated is

\[ P = 10! \times 2! \]

\[ P = 7\,257\,600 \]

From (a), the number of ways that they can be seated if they sit anywhere is 39,916,800. Thus, the number of ways that they can all be seated if two refuse to sit beside each other is

\[ P = 39\,916\,800 - 7\,257\,600 \]

\[ P = 32\,659\,200 \]

5. \( P(n, 2) = 30 \)

So \( n = 6 \)

After doing Activity 7, let the students recount the important things they learned about permutations. You may now ask them to visualize and cite situations where their knowledge of permutations can help them formulate conclusions and make wise decisions.

What to REFLECT on and UNDERSTAND

Ask the students to think deeper about permutations by doing Activity 8. In this activity, they will explain how to determine if a situation involves permutations, differentiate among the different kinds of permutations, and make a decision based on their knowledge of permutations.

Activity 8. Reason Out

**Answer Key**

1. A situation or problem involves permutations if the order of the objects is important.

2. The **permutations** of \( n \) distinct objects taken \( r \) at a time is obtained through the formula

\[ P(n, r) = \frac{n!}{(n-r)!} \]

Basically the objects are being arranged in a row.

The **circular permutations** of \( n \) objects refers to the different arrangements of the objects when arranged in a circle; it is obtained with the help of the formula

\[ P = (n - 1)! \]
Let students explain why this is \((n - 1)!\) as opposed to \(n!\) for non-circular permutations. For further clarification refer to the Learner’s Material page 295 of Module 6.

We use the words **distinguishable permutations** to refer to the different permutations of \(n\) objects when some of them are alike. It is calculated by using the formula

\[
P = \frac{n!}{p! q! r! \ldots},
\]

where \(n\) is the total number of objects, \(p\) objects are alike, \(q\) objects are alike, \(r\) objects are also alike, and so on.

3. a. \(P(n, n-1) = \frac{n!}{(n-(n-1))!} = \frac{n!}{n!} = \frac{1}{n!} = \frac{1}{n!} \)

b. \(P(n, n) = n!\) by definition.

c. The two answers are equal. This makes sense because in arranging \(n-1\) objects out of \(n\) objects, there is only 1 object or element left each time, and there is only one way to arrange the 1 object left. That is why there are equal number of ways in arranging \(n-1\) objects and \(n\) objects out of \(n\) given objects.

4. My knowledge of permutations will tell me that there are 24 possible arrangements of the numbers in the code. If I have enough time, I would try all of them, while making a systematic list to eliminate the wrong codes. If I am in a hurry, I would leave my bike with the security guard temporarily and find another way to go home.

**Activity 9. Journal Writing**

(Outputs will vary in content. Use these as a basis for further clarification when necessary.)

Give a formative test to the students before moving on to the next section.

**What to TRANSFER**

Let the students demonstrate their understanding of permutations by doing a practical task, in Activity 10. The activity may be done in small groups. In this activity, students will cite their own examples of situations where permutations are evident, formulate problems and solve them, and discuss how these situations and their knowledge of permutations can help them formulate conclusions and make wise decisions.

Assess students’ work using the rubric provided in the Learner’s Module.
Summary/Synthesis/Generalization:

This lesson was about permutations and their applications in real life. The lesson provided students with opportunities to identify situations that describe permutations and differentiate them from those that do not. The students were also given the chance to perform practical activities for them to further understand the topic. In addition, they were given the opportunity to formulate and solve problems on permutation and apply their knowledge in formulating conclusions and in making decisions. Their understanding of this lesson as well as the other Mathematics concepts previously learned will help them learn the next topic, Combinations.

Lesson 2: Combination

What to KNOW

Assess students’ knowledge of permutations. It is important to know the extent of their learning as this will help students’ understanding of combinations. Remind them that as they go through the lesson, they must always remember the important question: How does the concept of combinations help in formulating conclusions and in making wise decisions?

Ask students to perform Activity 1. This activity is a review lesson which can develop the students’ speed as well as their accuracy in answering the problems if the activity is done by means of Rotating Learning Stations. There can be six learning stations placed strategically in the classroom. In each station, you can place two of the problems for them to solve within a specified length of time. You can separate items 9 and 10 and place them in two different stations. The students may also move through the stations in small groups or in batches if the class size is large.

Activity 1. Recall-ection (Rotating Learning Stations)

<table>
<thead>
<tr>
<th>Answer Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 72</td>
</tr>
<tr>
<td>2. 360 360</td>
</tr>
<tr>
<td>3. 120</td>
</tr>
<tr>
<td>4. 5040</td>
</tr>
<tr>
<td>5. 1 108 800</td>
</tr>
<tr>
<td>6. 48</td>
</tr>
<tr>
<td>7. 840</td>
</tr>
<tr>
<td>8. 5760</td>
</tr>
</tbody>
</table>
9. Given the number of different dishes that our catering business can prepare, I would suggest that the dishes be combined in different ways. Moreover if we were to cater for an occasion which is three or more days long, I would suggest different orders of preparing the dishes so that it would not be predictable. On the financial side, I would also study which set of meals gives the best return or profit.

10. As an administrator in the school, I would recommend that schedules of students be made in such a way that their lunch breaks and dismissal time will not all fall at the same hour.

Answers to Guide Questions:

a. The number of ways asked for in each item was obtained by applying the concept of permutations in some, and the Fundamental Counting Principle in the others.

b. The situations in numbers 2, 4, 5, 6, 7, and 8 illustrate permutations, while the rest do not. In said items, order is important, while in the others, it is not.

Let the students answer Activity 2. In this activity, they will determine whether in doing some tasks, order or arrangement is important or not. It will lead them to identify which situations involve permutations, and which involve combinations.

Activity 2. Put Some Order Here

Answer Key

1. In tasks b, c, f, and h, order or arrangement is important.
   Examples:
   (b) A code of 1234 is different from a code of 2431 in a combination lock
   (c) “1st place – Jane, 2nd place – Belen, 3rd place – Kris” is different from “1st place – Kris, 2nd place – Jane, and 3rd place – Belen.”
   (f) A seating arrangement of Renz – Abby – Gelli – Grace is different from a seating arrangement of Grace – Abby – Gelli – Renz.
   (h) If your ATM card P.I.N. is 2753 but you pressed 2573, you will not be able to access your bank account.

2. In tasks a, d, e, g, i, and j, order or arrangement is not important.
   Examples:
   (a) You can choose to answer questions 1, 2, 3, 4, and 5, or questions 4, 6, 7, 8, and 9; it will not matter (assuming that they are worth the same number of points).
   (d) Committee members, with the exception of the leader, are not ranked among themselves. A committee composed of Rosalino, Rissa, Ramon, Clarita, and Melvin is the same as a committee composed of Rissa, Ramon, Clarita, Melvin, and Rosalino.
composed of Melvin, Rissa, Ramon, Rosalino, and Clarita.

(e) If six points, \(L, E, A, R, N,\) and \(S\) are on a plane and no three of them are collinear, we can name triangles such as \(\triangle RAN\) and \(\triangle ARN.\) In naming the triangles, the order of the letters does not matter.

(g) Since there was no mention of 1st, 2nd, or 3rd prize, then it is assumed that the prizes are of equal worth. Thus, drawing the numbers 3, 5, 10, 17, 23, and 28 is the same as drawing the numbers 23, 17, 10, 28, 3, and 5.

(i) From posters marked as A, B, C, D, E, and F, selecting posters A, D, and E is the same as selecting D, E, and A.

(j) Let \(S = \{1, 2, 3, 4, 5, 6\}.\)

If \(A = \{2, 4, 6\}\) and \(B = \{4, 6, 2\},\) then \(A\) and \(B\) are subsets of \(S.\) Moreover, \(A = B.\)

Let students do the next activity through Cooperative Learning. The activity gives students the opportunity to experience a hands-on task in which order does not matter, and thus involves the concept of combinations.

**Activity 3. Let’s Discover!**

**Answer Key**

Assume the available fruits are mango (M), guava (G), banana (B), pomelo (P), and avocado (A). Please use whatever fruits are available in your locale.

<table>
<thead>
<tr>
<th>Number of Fruits ((n))</th>
<th>Number of Fruits Taken at a Time ((r))</th>
<th>Different Selections/Combinations</th>
<th>Number of Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (mango (M), banana (B))</td>
<td>1</td>
<td>M ; B</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>M-B</td>
<td>1</td>
</tr>
<tr>
<td>3 (mango (M), banana (B), guava (G))</td>
<td>1</td>
<td>M ; B ; G</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>M-B; M-G; B-G</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>M – B – G</td>
<td>1</td>
</tr>
<tr>
<td>4 (mango(M), banana (B), guava (G), pomelo (P))</td>
<td>1</td>
<td>M ; B ; G ; P</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>M-B ; M-G ; M-P ; B-G ; B-P ; G-P</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>M-B-G ; M-B-P ; M-G-P ; B-G-P</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>M-B-G-P</td>
<td>1</td>
</tr>
<tr>
<td>5 (mango (M), banana (B), guava (G),</td>
<td>1</td>
<td>M ; B ; G ; P ; A</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>M-B ; M-G ; M-P ; M-A ; B-G ; B-P ; B-A ; G-P ; G-A ; P-A</td>
<td>10</td>
</tr>
<tr>
<td>Number of Fruits ($n$)</td>
<td>Number of Fruits Taken at a Time ($r$)</td>
<td>Different Selections/Combinations</td>
<td>Number of Combinations</td>
</tr>
<tr>
<td>------------------------</td>
<td>----------------------------------------</td>
<td>-----------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>pomelo (P) avocado (A))</td>
<td>3</td>
<td>M-B-G ; M-B-P ; M-B-A ; M-G-A ; B-G-P ; B-G-A ; G-P-A ; G-P-M ; P-A-M ; P-A-B</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>M-B-G-P ; M-B-G-A ; M-G-P-A ; B-G-P-A ; M-B-P-A</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>M-B-G-P-A</td>
<td>1</td>
</tr>
</tbody>
</table>

Results: (Summary)

<table>
<thead>
<tr>
<th>Number of Objects ($n$)</th>
<th>Number of Objects Taken at a Time ($r$)</th>
<th>Number of Possible Combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Answers to Guide Questions:
1. The order of selecting the objects does not matter.
2. For example, given four fruits mango, guava, banana, and pomelo, selecting three of them like mango, guava, and banana is the same as selecting banana, mango, and guava.
3. Each unique selection is called a combination.
4. The numbers of combinations are the entries in the Pascal’s Triangle. The pattern can also be described in the fourth column of the table below.
<table>
<thead>
<tr>
<th>Number of Objects( (n) )</th>
<th>Number of Objects Taken at a Time ( (r) )</th>
<th>Number of Possible Combinations</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2 = 2 There are 2 possible ways of selecting 1 object from 2 objects.</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>( \frac{(2)(1)}{2} ) = 1 Two positions to be filled, divided by the number of permutations of the 2 objects ((2!))</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>3 = 3 There are 3 possible ways of selecting 1 object from 3 objects ((3!)).</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>( \frac{(3)(2)}{2} ) = 3 Two positions to be filled, divided by the number of permutations of the 2 objects((2!))</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>( \frac{(3)(2)(1)}{6} ) = 1 Three positions to be filled, divided by the number of permutations of the 3 objects ((3!))</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>4 = 4 There are 4 ways of selecting 1 object from 4 objects.</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>( \frac{(4)(3)}{2} ) = 6 Two positions to be filled, divided by the number of permutations of the 2 objects ((2!))</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>( \frac{(4)(3)(2)}{6} ) = 4 Three positions to be filled, divided by the number of permutations of the 3 objects ((3!))</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>( \frac{(4)(3)(2)(1)}{24} ) = 1 Four</td>
<td></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Number of Objects ($n$)</th>
<th>Number of Objects Taken at a Time ($r$)</th>
<th>Number of Possible Combinations</th>
<th>Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>positions to be filled, divided by the number of permutations of the 4 objects ($4!$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>5</td>
<td>$5 = 5$ There are 5 ways of selecting 1 object from 5 objects.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>10</td>
<td>$\frac{(5)(4)}{2} = 10$ Two positions to be filled, divided by the number of permutations of the 2 objects ($2!$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>10</td>
<td>$\frac{(5)(4)(3)}{6} = 10$ Three positions to be filled, divided by the number of permutations of the 3 objects ($3!$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>5</td>
<td>$\frac{(5)(4)(3)(2)}{24} = 5$ Four positions to be filled, divided by the number of permutations of the 4 objects ($4!$)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td>$\frac{(5)(4)(3)(2)(1)}{120} = 1$ Five positions to be filled, divided by the number of permutations of the 5 objects ($5!$)</td>
</tr>
</tbody>
</table>

5. The answers in column 3 can thus be obtained by following the technique or pattern described in column 4 of the table above.

Let students briefly summarize what they learned from the activities done. Let them make connections between the activities and the current lesson, combinations. Then, let them read and study the notes and examples of combinations.
What to PROCESS

In this section, let the students apply the key concepts of combinations to answer the next activities.

Ask the students to do Activity 4. In this activity, they will determine which situations involve permutations and which involve combinations. Let them explain their answers.

Activity 4. Perfect Combination?

Answer Key

1. Situations 1, 4, 5, and 7 illustrate permutations. Situations 2, 3, 6, 8, 9, and 10 illustrate combinations.
2. The situations in which order is important involve permutations, while those in which order is not important involve combinations.

To help sharpen students’ computational skills, ask them to do Activity 5. In this activity, they will solve for \( C, n, \) or \( r \) as required.

Activity 5. Flex that Brain!

Answer Key

1. 56
2. 6
3. 2 and 6
4. 1
5. 7
6. 3
7. 13
8. 3 and 8
9. 28
10. 1001

Combinations involve selection from a set in which the order of choosing is not important. Let them do the next task, Activity 6, in pairs. In the activity, they will utilize the concept of combinations to simple real-life situations. Let them choose their partner.

Activity 6. Choose Wisely, Choose Me

Answer Key

1. 66
2. 99
3. 2 598 960
4. 252
5. 126
6. 350
7. \( 1.027 \times 10^{10} \)
8. 3150
9. 315
10. 504
Solutions to Problems in Activity 6:

We generally use the formula for combinations which is \( C(n, r) = \frac{n!}{(n-r)! \cdot r!} \).

1. \( C(12, 2) = 66 \)

2. The total number \( N \) of polygons is:
   \[
   N = C(7, 3) + C(7, 4) + C(7, 5) + C(7, 6) + C(7, 7)
   \]
   \[
   = \frac{7!}{4! \cdot 3!} + \frac{7!}{3! \cdot 4!} + \frac{7!}{2! \cdot 5!} + \frac{7!}{1! \cdot 6!} + \frac{7!}{0! \cdot 7!}
   \]
   \[
   = 35 + 35 + 21 + 7 + 1
   \]
   \[N = 99 \text{ polygons}\]

3. \( C(52, 5) = 2,598,960 \)

4. \( C(10, 5) = 252 \)

5. If out of the 10 problems you are required to solve number 10, then you only have 9 choices for the other four problems that you must solve.
   \[
   C(9, 4) = \frac{9!}{5! \cdot 4!}
   \]
   \[= 126 \text{ possible ways of selecting the four problems}\]

6. \( N = C(5, 2) \cdot C(7, 3) \)
   \[
   = \frac{5!}{3! \cdot 2!} \cdot \frac{7!}{4! \cdot 3!}
   \]
   \[= 10 \cdot 35 \]
   \[= 350 \text{ ways}\]

7. \( C(50, 10) = 1.027 \times 10^{10} \)

8. \( N = C(5, 2) \cdot C(7, 2) \cdot C(6, 2) \)
   \[
   = \frac{5!}{3! \cdot 2!} \cdot \frac{7!}{5! \cdot 2!} \cdot \frac{6!}{4! \cdot 2!}
   \]
   \[= 10 \cdot 21 \cdot 15 \]
   \[= 3,150 \text{ ways}\]

9. \( C(7, 2) \cdot C(6, 2) = 315 \)

10. \( C(10, 5) = \frac{10!}{5! \cdot 5!} \)
    \[= 252 \text{ different sets of gowns}\]

\[252 \text{ sets} \times \frac{2\text{days}}{\text{set}} = 504 \text{ days before she runs out of a new set}\]
Let the students master their skill in calculating combinations. Ask them to do Activity 7. This activity provides more opportunities for them to apply the concept in solving real-life problems.

Activity 7. Collect and Select and Arrange!

<table>
<thead>
<tr>
<th>Answer Key</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 103 680</td>
</tr>
<tr>
<td>2. 240</td>
</tr>
<tr>
<td>3. 325</td>
</tr>
<tr>
<td>4. 277 200</td>
</tr>
<tr>
<td>5. 35</td>
</tr>
<tr>
<td>6. 160</td>
</tr>
<tr>
<td>7. 224</td>
</tr>
<tr>
<td>8. 90</td>
</tr>
<tr>
<td>9. 302 400</td>
</tr>
<tr>
<td>10. 135</td>
</tr>
</tbody>
</table>

Solutions to Problems in Activity 7:

We generally use the formula for combinations which is \[ C(n, r) = \frac{n!}{(n-r)! \cdot r!} \]

and the formula for permutations which is \[ P(n, r) = \frac{n!}{(n-r)!} \].

1. \[ N = 5! \cdot 4! \cdot 3! \cdot 3! \]
   = 103 680 ways

2. If there are 6 students but two must always sit beside each other, then it is as if there are only 5 students to be arranged. The 5 students can be arranged in 5! ways, while the two who insist on sitting beside each other can arrange themselves in 2! ways. Thus, the total number of ways of arranging all of them is:
   \[ N = 5! \cdot 2! \]
   = 240 ways

3. \[ C(52, 5) = 300 \]

4. \[ n=12, \quad p=3, \quad q=4, \quad r=2, \quad s=3 \]
   \[ P = \frac{n!}{p! \cdot q! \cdot r! \cdot s!} \]
   \[ P = \frac{12!}{3! \cdot 4! \cdot 2! \cdot 3!} \]
   \[ P = 277 200 \]

5. \[ C(26, 2) = 325 \]

6. \[ C(4, 1) \cdot C(8, 1) \cdot C(5, 1) = 160 \]

7. From 6 red and 8 green marbles, picking three marbles, at least 2 of which are green, implies that there are either 2 green marbles and 1 red marble
picked, or there are 3 green marbles and no red marble picked. The number of ways \( N \) of picking such is given by:

\[
N = C(8, 2) \cdot C(6, 1) + C(8, 3) \cdot C(6, 0)
\]

\[
= \frac{8!}{6!2!} \cdot \frac{6!}{5!1!} + \frac{8!}{5!3!} \cdot \frac{6!}{0!6!}
\]

\[
= (28)(6) + (56)(1)
\]

\[
= 168 + 56
\]

\[
= 224 \text{ ways}
\]

8. \textit{Number of rays} = P(10, 2)

\[
= \frac{10!}{8!}
\]

\[
= 90 \text{ rays}
\]

9. The first step which is selecting seven out of ten books involves combinations, while the second step which is arranging only five out of the seven books involves permutations. The number of ways \( N \) of doing this is:

\[
N = C(10, 7) \cdot P(7, 5)
\]

\[
= \frac{10!}{7!3!} \cdot \frac{7!}{2!}
\]

\[
= 120 \cdot 2520
\]

\[
= 302400 \text{ ways}
\]

10. \( C(7,1) + C(7,2) + C(7,3) + C(7,4) + C(7,5) + C(7,6) + C(7,7) \)

\[
= 148
\]

\textbf{What to REFLECT on and UNDERSTAND}

Let the students do Activity 8. This activity provides an opportunity for them to examine and evaluate their own understanding of the lesson by solving problems which involve both permutations and combinations.

\textbf{Activity 8. I Know Them So Well}

\textbf{Answer Key}

1. A situation involves combinations if it consists of task/tasks of selecting from a set and the order or arrangement is not important.
2. Joy is not correct because she only calculated the number of triangles that can be formed \([C(7,3)]\). She did not include the number of other polygons, namely, quadrilateral, pentagon, hexagon, or heptagon.
3. a. 17,325  
b. 3,991,680

4. a. 42  
b. 10  
c. 2,520  
d. 60  
e. 73

Solutions:
3. a. \( \binom{12}{8} \cdot \binom{7}{4} = 17,325 \) 
b. \( P(12, 7) = 3,991,680 \)

4. There are 28 participants, four in each of the seven groups.
   a. In the eliminations:
      \[ C(4, 2) = \frac{4!}{2! \cdot 2!} = 6 \text{ elimination games in each group} \]
      \[ 6 \text{ games/group} \times 7 \text{ groups} = 42 \text{ games in all} \]
   b. In the final round:
      \[ C(5, 2) = \frac{5!}{3! \cdot 2!} = 10 \text{ games} \]
   c. In the semi-finals:
      \[ P(7, 5) = (7)(6)(5)(4)(3) = 2520 \text{ ways of coming up with the top 5 out of seven} \]
   d. \( P(5, 3) = 60 \)
   e. \( C(4, 2) \times 7 = 42 \)  
      \( C(7, 2) = 21 \)  
      \( C(5, 2) = 10 \)  
      Total = 73

Activity 9. Journal Writing

(Outputs will vary in content. Use these as basis for further clarification when necessary.)

Find out how well the students understood the lesson by giving them a formative test before they proceed to the next section.

What to TRANSFER

Give the students opportunities to apply their learning to real-life situations and demonstrate their understanding of combinations by doing a practical task. Let them perform Activity 10. In this activity, they will give their own examples of real-life situations that illustrate combinations, formulate problems related to these situations and solve the problems.
Summary/Synthesis/Generalization

This lesson was about combinations and its applications in real life. Through the lesson, students were able to identify situations that describe combinations and differentiate them from those that do not. They were also given the opportunity to perform practical activities to further understand the topic, formulate related real-life problems, and solve these problems. They also applied their knowledge of this concept in formulating conclusions and in making wise decisions.
SUMMATIVE TEST

Part I
Choose the letter that you think best answers the question

Note: Students may be allowed to use their calculators, but find a way to ensure that they really know the calculations involved. It is suggested that you require students to show solutions for some of the items.

1. What do you call the different arrangements of the objects of a group?
   A. selection   C. permutation
   B. differentiation   D. combination

2. Which situation illustrates permutation?
   A. forming a committee of councilors
   B. selecting 10 questions to answer out of 15 questions in a test
   C. choosing 2 literature books to buy from a variety of choices
   D. assigning rooms to conference participants

3. It is the selection of objects from a set.
   A. combination   C. permutation
   B. differentiation   D. distinction

4. Which of the following situations illustrates combination?
   A. arranging books in a shelf
   B. drawing names from a box containing 200 names
   C. forming different numbers from 5 given digits
   D. forming plate numbers of vehicles

5. Which of the following situations does NOT illustrate combination?
   A. selecting fruits to make a salad
   B. assigning telephone numbers to homes
   C. choosing household chores to do after classes
   D. selecting posters to hang in the walls of your room

6. Which of the following expressions represents the number of distinguishable permutations of the letters of the word CONCLUSIONS?
   A. $11!$
   B. $\frac{11!}{8!}$
   C. $\frac{11!}{2! 2! 2!}$
   D. $\frac{11!}{2! 2! 2! 2!}$
7. A certain restaurant allows you to assemble your own vegetable salad. If there are 8 kinds of vegetables available, how many variations of the salad can you make containing at least 5 vegetables?
A. 56    B. 84    C. 93    D. 96

8. Calculate \( P(12, 4) \).
A. 40 320    B. 11 880    C. 990    D. 495

9. How many different 3-digit numbers can be formed from the digits 1, 3, 4, 6, 7, 9 if repetition of digits is not allowed?
A. 840    B. 720    C. 360    D. 120

10. Miss Cruz plotted some points on the board, no three of which are collinear. When she asked her student to draw all the possible lines through the points, he came up with 45 lines. How many points were on the board?
A. 10    B. 9    C. 8    D. 7

11. If \( P(9, r) = 504 \), what is \( r \)?
A. 7    B. 6    C. 5    D. 3

12. If \( P(n, 4) = 17 160 \), then \( n = _____ \).
A. 9    B. 11    C. 13    D. 14

13. If \( x = P(7, 4) \), \( y = P(8, 4) \), and \( z = P(9, 3) \), arrange \( x \), \( y \), and \( z \) from smallest to greatest.
A. \( x, y, z \)    B. \( z, x, y \)    C. \( y, x, z \)    D. \( x, z, y \)

14. Calculate \( \frac{7!}{3! \cdot 2!} \).
A. 420    B. 840    C. 1680    D. 2520

15. Which of the following can be a value of \( r \) in \( C(15, r) = 1365 \)?
A. 6    B. 5    C. 4    D. 3

16. If \( C(n, 5) = 252 \), then \( n = _____ \).
A. 7    B. 8    C. 9    D. 10

17. Calculate: \( C(20, 5) \)
A. 6840    B. 15 504    C. 116 280    D. 1 860 480

18. Let \( a = C(7, 4) \), \( b = C(7, 5) \), \( c = C(7, 6) \) and \( d = C(7, 7) \). If there are 7 points on the plane, no three of which are collinear, what represents the total number of polygons that can be formed with at least 5 sides?
A. \( a + b \)    C. \( a + b + c \)
B. \( c + d \)    D. \( b + c + d \)
19. Find $C(18, 4)$.
   A. 2400   B. 3060   C. 4896   D. 73440

20. Evaluate: $C(25, 4) + C(30, 3) + C(35, 2)$
   A. 17900   B. 17305   C. 16710   D. 4655

21. In how many different ways can 7 potted plants be arranged in a row?
   A. 5040   B. 2520   C. 720   D. 210

22. In how many different ways can 10 different-colored horses be positioned in a carousel?
   A. 504   B. 4032   C. 362880   D. 3628800

23. In how many possible ways can Juan answer a 10-item matching type quiz if there are also 10 choices and he answers by mere guessing?
   A. 3628800   B. 40320   C. 720   D. 10

24. Khristelle was able to calculate the total number of 3-digit numbers that can be formed from a given set of non zero digits, without repetition. If there were 60 numbers in all, how many digits were actually given?
   A. 8   B. 7   C. 6   D. 5

25. How many different rays can be formed from 8 distinct points on a plane, no three of which lie on the same line?
   A. 56   B. 28   C. 26   D. 4

26. If a committee of 8 members is to be formed from 8 sophomores and 5 freshmen such that there must be 5 sophomores in the committee, which of the following is/are true?
   I. The 8 committee members can be selected in 1287 ways.
   II. The 5 sophomores can be selected in 56 ways.
   III. The 3 freshmen can be selected in 10 ways
   A. I only   B. I and II   C. II and III   D. I, II, and III

27. In a gathering, each of the guests shook hands with everybody else. If a total of 378 handshakes were made, how many guests were there?
   A. 30   B. 28   C. 25   D. 23

28. If 4 marbles are picked randomly from a jar containing 8 red marbles and 7 blue marbles, in how many possible ways can at least 2 of the marbles picked be red?
   A. 1638   B. 1568   C. 1176   D. 1050
Part II

Read and understand the situations below then answer or perform what are asked. Your answers will be rated based on the Rubric for Problem Formulated and Solved.

Assume the role of the class president in your play presentation. As part of the safety awareness campaign in your school, your teacher assigns you to take charge of the class activity. You need to make a plan about how you will carry out the activity. You are to assign some group leaders in your class to make a project about road safety, disaster prevention, risk reduction, and the like.

1. If there must be five groups, whom will you assign to be the group leaders? Why?
2. Enumerate five different possible activities that your class can make (e.g., exhibit, poster-making, etc.)
3. Explain how you will do your groupings and your task assignment.
4. What mathematical concepts can you relate with this situation?
5. Formulate 2 problems involving these mathematics concepts or principles.
6. Write the equation(s) that describe the situation or problem.
7. Solve the equation(s) and the problems formulated.

Summative Test

Answer Key

<table>
<thead>
<tr>
<th>I.</th>
<th></th>
<th>11. D</th>
<th></th>
<th>21. A</th>
</tr>
</thead>
</table>

II. Check students’ answers using the Rubric on Problems Formulated and Solved.

Solutions to Selected Problems in the Pre-Assessment Part I:

5. \(5 \times 4 \times 3 \times 2 = 120\)

6. Eight people around a circular table, with two of them insisting on sitting beside each other:
   
   It is as if there are seven people, \(n = 7\).
\[ P = (7 - 1)! \cdot 2! \quad \text{The 2! is the number of permutations of the said two people.} \]
\[ P = 6! \cdot 2! \]
\[ = 1440 \text{ ways} \]

8. Observations show (aside from the formula in Geometry) that the number of diagonals \( D \) of an \( n \)-sided polygon is equal to \( C(n, 2) - n \).
   If \( n = 9 \), then \[ D = C(9, 2) - 9 \]
   \[ = \frac{9!}{7! \cdot 2!} - 9 \]
   \[ = 36 - 9 \]
   \[ = 27. \]
   Thus, \( n = 9 \), and it is a nonagon.

10. If \( r = 4 \), then \[ P(9, 4) = (9)(8)(7)(6) \]
   \[ = 3024 \]
   Thus, \( r = 4 \).

11. \[ P(12, 3) = 1320 \]

13. \[ P(n, 4) = 5040 \]
   If \( n = 10 \), then \[ P(10, 4) = (10)(9)(8)(7) \]
   \[ = 5040. \]
   Thus, \( n = 10 \).

15. \[ P = \frac{8!}{2! \cdot 2!} = 10080 \]

16. \[ P(10, 5) = 30240 \]

17. \[ P = 4! \cdot 3! \]

18. Ten chairs in a row:

   Seat number: 1 2 3 4 5 6 7 8 9 10

   The five consecutive chairs may be 1-2-3-4-5, 2-3-4-5-6, 3-4-5-6-7, 4-5-6-7-8, 5-6-7-8-9, or 6-7-8-9-10. There are six possibilities of using 5 consecutive chairs.

   The five students can be seated in 5! ways, multiplied by 6.
   Therefore, \( N = 5! \cdot 6 \)
   \[ = 720 \text{ ways} \]

25. \[ C(3, 1) \cdot C(7, 2) \cdot C(4, 1) \cdot C(4, 2) = 1512 \]
26. \[ N = C(9, 3) \cdot C(9, 4) \]
\[ = \frac{(9)(8)(7)}{3!} \cdot \frac{(9)(8)(7)(6)}{4!} \]
\[ = 10584 \]

27. To start the solution means to combine any two of the \( n \) equations.

\[ C(n, 2) = 10 \]
\[ \frac{n(n-1)}{2!} = 10 \]
\[ \frac{n(n-1)}{2} = 10 \]
\[ n^2 - n = 20 \]
\[ n^2 - n - 20 = 0 \]
\[ (n-5)(n+4) = 0 \]
Thus \( n = 5 \). (\( n \) cannot be \(-4\) of course.)

Solutions for Selected Problems in the Summative Test:

22. Circular permutation where \( n = 10 \):
\[ P = (n-1)! \]
\[ = (10-1)! \]
\[ = 9! \]
\[ = 362880 \]

27. \[ C(n, 2) = 378 \]
\[ \frac{n(n-1)}{2!} = 378 \]
\[ n(n-1) = 756 \]
\[ n^2 - n - 756 = 0 \]
\[ (n-28)(n+27) = 0 \]
\[ n = 28. \]

28. Either 2 of the marbles picked are red, or 3 of them are red, or 4 of them are.
\[ N = C(8, 2) \cdot C(7, 2) + C(8, 3) \cdot C(7, 1) + C(8, 4) \cdot C(7, 0) \]
\[ = \frac{8!}{6! \cdot 2!} \cdot \frac{7!}{5! \cdot 2!} + \frac{8!}{5! \cdot 3!} \cdot \frac{7!}{6! \cdot 1!} + \frac{8!}{4! \cdot 4!} \cdot \frac{7!}{7! \cdot 0!} \]
\[ = 28 \cdot 21 + 56 \cdot 7 + 70 \cdot 1 \]
\[ = 588 + 392 + 70 \]
\[ = 1050 \]
GLOSSARY OF TERMS

Circular permutation – the different possible arrangements of objects in a circle. The number of permutations $P$ of $n$ objects around a circle is given by $P = (n - 1)!$

Combinations – the number of ways of selecting from a set when the order is not important. The number of combinations of $n$ objects taken $r$ at a time is given by $C(n, r) = \frac{n!}{(n-r)!r!}$, $n \geq r$

Distinguishable permutations – the permutations of a set of objects where some of them are alike. The number of distinguishable permutations of $n$ objects when $p$ are alike, $q$ are alike, $r$ are alike, and so on, is given by $P = \frac{n!}{p!q!r!...}$.

Fundamental Counting Principle – states that if activity A can be done in $n_1$ ways, activity B can be done in $n_2$ ways, activity C in $n_3$ ways, and so on, then activities A, B, and C can be done simultaneously in $n_1 \cdot n_2 \cdot n_3 \cdots$ ways.

Permutations – refers to the different possible arrangements of a set of objects. The number of permutations of $n$ objects taken $r$ at a time is: $P(n, r) = \frac{n!}{(n-r)!}$, $n \geq r$.

$n$ - Factorial – the product of the positive integer $n$ and all the positive integers less than $n$. $n! = n(n-1)(n-2) \cdots (3)(2)(1)$.

References:

Books:
Websites Links as References:


Module 7: Probability of Compound Events

A. Learning Outcomes

Content Standard:
The learner demonstrates understanding of key concepts of combinatorics and probability.

Performance Standard:
The learner is able to use precise technique and probability in formulating conclusions and in making decisions.

Unpacking the Standard for Understanding

<table>
<thead>
<tr>
<th>Subject: Mathematics 10</th>
<th>Learning Competencies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quarter:</strong> Third Quarter</td>
<td>1. Illustrate the probability of a union of two events and intersection of events</td>
</tr>
<tr>
<td><strong>Topic:</strong> Probability of compound events, independent and dependent events, conditional probability</td>
<td>2. Illustrate and find probability of mutually exclusive events</td>
</tr>
<tr>
<td><strong>Lessons:</strong></td>
<td>3. Illustrate independent and dependent events</td>
</tr>
<tr>
<td>1. Union and Intersection of Events</td>
<td>4. Find probability of independent and dependent events</td>
</tr>
<tr>
<td>2. Independent and Dependent Events</td>
<td>5. Identify conditional probability</td>
</tr>
</tbody>
</table>

Writer: Allan M. Canonigo

Essential Understanding: The learners will understand that there are real-life situations where counting techniques, probability of compound events, probability of independent events, and conditional probability are necessary in order to make sound decisions.

Essential Question: How do counting techniques, probability of compound events, probability of independent events, and conditional probability help in formulating conclusions and in making decisions in real life?

Transfer Goal: The learners will recognize real-life situations where key concepts of counting technique and probability of compound events, probability of independent events, and conditional probability can be applied in formulating conclusions and in making decisions.
B. Planning for Assessment

Product/Performance
The following are products and performances that students are expected to come up with in this module.

1. Venn diagrams of union and intersection of two or more events
2. Situations in real life where mutually exclusive and non-mutually exclusive events are illustrated
3. Situations in real life where dependent and independent events are illustrated
4. Situations in real life which illustrate conditional probabilities
5. Role playing a situation which entails decision making and which applies the concepts of independent and dependent events and conditional probabilities
6. Investigating real phenomena which need decision making using conditional probabilities

<table>
<thead>
<tr>
<th>TYPE</th>
<th>KNOWLEDGE</th>
<th>PROCESS/ SKILLS</th>
<th>UNDERSTANDING</th>
<th>PERFORMANCE/ REASONING</th>
</tr>
</thead>
</table>
| Pre-assessment/ Diagnostic | Part 1  
Identifying probability of simple and compound events; independent and dependent events, mutually exclusive events; and conditional probabilities | Part 1  
Solving probability of simple and compound events  
Using the Venn diagram to solve probability involving union and intersection of events  
Applying counting techniques in solving probability of compound events  
Solving probability of dependent and independent events | Part 1  
Identifying real life situations which illustrate conceptual understanding of probability of compound events, mutually exclusive events, independent and dependent events, and conditional probabilities  
Solving problems involving probability of independent and dependent events, mutually exclusive events, and conditional probabilities |                                                     |
| Part II Problem Solving | Part II Problem Solving  
Identifying concepts on probability to solve real-life problems | Part II Problem Solving  
Identifying options or alternative solutions in solving real-life problems | Part II Problem Solving  
Explaining how the concepts of probability (such as independent and dependent events, conditional probability) are used in making decisions | Part II Problem Solving  
Making wise decision in a varsity try out and trying to win in a game show |
<table>
<thead>
<tr>
<th>TYPE</th>
<th>KNOWLEDGE</th>
<th>PROCESS/SKILLS</th>
<th>UNDERSTANDING</th>
<th>PERFORMANCE/REASONING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formative</td>
<td>Quiz: Lesson 1&lt;br&gt;Identifying simple and compound events&lt;br&gt;Recognizing union and intersection of events&lt;br&gt;Recognizing mutually and non-mutually exclusive events</td>
<td>Quiz: Lesson 1&lt;br&gt;Using the tree diagram in listing the elements of the sample space of a given experiment&lt;br&gt;Using the Venn diagram to illustrate union and intersection of events&lt;br&gt;Using the Venn diagram to illustrate mutually exclusive events&lt;br&gt;Using combination and permutation in determining the number of ways an event can occur and in listing the elements of the sample space</td>
<td>wise decisions.</td>
<td>Formulating alternatives or options in making decisions and in evaluating the merit of each option</td>
</tr>
</tbody>
</table>

Quiz: Lesson 1<br>Solving probability of simple and compound events<br>Solving probability involving union and intersection of events<br>Solving probability involving mutually exclusive events<br>Solving probability of events involving combination and permutation<br>Formulating and describing situation in real life involving events that are mutually exclusive and not mutually exclusive<br>Justifying why a real life phenomenon involves mutually exclusive events<br>Determining the different ways of how the concept of probability is used in print media and entertainment industry
<table>
<thead>
<tr>
<th>TYPE</th>
<th>KNOWLEDGE</th>
<th>PROCESS/SKILLS</th>
<th>UNDERSTANDING</th>
<th>PERFORMANCE/REASONING</th>
</tr>
</thead>
</table>
| Formative | Quiz: Lesson 2  
Recognizing independent and dependent events | Quiz: Lesson 2  
Using Venn diagram to illustrate independent and dependent events | Quiz: Lesson 2  
Stating and explaining the conditions why two events are independent or dependent | Formulating and describing situations or problems in real life involving events that are independent and dependent |
| Formative | Quiz: Lesson 3  
Recognizing conditional probabilities  
Identifying certain conditions in a situation involving probability | Quiz: Lesson 3  
Using Venn diagram to show the relationship of the events involving conditional probability | Quiz: Lesson 3  
Stating and explaining the conditions in a problem involving conditional probability | Solving problems involving conditional probability  
Formulating and describing situations or problems in real life involving conditional probability |
| Summative | Part 1  
Identifying probability of simple and compound events, independent and dependent events, mutually exclusive events; | Part 1  
Solving probability of simple and compound events  
Using the Venn diagram to solve probability involving union and intersection | Part 1  
Identifying real life situations which illustrate conceptual understanding of probability of compound events, mutually exclusive events. |  |

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<table>
<thead>
<tr>
<th>TYPE</th>
<th>KNOWLEDGE</th>
<th>PROCESS/SKILLS</th>
<th>UNDERSTANDING</th>
<th>PERFORMANCE/REASONING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>and conditional probabilities</td>
<td>of events</td>
<td>independent and dependent events, and conditional probabilities</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Applying counting techniques in solving probability of compound events</td>
<td>Solving problems involving probability of independent and dependent events, mutually exclusive events, and conditional probabilities</td>
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<td></td>
<td></td>
<td>Solving probability involving mutually exclusive events</td>
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<td></td>
<td></td>
<td>Solving probability of dependent and independent events</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Part II</td>
<td>Problem Solving</td>
<td>Identifying concepts of probability that are applied in solving real life problems</td>
<td>Explaining how the concepts of probability (such as independent and dependent events, conditional probability) are used in making wise decisions.</td>
<td>Making wise decision in a varsity try out and trying to win in a game show</td>
</tr>
<tr>
<td></td>
<td>Identifying options or alternative solutions in solving real-life problems</td>
<td></td>
<td></td>
<td>Formulating alternatives or options in making decisions and in evaluating the merit of each option</td>
</tr>
<tr>
<td>Self-Assessment</td>
<td>Journal Writing</td>
<td>Expressing understanding of the concepts of compound probability, probability of mutually exclusive events, independent events, and conditional probability</td>
<td>Expressing critical analysis of the process of problem solving and in determining the options for decision making in a particular situation</td>
<td></td>
</tr>
</tbody>
</table>
## Assessment Matrix (Summative Test)

<table>
<thead>
<tr>
<th>Levels of Assessment</th>
<th>What will I assess?</th>
<th>How will I assess?</th>
<th>How will I give mark?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Knowledge 15%</strong></td>
<td>The learners demonstrate understanding of key concepts of combinatorics and probability. Illustrate the probability of a union of two events and intersection of events.</td>
<td>Paper and Pencil Test Part I Items 1, 3</td>
<td>1 point for every correct answer</td>
</tr>
<tr>
<td><strong>Process/Skills 25%</strong></td>
<td>Illustrate and find probability of mutually exclusive events. Illustrate independent and dependent events. Find probability of independent and dependent events. Identify conditional probability. Solve problems on conditional probability.</td>
<td>Part I Items 2, 4, 6, 7, 8, 9, 10 Part II Items 11, 17 Part II Item 18</td>
<td>1 point for every correct answer 1 point for every correct answer 3 points in all 1 point for every correct answer to 2 questions and 1 point for the justification.</td>
</tr>
<tr>
<td><strong>Product/Performance 30%</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
C. Planning for Teaching and Learning

This module covers key concepts of probability of compound events, mutually and not mutually exclusive events, independent and dependent events, and conditional probability. Each lesson starts with an activity that helps the students recall their previous lessons. Each activity has a set of questions or problems or tasks that require them to apply their previous knowledge of probability theory, counting techniques, and sets. After doing the activity, they are provided with questions that help them reflect on their learning. As the lesson progresses, some illustrations and examples are provided for students to make sense of their learning and to connect the concepts that they learned to the previous activities.

Lesson 1 of this module starts with recalling of students’ understanding of the concept of the probability of simple events. Eventually, they will identify and differentiate simple events from compound events. As they work on the different activities, they will apply the concepts of sets, Venn diagram, and counting techniques in solving the probability of compound events and in illustrating union and intersection of events. Putting together these concepts, they will illustrate and solve problems on probability involving mutually exclusive and not mutually exclusive events.

Lesson 2 of this module is on independent and dependent events. The students will recognize and differentiate independent from dependent events. They will also analyze and classify real-life situations as independent and dependent. To summarize their own understanding, they will formulate their own problems involving independent events.

Lesson 3 of this module is a lesson on one of the most important concepts in the theory of probability. This concept of probability is based on some questions such as: What is the chance that a team will win the game now that they have taken the first point? What is the chance that a child smokes if the household has two parents who smoke? These few questions may lead the students to the concept of conditional probability. This lesson exposes the students to situations in which they could make prediction or make decision as they solve problems on conditional probability.
As an introduction to the lesson, ask the students to look and observe the pictures found in their module. You may also post these pictures on the board. However, you may use pictures similar to the one shown below.

What do these different pictures tell us about different phenomena? Can you think of a situation that you are not certain whether it will happen or not and when it will occur? What are the necessary preparations that might be done to minimize the impact of such phenomena?

Ask the students to talk about their experiences or their observations when a certain phenomenon happened. Ask them about the importance of preparation in time of this event.

Objectives:

After the students have gone through the lessons of this module, they are expected to:

1. illustrate the probability of a union of two events and intersection of two events;
2. illustrate and find probability of mutually exclusive events;
3. illustrate independent and dependent events;
4. find probability of independent and dependent events;
5. identify conditional probability; and
6. solve problems on conditional probability.

PRE-ASSESSMENT

Check students’ prior knowledge, thinking skills, and understanding of mathematics concepts in relation to probability theory. The result of the diagnostic assessment will help you determine the students’ background knowledge on probability theory.
### Answer Key

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>B</td>
<td>11.</td>
<td>D</td>
</tr>
<tr>
<td>2.</td>
<td>D</td>
<td>12.</td>
<td>A</td>
</tr>
<tr>
<td>4.</td>
<td>D</td>
<td>14.</td>
<td>A</td>
</tr>
<tr>
<td>5.</td>
<td>A</td>
<td>15.</td>
<td>B</td>
</tr>
<tr>
<td>6.</td>
<td>A</td>
<td>16.</td>
<td>A</td>
</tr>
<tr>
<td>7.</td>
<td>C</td>
<td>17.</td>
<td>D</td>
</tr>
<tr>
<td>8.</td>
<td>D</td>
<td>18. a.)</td>
<td>A</td>
</tr>
<tr>
<td>9.</td>
<td>A</td>
<td>b.)</td>
<td>The number of dolphins remains the same after 6 months.</td>
</tr>
</tbody>
</table>

### Part II Problem Solving

19. Let: $g =$ probability of winning against the good player, $G$
   $t =$ probability of winning against the top player, $T$
   $1 - g =$ probability of losing when playing with the good player, $G$
   $1 - t =$ probability of losing when playing with the top player, $T$

One option is the sequence $TGT$. The result is given below:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>G</td>
<td>T</td>
<td>Probability</td>
</tr>
<tr>
<td>Win</td>
<td>Win</td>
<td>Win</td>
<td>$tgt$</td>
</tr>
<tr>
<td>Win</td>
<td>Win</td>
<td>Loss</td>
<td>$tg(1-t)$</td>
</tr>
<tr>
<td>Loss</td>
<td>Win</td>
<td>Win</td>
<td>$(1-t)gt$</td>
</tr>
</tbody>
</table>

Pertinent to the previous discussion is the observation that the first two rows naturally combine into one: the probability of the first two wins is $P(WinWin) = tgt + tg(1-t) = tg$,

which is simply the probability of winning against both $T$ and $G$ (in the first two games in particular).

Since winning the first two games and losing the first game but winning the second and the third are mutually exclusive events, the Sum Rule applies. Gaining acceptance playing the $TGT$ sequence has the total probability of $P(TGT) = tg + (1-t)tg = tg(2-t)$.

Similarly, the probability of acceptance for the GTG schedule is based on the following table:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>T</td>
<td>G</td>
<td>Probability</td>
</tr>
<tr>
<td>Win</td>
<td>Win</td>
<td>Win</td>
<td>$gt$</td>
</tr>
<tr>
<td>Loss</td>
<td>Win</td>
<td>Win</td>
<td>$(1-g)tg$</td>
</tr>
</tbody>
</table>

The probability on this case is found to be $P(GTG) = gt + (1-g)gt = gt(2-g)$.

Assuming that the top member $T$ is a better player than just the good one $G$, $t < g$. But then, $gt(2-g) < tg(2-t)$. In other words, $P(GTG) < P(TGT)$.

You have a better chance of being admitted to the club by playing the apparently more difficult sequence $TGT$ than the easier one $GTG$. Perhaps there is a moral to the story/problem: the more difficult tasks offer greater rewards.
20. Let:

\[ V = \text{event that a student has more than one viand} \]
\[ F = \text{event that a student prefers fish as viand} \]

\[
P(\text{1 viand that is a fish}) = 1 - P(V) - P(F) + P(F \cap V) = 1 - 0.70 - 0.20 + .15 \cdot 0.70 = 0.205
\]

**LEARNING GOALS AND TARGETS**

After going through this module, the students should be able to demonstrate understanding of the key concepts of probability of compound events, mutually exclusive events, and independent events, and of conditional probability. With these knowledge and skills, they should be able to use probability in formulating conclusions and in making decisions.

**Lesson 1: Union and Intersection of Events**

**What to KNOW**

Begin Lesson 1 of this module by assessing students’ knowledge and skills of the different mathematics concepts related to counting techniques and probability of simple events as well as concepts of sets they previously studied. Tell them that these knowledge and skills are important in understanding the probability of compound events. As they go through this lesson, encourage the students to think of this question, *Why do you think is the study of probability so important in making decisions in real life?*

**Activity 1: Recalling Probability**

This is an activity to help students recall the concept of probability of simple events which they have taken up in Grade 8.

<table>
<thead>
<tr>
<th><strong>Answer Key</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Sample space: {1, 2, 3, 4, 5, 6}</td>
</tr>
<tr>
<td>a. The probability of obtaining a 5 is (\frac{1}{6}).</td>
</tr>
<tr>
<td>b. The probability of obtaining a 6 is (\frac{1}{6}).</td>
</tr>
<tr>
<td>c. The probability of obtaining an odd number is (\frac{3}{6}) or (\frac{1}{2}).</td>
</tr>
</tbody>
</table>
2. Sample Space:
   \{\text{red, red, red, yellow, yellow, yellow, yellow, blue, blue}\}
   
   a. The probability that a yellow ball is picked at random is \(\frac{5}{10}\) or \(\frac{1}{2}\).
   
   b. The probability that a red ball is picked at random is \(\frac{3}{10}\).

   Ask the students to list the sample space for each and ask them to explain their answers. Point out that these are simple events. Help them recall by discussing the following:

   When the sample space is finite, any subset of the sample space is an event. An event is any collection of outcomes of an experiment. Any subset of the sample space is an event. Since all events are sets, they are usually written as sets (e.g., \{1, 2, 3\}).

   **Probability of Simple Events:** An event \(E\), in general, consists of one or more outcomes. If each of these outcomes is equally likely to occur, then

   \[
   \text{Probability of event } E = P(E) = \frac{\text{number of ways the event can occur}}{\text{number of possible outcomes}}
   \]

   When you roll a die, you get anyone of these outcomes: 1, 2, 3, 4, 5, or 6. This is the sample space, the set of all outcomes of an experiment. Thus, we say that an event is a subset of the sample space. And so the probability of an event \(E\) can also be defined as

   \[
   E = P(E) = \frac{\text{number of outcomes in the event}}{\text{number of outcomes in the sample space}}
   \]

**Activity 2: Understanding Compound Events**

1. Sample space: For easier listing, we use the following: \(\text{fr}\) – fried rice; \(\text{sr}\) – steamed rice; \(\text{ca}\) - chicken adobo; \(\text{p}\) - *pinakbet*; \(\text{pj}\) – pineapple juice; and \(\text{oj}\) – orange juice

   \{(\text{fr, ca, pj}), (\text{fr, ca, oj}), (\text{fr, p, pj}), (\text{fr, p, oj}), (\text{sr,ca, pj}), (\text{sr, ca, oj}), (\text{sr, p, pj}), (\text{sr, p, oj})\}

   There are a total of 8 possible outcomes.

2. \{(\text{fr, ca, pj}), (\text{fr, p, pj}), (\text{sr,ca, pj}), (\text{sr, p, pj})\}

3. There are 4 outcomes in selecting any lunch with pineapple juice.

4. There are 2 outcomes for selecting a lunch with steamed rice and with pineapple juice.

5. There are 2 outcomes for selecting a lunch with chicken adobo and pineapple juice.

6. There are 2 events for selecting a lunch with chicken adobo and pineapple juice.

7. \(\frac{4}{8}\) or \(\frac{1}{2}\)

8. \(\frac{2}{8}\) or \(\frac{1}{4}\)
Based on the answers of the students, ask them the following questions:

a. What does the tree diagram tell you?
The tree diagram shows the total number of outcomes.

b. How did you determine the sample space?
The sample space is obtained by listing all the outcomes that are obtained using the tree diagram.

c. Differentiate an outcome from a sample space. Give another example of an outcome.
An outcome is an element of a sample space. One example is \((sr, p, pj)\).

d. Aside from a tree diagram, how else can you find the total number of possible outcomes?
The total number of possible outcomes can also be found using the fundamental counting principle (multiplication rule).

The students should be able to point out that the tree diagram is very helpful in listing all the possible outcomes in a sample space. This will help them recall their lesson in Grade 8 about counting techniques using a tree diagram. Help them recognize that the events in this activity are not like the events in the previous activity. This activity will help them differentiate simple events from compound events.

**Simple Events:** Any event which consists of a single outcome in the sample space is called an elementary or simple event.

**Compound Events:** Events which consist of more than one outcome are called compound events. A compound event consists of two or more simple events.

Remind the students that it is often useful to use a Venn diagram to visualize the probabilities of events. In Activity 2, students explore the use of a Venn diagram to determine the probabilities of individual events, the intersection of events, and the complement of an event. To understand more about the probability of union and intersection of events, ask students to proceed to Activity 3. Discuss the illustrative example or give similar examples to help them understand the concept of Venn diagram.

**Activity 3: Intersection and Union of Events**
1. The total number of students in the senior class is 345.
2. \[ \frac{159}{345} \]
3. \[ \frac{227}{345} \]
4. \[ \frac{30}{345} \]
Activity 4: Taking Chances with Events \( A \) or \( B \)

The next activity will help students understand the concepts of events which are mutually exclusive and which are not mutually exclusive. Prior to answering this activity, remind the students to try to figure out events which are mutually exclusive and which are not.

1. a. \[ P(7 \text{ or } 15) = \frac{1}{15} + \frac{1}{15} = \frac{2}{15} \]
   
   b. \[ P(5 \text{ or a number divisible by 3}) = \frac{1}{15} + \frac{5}{15} = \frac{6}{15} \text{ or } \frac{2}{3} \]
   
   c. \[ P(\text{even or a number divisible by 3}) = \frac{7}{15} + \frac{5}{15} = \frac{10}{15} \text{ or } \frac{2}{3} \]
   
   d. \[ P(\text{a number divisible by 3 or 4}) = \frac{5}{15} + \frac{3}{15} = \frac{8}{15} \]

2. \[ P(\text{red or yellow}) = \frac{14}{44} + \frac{18}{44} = \frac{32}{44} \text{ or } \frac{8}{11} \]

3. \[ P(\text{dog or cat}) = \frac{2107}{5200} + \frac{807}{5200} = \frac{303}{5200} = \frac{2611}{5200} \]

The students should be able to recognize problems on probability which involve mutually exclusive and not mutually exclusive events. They should be able to tell that events that cannot occur at the same time are called mutually exclusive events.

Present the Venn diagram and ask them to observe events \( A \) and \( B \). Guide them so they could tell that these two events illustrated are mutually exclusive. In Activity 4, you may point out the event, getting 5 or a number divisible by 3 in the set \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\} \) as an example of mutually exclusive event.

If two events, \( A \) and \( B \), are mutually exclusive, then the probability that either \( A \) or \( B \) occurs is the sum of their probabilities. In symbols,

\[ P(A \text{ or } B) = P(A) + P(B) \]

Consider the set \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\} \). The numbers divisible by 3 in the given set are 3, 6, 9, 12, and 15. Thus, \( \{3, 6, 9, 12, 15\} \) is a subset of the given set. Also, the numbers divisible by 4 in the same set are 4, 8, and 12. So, \( \{4, 8, 12\} \) is also a subset of the given set \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\} \). Notice that both subsets contain a common element, 12. Thus, the event of getting a number divisible by 3 or the event of getting a number divisible by 4 in the set \( \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\} \) are not mutually exclusive.

The Venn diagram below shows events \( A \) and \( B \) which are not mutually exclusive because \( A \) and \( B \) intersect. Note that there are outcomes that are common to \( A \) and \( B \), which is the intersection of \( A \) and \( B \).
If two events, $A$ and $B$, are not mutually exclusive, then the probability that either $A$ or $B$ occurs is the sum of their probabilities decreased by the probability of both occurring. In symbols,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

To find out whether students have understood the concept of mutually exclusive events, ask them to do Activity 5.

**Activity 5: More Exercises on Mutually Exclusive and Not Mutually Exclusive Events**

1. a. $P(\text{chocolate or coffee}) = \frac{10}{30} + \frac{8}{30} = \frac{18}{30}$ or $\frac{3}{5}$
   
   b. $P(\text{caramel or not coffee}) = \frac{11}{15}$
   
   c. $P(\text{coffee or caramel}) = \frac{2}{3}$
   
   d. $P(\text{chocolate or not caramel}) = \frac{9}{10}$

2. $P(\text{blue or red shirt}) = \frac{3}{5}$

3. $P(\text{black pair of pants or red shirt}) = \frac{1}{15}$

4. Let $P(Q)$ be the probability that a license plate contains a double letter and an even number

   $$P(Q) = \frac{26 \cdot 1 \cdot 10 \cdot 10 \cdot 5}{26 \cdot 26 \cdot 10 \cdot 10}$$

   $$P(Q) = \frac{5}{26 \cdot 10} \text{ or } \frac{1}{52}$$

**Activity 6: Mutually Exclusive or Not?**

In previous lessons, the students learned about counting techniques and they were able to differentiate permutation from combination.

1. Mutually Exclusive: $\frac{69}{81}$ or $\frac{23}{27}$

2. Not - mutually exclusive: $\frac{188}{240}$ or $\frac{47}{60}$
3. Non-Mutually Exclusive: \( \frac{1}{4} \)

4. Mutually Exclusive: 75%

In the next activity, ask the students to observe how the concepts of permutation and combination are used in solving probability problems.

**Activity 7: Counting Techniques and Probability of Compound Events**

Guide students so they can point out the problems in this activity which involve concepts of combination and permutation. Then, let them use these concepts in determining the sample space and in determining the events.

1. a. \( \frac{20}{48} \) or \( \frac{5}{12} \)

   \[ \frac{28}{48} \]

   b. \( \frac{4}{12} \) or \( \frac{7}{12} \)

2. a. \( \frac{28C3}{48C3} = \frac{28 \cdot 27 \cdot 26}{3 \cdot 2 \cdot 1} \)

   \[ \frac{28 \cdot 27 \cdot 26}{48 \cdot 47 \cdot 46} \]

   \[ \frac{7 \cdot 9 \cdot 13}{4 \cdot 47 \cdot 23} \]

   \[ \frac{819}{4,324} \text{ or } 0.189 \]

   b. \( \frac{28C1 \cdot 20C2}{48C3} \)

3. \( \frac{28C1 \cdot 48C2}{48C3} \)

To help them reflect, you may go through the following questions:

a. In finding the probability of each event above, what concepts are needed? *The use of counting techniques, permutation, and combination*

b. Differentiate the event required in question 1 from questions 2 and 3. *Questions 2 and 3 can be solved using permutation and combination.*

c. Compare the events in questions 2 and 3. What necessary knowledge and skills do you need to get the correct answer? How did you compute for the probability of an event in each case?
What to REFLECT on and UNDERSTAND

At this time students must already have a clear understanding of the concept of probability and other concepts such as counting techniques, combination, and permutation. Help them reflect and further apply their understanding of compound events, as well as of mutually and not mutually exclusive events as they go through the next activity.

Activity 8: A Chance to Further Understand Probability

This activity can be answered in small groups. You may ask students to work in groups and discuss among themselves the following questions. Ask a representative from each group to discuss one question.

1. How does a simple event differ from a compound event?

   Any event which consists of a single outcome in the sample space is called an elementary or simple event. On the other hand, events which consist of more than one outcome are called compound events. A compound event consists of two or more simple events.

2. Differentiate mutually exclusive events from non-mutually exclusive events.

   Mutually exclusive events are two or more events having no common elements, while the events which are not mutually exclusive are two or more events which have common elements.

3. Suppose there are three events A, B, and C that are not mutually exclusive. List all the probabilities you would need to consider in order to calculate \( P(A \text{ or } B \text{ or } C) \). Then, write the formula you would use to calculate the probability.

   The probabilities needed:
   \[ P(A \cap B \cap C), P(A \cap B), P(B \cap C), P(A \cap C), P(A), P(B), P(C) \]

   The formula:
   \[
   P(A \cup B \cup C) = P(A) + P(B) + P(C) - [P(A \cap B) + P(B \cap C) + P(A \cap C) - P(A \cap B \cap C)]
   \]

4. Explain why subtraction is used when finding the probability of two events that are not mutually exclusive.

   Two or more events which are not mutually exclusive are events having common elements. So if A and B are two events which are not mutually exclusive, then \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
   where \( A \cap B \) are the common elements.

What to TRANSFER

This time, students should already know how to apply what they have learned in real-life situations. You can ask them to do certain tasks that will demonstrate their understanding of probability of compound events, mutually exclusive events, and non-mutually exclusive events.
Activity 9: Where in the Real World?

Answer the following questions. Write a report of your answers using a minimum of 120 words. Be ready to present your answers in class.

1. Describe a situation in your life that involves events which are mutually exclusive and not mutually exclusive. Explain why the events are mutually exclusive or not mutually exclusive.

- The children of two different families are mutually exclusive because it is impossible for both families to have a common child.
- You and your friends are eating in a restaurant. The event that you and your friends ordering the same food or drinks (e.g., rice, drinks) may not be mutually exclusive because you may be ordering the same drink.

2. Think about your daily experience. How is probability portrayed in your favorite newspapers, television shows, and radio programs? What are your general impressions of the ways in which probability is used in the print media and entertainment industry?

Few examples in which probability is used in the media are as follow:

- In advertisements: 9 out of 10 dentists surveyed prefer a specific brand of toothpaste.
- In news/weather: There is a 30% chance of rain today.
- In sports: “A certain basketball player” has a “shooting average of” 0.89 (0.89 indicates this person’s chances of shooting the ball).

In the case of advertising, the data provided are used as a means to convince the audience (e.g., viewers) to use the product. This is always the case for the entertainment industry. They use certain data to show trends because people tend to follow trends. Be sure to remind students to be critical about what the media is telling us. This is a good opportunity for students to realize the importance of responsible use of data.

The answers provided may serve as your guide in engaging the students in a more fruitful discussion of the application of probability in real life.

Summary/Synthesis/Generalization:

In this lesson, students were able to recall and use their knowledge and understanding of the concept of the probability of simple events in solving problems involving probability of compound events. The different activities required them to make connections and apply concepts of union and intersection of sets, specifically using Venn diagrams to illustrate mutually exclusive as well as non-mutually exclusive events. Also, students were given problems that required them to use concepts which they have previously learned on permutation and combination in solving real-life problems. Most importantly, the
different tasks and activities were opportunities for them to use their reasoning and mathematical skills in solving real-life problems.

Lesson 2: Independent and Dependent Events

In Lesson 1 of this module, students learned about the basic concepts of the probability of compound events. In this lesson, you may start by showing a coin and ask whether the outcome of the flip of a fair coin is independent of the outcomes of the flips that came before it.

What to KNOW

Activity 1: Understanding Independent and Dependent Events

Consider the situations below and answer the questions that follow.

Situation 1:

a. \( P(\text{blue, blue}) = \frac{12}{35} \cdot \frac{12}{35} \)

b. \( P(\text{red, yellow}) = \frac{14}{35} \cdot \frac{9}{34} \)

Situation 2:

a. \( P(\text{red, blue}) = \frac{14}{35} \cdot \frac{12}{34} \)

b. \( P(\text{yellow, yellow}) = \frac{9}{35} \cdot \frac{8}{34} \)

After the activity, you may go through the following questions to help students reflect on their own answers and solutions.

Reflect:

a. Compare the process of getting the probabilities in each of the situations above.

   In situation 1, the ball was put back inside the box before getting the second ball. In situation 2, the ball was not put back inside the box.

b. In situation 1, is the probability of obtaining the second ball affected after getting the first ball? What about in situation 2?

   In situation 1, the probability of getting the second ball was the same as the probability of drawing a red ball in the first draw. On the other hand, the probability of obtaining the second ball was affected since the ball was not put back inside the box. Thus, the number of ball was changed.

c. What conclusion can you make about the events in the given situations? How are these events different?

   This activity should help students understand the concept of dependent and independent events. You may discuss the following if necessary.

Independent and Dependent Events: In situation 1, the probability of getting a blue ball in the second draw is not affected by the probability of drawing a red ball in the first draw, since the first ball is put back inside the box prior to the second draw. Thus, the two events are independent of each other. The two events are independent if the result of one event does not affect the result of the other event.
Example: When a coin is tossed and a die is rolled, the event that a coin shows up head and the event that a die shows up a 5 are independent events.

Two events are independent if the occurrence of one of the events gives us no information about whether or not the other event will occur; that is, the events have no influence on each other.

If two events, \( A \) and \( B \), are independent, then the probability of both events occurring is the product of the probability of \( A \) and the probability of \( B \). In symbols,

\[
P(A \text{ and } B) = P(A) \cdot P(B)
\]

When the outcome of one event affects the outcome of another event, they are dependent events. In situation 2 above, if the ball was not placed back in the box, then drawing the two balls would have been dependent events. In this case, the event of drawing a yellow ball in the second draw is dependent on the event of drawing a yellow ball in the first draw.

Example: A box contains 7 white marbles and 7 red marbles. What is the probability of drawing 2 white marbles and 1 red marble in succession without replacement?

In the first draw, the probability of getting a white marble is \( \frac{7}{14} \). In the second draw, the probability of getting a white marble is \( \frac{6}{13} \). Then in the third draw, the probability of getting a red marble is \( \frac{7}{12} \). So,

\[
P(1 \text{ white } 1 \text{ white } 1 \text{ red}) = \frac{7}{14} \cdot \frac{6}{13} \cdot \frac{7}{12} = \frac{7}{52}
\]

If two events, \( A \) and \( B \), are dependent, then the probability of both events occurring is the product of the probability of \( A \) and the probability of \( B \) after \( A \) occurs. In symbols,

\[
P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)
\]

The symbols, \( P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A) \) is used in this lesson to show that two events \( A \) and \( B \) are independent. The symbol, \( P(B \text{ following } A) \) means “the probability of \( B \) following the occurrence of \( A \).” In the sequence of the lesson, the concept of conditional probability is discussed after the lesson on dependent and independent events.

**What to PROCESS**

This section requires students to use the mathematical ideas that they learned from the previous activities and from the discussion. You may ask students to answer the problems. Encourage them to present different solutions to the problems.
Activity 2: More on Independent and Dependent Events

1. \( P(\text{blue, yellow}) = \frac{6}{21} \cdot \frac{4}{21} = \frac{8}{147} \)

2. \( P(\text{milk chocolate, white chocolate}) = \frac{5}{46} \)

3. \( P(\text{green, gray}) = \frac{6}{203} \)

See to it that students are able to answer all the questions correctly. If not, find out their difficulty and help them understand independent and dependent events. By now, they should clearly distinguish the difference between independent events and dependent events.

The next activity requires students to determine whether the events are independent or dependent.

Activity 3: Which Events Are Independent?

1. The events are dependent. Let \( P(a, a) \) be the event that 2 stuffed animals are chosen:
   \[ P(a, a) = \frac{8}{23} \cdot \frac{7}{22} = \frac{28}{253} \]

2. The events are dependent. Let \( P(b, a) \) be the event that Dominic chose a banana, then an apple:
   \[ P(b, a) = \frac{5}{20} \cdot \frac{6}{19} = \frac{3}{38} \]

3. The events are independent. Let \( P(\text{blue, blue}) \) be the event that Nick’s pick is a blue pen in either the first or second pick:
   \[ P(\text{blue, blue}) = \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81} \]

What to REFLECT on and UNDERSTAND

Prior to the next activity, you may ask students to write a short reflection paper that critically expresses their understanding of independent and dependent events.

Activity 4: Probability of Independent and Dependent Events

1. \[ \frac{10}{28} \cdot \frac{10}{28} \cdot \frac{5}{28} = \frac{125}{5488} \]

2. \[ \frac{10}{19} \cdot \frac{9}{18} = \frac{10}{38} \quad \text{or} \quad \frac{5}{19} \]
Activity 5: Are the Events Independent or Dependent?

This time, students need to reflect on their understanding of dependent and independent events. Guide them in answering the questions by asking them to work in groups and discuss their answers. After this group activity, ask the representative from selected groups to present their answers.

1. What makes an event independent?

Two events are independent if the occurrence of one event does not affect the occurrence of the other (e.g., random selection with replacement).

2. Differentiate a dependent event from an independent event.

Independent events are events for which the probability of any one event occurring is unaffected by the occurrence or non-occurrence of any of the other events. On the other hand, two events are dependent if the occurrence of one event affects the occurrence of the other.

Then, you go through the following questions:

What new realizations do you have about the probability of a dependent event? How would you make connections of this topic to other topics that you previously learned? How would you use these concepts in real life?

Journal Writing: Ask students to write their realizations and answers to these questions on a piece of paper that will serve as a reflection paper.

What to TRANSFER

This section provides students with opportunities to apply what they have learned in this lesson to real-life situations. Ask them to discuss in pairs or in small groups.

Activity 6: Where in the Real World?

1. Describe a situation in your life that involves dependent and independent events. Explain why the events are dependent or independent.

Possible Answer:
An example of dependent event: Parking illegally and getting a parking ticket. Parking illegally increases your chances of getting a ticket.

An example of independent events: Meeting your friend on your way home and finding a 5-peso coin. (Your chance of finding a 5-peso coin does not depend on your meeting of friends.)

2. Formulate your own problems involving independent and dependent events.
Possible Answers:

Sample Problem on Dependent Events: Mira, Jose, and Ruth go to a restaurant and order a sandwich. The menu has 10 types of sandwiches and each of them is equally likely to order any type. What is the probability that each of them orders a different type?

Sample Problem on Independent Events: A dresser drawer contains one pair of socks with each of the following colors: blue, brown, white, and black. Each pair is folded together in a matching set. You reach into the sock drawer and choose a pair of socks without looking. You replace this pair and then choose another pair of socks. What is the probability that you will choose the red pair of socks both times?

Summary/Synthesis/Generalization:

In this lesson, students were introduced to the concept of independent and dependent events. It is important to emphasize that two events are independent if the occurrence of one of the events gives no information about whether or not the other event will occur; that is, the events have no influence on each other. The problems that they solved required them to apply their knowledge and skills learned in the previous lesson which helped them formulate their own real-life problems. Their understanding of this lesson will also facilitate their learning of the next lesson, which is conditional probability.

Lesson 3: Conditional Probability

Conditional probability plays a key role in many practical applications of probability. In these applications, important conditional probabilities are often drastically affected by seemingly small changes in the basic information from which the probabilities are derived.

In this lesson, the focus is on conditional probability. To understand conditional probability, you may ask students to answer Activity 1.

Activity 1: Probability of an Event Given Certain Conditions

1. Tree diagram

Let \( g_1, g_2, g_3 \), be the three nondefective batteries and \( d \) be the defective battery.
2. Sample Space:
\[ \{ (g_1, g_2), (g_1, g_3), (g_1, d), (g_2, g_1), (g_2, g_3), (g_2, d), (g_3, g_1), (g_3, g_2),
(g_3, d), (d, g_1), (d, g_2), (d, g_3) \} \]

3. \[ P(\text{second is } g) = \frac{9}{12} \text{ or } \frac{3}{4} \]

4. \[ P(g_2 | g_1) = \frac{P(g_2 \cap g_1)}{P(g_1)} = \frac{6}{12} = \frac{2}{3} \]

In Activity 1, guide the students in pointing out that a condition was given when they were asked to find the probability of an event. This shows an example of probability involving conditions which is referred to as conditional probability. To understand conditional probability further, ask students to proceed to Activity 2.

**Activity 2: More on Conditional Probability**

1. Two events are dependent if the probability that one event occurs changes based on whether the other event has occurred. Find the probability of \( P \) given that \( M \) has occurred and see if it is different from the probability of \( P \). Note that two events are independent if \( P(P|M) = P(P) \) or \( P(M|P) = P(M) \) or \( P(P \text{ and } M) = P(P)P(M) \).

\[ P(P|M) = \frac{24}{40} \text{ or } \frac{3}{5} \text{ and } P(P) = \frac{36}{60} \text{ or } \frac{3}{5} \]

Since these probabilities are the same, events \( P \) and \( M \) are independent. Emphasize that \( P \) inside the parentheses represents the event “Pass.” Ask the students to refer to the given table where this symbol was used to denote the event “Pass.”

2. Events \( P \) and \( F \) are also independent since \( P(P|F) = \frac{36}{60} = \frac{3}{5} \)

\[ \text{and } P(P) = \frac{60}{100} = \frac{3}{5} \]

Again, emphasize that \( P \) inside the parentheses represents the event “Pass.” Ask the students also to refer to the given table where this symbol was used to denote the event “Pass.”

3. There are 40 males. Of these 40 males, 24 passed the proficiency examination

so, \( P(P|M) = \frac{24}{40} \) or 0.60.

4. There are 60 people that passed the proficiency examination. Of these 60 people, 24 are male,

so, \( P(M|P) = \frac{24}{60} \) or 0.40.

5. \( P(F|P) = \frac{36}{60} \) or 0.60.
The usual notation for "event $A$ occurs given that event $B$ has occurred" is "$A|B$" ($A$ given $B$). The symbol $|$ is a vertical line and does not imply division. $P(A|B)$ denotes the probability that event $A$ will occur given that event $B$ has occurred already. We define conditional probability as follows:

For any two events $A$ and $B$ with $P(B) > 0$, the conditional probability of $A$ given that $B$ has occurred is defined by

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

When two events, $A$ and $B$, are dependent, the probability of both events occurring is $P(A \text{ and } B) = P(B) \cdot P(A|B)$. Also, $P(A \text{ and } B) = P(A) \cdot P(B|A)$.

**Sample Problem:** A mathematics teacher gave her class two tests. Twenty-five percent of the class passed both tests and 42% of the class passed the first test. What percent of those who passed the first test also passed the second test?

**Solution:** This problem involves a conditional probability since it asks for the probability that the second test was passed given that the first test was passed.

$$P(\text{Second|First}) = \frac{P(\text{First and Second})}{P(\text{First})} = \frac{0.25}{0.42} = \frac{25}{42} = 0.60 \text{ or } 60\%$$

**Activity 3: Conditional Probability of Independent Events**

1. $P(X) = 0.15$
2. $P(A) = 0.40$
3. $P(A \cap X) = 0.06$
4. $P(X|A) = \frac{P(X \cap A)}{P(A)} = \frac{0.06}{0.40} = 0.15$

Take note that items 1 and 4, $P(X)$ and $P(X|A)$ are both equal to 0.15.

Notice that the occurrence of event $A$ gives no information about the probability of event $X$. The events $X$ and $A$ are independent events.

Two events $A$ and $B$ are said to be independent if either:

i. $P(A|B) = P(A)$, i.e., $P(B|A) = P(B)$, or equivalently,

ii. $P(A \cap B) = P(A) \cdot P(B)$.

Probabilities are usually very sensitive to the information given as a condition. Sometimes, however, a probability does not change when a condition
is supplied. If the extra information provided by knowing that an event \( B \) has occurred does not change the probability of \( A \), that is, if \( P(A|B) = P(A) \), then events \( A \) and \( B \) are said to be independent. Since

\[
P(A|B) = \frac{P(A \cap B)}{P(B)}
\]

Sometimes a conditional probability is known, and we want to find the probability of an intersection. By rearranging the terms in the definition of conditional probability and considering the definition of independence, we obtain the Multiplicative Rule which is

\[
P(A \cap B) = P(A) \cdot P(B)
\]

Let us look at some other problems in which you are asked to find a conditional probability in Activity 4.

What to PROCESS:

This section requires you to use the mathematical ideas you learned from the previous activities and from the discussion. Answer the problems in the following activities in different ways when possible.

Activity 4: Conditional Probability Independent and Dependent Events

1. a. \( P(S \cap Q) = P(S|Q) \cdot P(Q) \)

\[
= 0.4 \cdot 0.5 = 0.2
\]

b. \( P(Q|S) = \frac{P(S \cap Q)}{P(S)} \)

\[
= \frac{0.2}{0.3} = \frac{2}{3}
\]

c. \( P(S'|Q) = \frac{P(S' \cap Q)}{P(Q)} \)

\[
= \frac{P(Q) - P(S \cap Q)}{P(Q)}
\]

\[
= \frac{0.5 - 0.2}{0.5} = 0.6
\]

d. \( P(S|Q') = \frac{P(S \cap Q')}{P(Q')} \)

\[
= \frac{P(S) - P(S \cap Q)}{1 - P(Q)}
\]

\[
= \frac{0.3 - 0.2}{1 - 0.5} = 0.2
\]
2. a. The event \( T \) is just the union of \( S \) and \( Q \), so

\[
P(T) = P(S \cup Q) = P(S) + P(Q) - P(S)P(Q)
\]

\[= 0.44
\]

b. \( P \ R = P \ S \cup Q - P \ S \ P \ Q \)

\[= 0.38
\]

c. \( P \ S | R = \frac{P \ S \cap R}{P \ R} = \frac{7}{19}
\]

d. \( P \ R | S = \frac{P \ R \cap S}{P \ S} = 0.7
\]

e. \( S \) and \( R \) are not independent since \( P \ S | R \neq P \ S \), but it should be \( P(R|S) = P(R) \)

Reflect:

a. What do you notice about the conditional probability of independent events?

b. How about the conditional probability of dependent events?

Activity 5: Solving Problems Involving Conditional Probability

1. \( P \text{ second is G | at least one G} = \frac{2}{3} \)

<table>
<thead>
<tr>
<th>First Child</th>
<th>Younger Child</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>( (B, G) )</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>( (G, B) )</td>
</tr>
<tr>
<td>G</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>( (G, G) )</td>
</tr>
</tbody>
</table>

There are three outcomes which are \( (B, G) \), \( (G, B) \), and \( (G, G) \). For the probability that the younger child is a girl, we have \( (B, G) \) and \( (G, G) \). So, there are 2 out of three possible outcomes.

2. Let \( W = \) the event that a fan waved a banner, and \( A = \) the event that a fan cheered for team \( A \)

\[
\text{So, } P(W|A) = \frac{0.20}{0.80} = \frac{1}{4}
\]

What to REFLECT on and UNDERSTAND:

This activity will help you find out how much students learned from the previous discussion on conditional probability.
Activity 6: Using Venn Diagram

Consider the Venn diagram below.

1. The Venn diagram shows two events $A$ and $B$ with their intersection.

![Venn Diagram](image)

2. To find $P(B|A)$, divide the probability of the intersection of events $A$ and $B$ by the probability of event $A$.

   To get the conditional probability, divide the overlap of the two circles by the circle on the left. $P(B|A) = P(A \text{ and } B)$ divided by $P(A)$.

3. At San Isidro High School, the probability that a student joins Technology Club and Mathematics Club is 0.087. The probability that a student joins Technology Club is 0.68. What is the probability that a student joins the Mathematics Club given that the student is a member of the Technology Club?

What to TRANSFER

This section is an opportunity for students to apply what they have learned in this lesson to real-life situations. Ask students to do the task in Activity 6 to demonstrate their understanding of conditional probability.

Activity 7: Probability in Real Life

Choose your own topic of study or choose from four recommended topics. Write a research report. Focus on the question that follows:

**How can we use statistics and probability to make informed decisions about any of the following topics?**

**Recommended Topics:**
1. Driving and cell phone use
2. Diet and health
3. Professional athletics
4. Costs associated with a college education

The research report should contain the following:

1. Situation (Problem situation in real life about the topic. This includes analysis of the impact of the problem if not properly addressed)
2. Problem Solution (Suggested solution illustrating how statistics and probability can be applied in minimizing the impact of the problem)
3. Strategies on how to inform and convince others about the situation
4. Reflections and insights which contain a critical analysis

**Note:** Guide your students in formulating criteria on how the report should be graded. The objective of this research report is to demonstrate their understanding and apply their knowledge and skills on probability in solving real-life problems specifically on decision making.

**Summary/Synthesis/Generalization:**

This lesson was about conditional probability and their applications in real-life. The lesson provided the students with different opportunities to make connections, to use their reasoning ability in solving problems on conditional probability, and eventually communicate their research findings. Their understanding of this lesson and other previously learned concepts and principles will help them in making decisions when confronted with real-life problems involving probability.
SUMMATIVE TEST

Answer the following by choosing the letter of the correct answer.

1. The probability of heads landing up when you flip a coin is $\frac{1}{2}$. What is the probability of getting tails if you flip it again?
   A. $\frac{1}{4}$  
   B. $\frac{1}{3}$  
   C. $\frac{1}{2}$  
   D. $\frac{3}{4}$

2. Suppose you roll a red die and a green die. The probability that the sum of the numbers on the dice is equal to 9 is $\frac{4}{36}$ since there are 4 of the 36 outcomes where the sum is 9. What if you see that the red die shows the number 5, but you still have not seen the green die? What then are the chances that the sum is 9?
   A. $\frac{1}{6}$  
   B. $\frac{1}{4}$  
   C. $\frac{1}{3}$  
   D. $\frac{2}{3}$

3. If a coin is tossed 3 times, what is the probability that all three tosses come up heads given that at least two of the tosses come up heads?
   A. $\frac{1}{6}$  
   B. $\frac{1}{4}$  
   C. $\frac{1}{3}$  
   D. $\frac{3}{8}$

4. Among a large group of patients recovering from shoulder injuries, it was found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.
   A. 0.26  
   B. 0.38  
   C. 0.40  
   D. 0.48

5. A class has the following grade distribution:

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>5</td>
</tr>
<tr>
<td>90</td>
<td>14</td>
</tr>
<tr>
<td>85</td>
<td>7</td>
</tr>
<tr>
<td>80</td>
<td>9</td>
</tr>
<tr>
<td>75</td>
<td>8</td>
</tr>
</tbody>
</table>

Suppose that a student passes the course if she or he gets a grade of 80. If a student is randomly picked from this class, what is the probability that the student’s grade is 95 if it is known that the student is passing the course?
   A. $\frac{5}{35}$  
   B. $\frac{5}{43}$  
   C. $\frac{5}{40}$  
   D. $\frac{5}{28}$
6. Barbara, Carol, Alice, Perla, and Sabrina are competing for two roles in a play. Assume that the two to get roles will be randomly chosen from the five girls. What is the conditional probability that Perla gets a role if we know that Carol does not get a role?

A. $\frac{1}{4}$  
B. $\frac{1}{3}$  
C. $\frac{1}{2}$  
D. $\frac{3}{4}$

For numbers 7 to 8. Two men and three women are in a committee. Two of the five are to be chosen to serve as officers.

7. If the officers are chosen randomly, what is the probability that both officers will be women?

A. $\frac{3}{4}$  
B. $\frac{1}{3}$  
C. $\frac{3}{8}$  
D. $\frac{3}{10}$

8. What is the probability that both officers will be women given that at least one is a woman?

A. $\frac{3}{4}$  
B. $\frac{1}{3}$  
C. $\frac{3}{8}$  
D. $\frac{3}{10}$

9. Mario has 5 blocks of different colors in a bag. One block is red, one is yellow, one is green, one is blue, and one is black. Mario pulls out a block, looks at it, and puts it back in the bag. If he does this 3 times, what is the probability that the 3 blocks selected are all of the same color?

A. $\frac{5}{5^3}$  
B. $\frac{1}{5^3}$  
C. $\frac{4}{5^3}$  
D. $\frac{5}{4 \times 5}$

10. In a small town with two schools, 1000 students were surveyed if they had mobile phone. The results of the survey are shown below:

<table>
<thead>
<tr>
<th></th>
<th>With Mobile Phone</th>
<th>Without Mobile Phone</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>School A</td>
<td>365</td>
<td>156</td>
<td>521</td>
</tr>
<tr>
<td>School B</td>
<td>408</td>
<td>71</td>
<td>479</td>
</tr>
<tr>
<td>Total</td>
<td>773</td>
<td>227</td>
<td>1000</td>
</tr>
</tbody>
</table>

What is the probability that a randomly selected student has a mobile phone given that the student attends School B?

A. $\frac{521}{1000}$  
B. $\frac{408}{1000}$  
C. $\frac{408}{479}$  
D. $\frac{408}{521}$
Part II. Problem Solving

Solve the following. Show complete solutions.

10. James and Jenny are playing games. James places tiles numbered 1 to 50 in a bag. James select a tile at random. If he selects a prime number or a number greater than 40, then he wins. What is the probability that James will win on his first turn?

For numbers 12 to 15, use the following situation:

The international club of a school has 105 members, many of whom speak multiple languages. The most commonly spoken languages in the club are English, Korean, and Chinese. Use the Venn diagram below to determine the probability of selecting a student who:

11. Does not speak English.

12. Speaks Korean given that he/she speaks English.

13. Speaks English given that he/she speaks Chinese.


15. Billy, Raul, and Jose are in a bicycle race. If each boy has an equal chance of winning, find each probability below. Draw a tree diagram to answer each question.
   a. Jose wins the race.
   b. Raul finishes last.
   c. Jose, Raul, and Billy finish first, second, and third, respectively

16. There are four batteries, and one is defective. Two are to be selected at random for use on a particular day. Find the probability that the second battery selected is not defective, given that the first was defective.

17. Suppose that a foreman must select one worker from a pool of four available workers (numbered 1, 2, 3, and 4) for a special job. He selects the worker by mixing the four names and randomly selecting one. Let A denote the event that worker 1 or 2 is selected, let B denote the event that worker 1 or 3 is selected, and let C denote the event that worker 1 is selected. Are A and B independent? Are A and C independent? Justify your answer.

18. Blood type, the best known of the blood factors, is determined by a single allele. Each person has blood type A, B, AB, or O. Type O represents the absence of a factor and is recessive to factors A and B. Thus, a person with type A blood may be either homozygous (AA) or heterozygous (AO) for this allele; similarly, a person with type B blood may be either homozygous (BB) or heterozygous (BO). Type AB occurs if a person is given an A factor by one parent and a B factor by the other parent. To have type O blood, an individual
must be homozygous O (OO). Suppose a couple is preparing to have a child. One parent has blood type AB, and the other is heterozygous B. What are the possible blood types that the child will have and what is the probability of each?

19. A driver knows that there are traffic lights around the corner. There is a red light, a yellow light, and a green light. He thinks that since there are three lights, his probability of encountering a red light or a yellow light is \( \frac{2}{3} \). Is the driver right? Explain.

20. **Trying Your Chance in a Game Show**

You are a contestant in a game show. You will win if you select the door behind which the prize is hidden. Suppose you have selected a door. Before opening your choice, the emcee selected and opened one of the two closed doors revealing an empty stage. Will you change your original choice and select the only remaining door? How will your chances change if you switch?

---

**Summative Test**

**Answer Key**

<table>
<thead>
<tr>
<th>Part I</th>
<th>Part II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. C</td>
<td>11. ( \frac{23}{50} )</td>
</tr>
<tr>
<td>2. A</td>
<td>12. ( \frac{16}{105} )</td>
</tr>
<tr>
<td>3. B</td>
<td>13. ( \frac{41}{89} )</td>
</tr>
<tr>
<td>4. D</td>
<td>14. ( \frac{21}{26} )</td>
</tr>
<tr>
<td>5. A</td>
<td>15. ( \frac{33}{105} )</td>
</tr>
<tr>
<td>6. A</td>
<td>16. a.) ( \frac{1}{3} )</td>
</tr>
<tr>
<td>7. D</td>
<td>b.) ( \frac{1}{3} )</td>
</tr>
<tr>
<td>8. B</td>
<td>c.) ( \frac{1}{6} )</td>
</tr>
<tr>
<td>9. B</td>
<td></td>
</tr>
<tr>
<td>10. C</td>
<td></td>
</tr>
</tbody>
</table>
17. 1

18. The name is selected at random, so we can assign a probability of \( \frac{1}{4} \) to each individual worker. Then \( P(A) = \frac{1}{2} \), \( P(B) = \frac{1}{2} \), and \( P(C) = \frac{1}{4} \). The intersection \( AB \) contains only worker 1, \( P(A \text{ and } B) = \frac{1}{4} \). Now, \( P(A \text{ and } B) = \frac{1}{4} = P(A)P(B) \), so \( A \) and \( B \) are independent. Since intersection \( AC \) also contains only worker 1, \( P(A \text{ and } C) = \frac{1}{4} \). But, \( P(A \text{ and } C) = \frac{1}{4} \) which is not equal to \( P(A)P(C) = \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8} \), so \( A \) and \( C \) are not independent.

19. Solution:

\[
\begin{array}{cc}
A & B \\
B & O
\end{array}
\]

\[
\begin{array}{cc}
A & B \\
B & O
\end{array}
\]

The four possible outcomes are \( AB \), \( AO \), \( BB \), and \( BO \) which are equally likely. The probability that the child will have type B blood is 0.5 because \( BB \) and \( BO \) are both expressed as type B. The probabilities of type AB and type A (\( AO \)) are each 0.25.

20. No. The time span for the yellow light to turn on is less than the time span for the other lights to turn on.

21. Here are the key points to understand the problem in this Game show:

If there are two choices and you know nothing about them, then the probability of each choice to contain a prize is 0.5. The flaw in this Game Show is not taking the emcee’s hints into account, thinking the chances are the same before and after. The goal is not to understand this puzzle — it is to realize how subsequent actions and information challenge previous decisions.

Solution: There is a \( \frac{1}{3} \) chance that you will get the door with the prize, and a \( \frac{2}{3} \) chance that you will miss the prize. If you do not switch, the probability that you will get the prize is \( \frac{1}{3} \). However, if you missed, then the prize is behind one of the remaining two doors (with the probability
of $\frac{2}{3}$). Furthermore, of these two, the emcee will open the empty one, leaving the prize door closed. Therefore, if you miss and then switch, you are certain to get the prize. Summing up, if you do not switch, your chance of winning is $\frac{1}{3}$ whereas if you do switch your chance of winning is $\frac{2}{3}$.

Glossary of Terms

**Complement of an Event** – a set of all outcomes that are NOT in the event. If $A$ is the event, the complement of the event $A$ is denoted by $A'$

**Compound Events** – a composition of two or more simple events

**Conditional Probability** – The **conditional probability** of event $B$ given $A$ is the probability that the event $B$ will occur given that event $A$ has already occurred. This probability is written as $P(B|A)$ and read as the **probability of $B$ given $A$**. In the case where events $A$ and $B$ are **independent** (where event $A$ has no effect on the probability of event $B$), the conditional probability of event $B$ given event $A$ is simply the probability of event $B$, that is, $P(B)$.

**Dependent Events** – Two events are dependent if the occurrence of one event does affect the occurrence of the other (e.g., random selection without replacement).

**Events** – a set of possible outcomes resulting from a particular experiment. For example, a possible event when a single six-sided die is rolled is $\{5, 6\}$, that is, the roll could be a 5 or a 6. In general, an event is any subset of a sample space (including the possibility of an empty set).

**Independent Events** – events for which the probability of any one event occurring is unaffected by the occurrence or non-occurrence of any of the other events. Formally, $A$ and $B$ are independent if and only if $P(A|B) = P(A)$.

**Intersection of Events** – a set that contains all of the elements that are in both events. The intersection of events $A$ and $B$ is written as $A \cap B$.

**Mutually Exclusive Events** – events that have no outcomes in common. This also means that if two or more events are mutually exclusive, they cannot happen at the same time. This is also referred to as disjoint events.

**Union of Events** – a set that contains all of the elements that are in at least one of the two events. The union is written as $A \cup B$.

**Venn Diagram** – A diagram that uses circles to represent sets, in which the relations between the sets are indicated by the arrangement of the circles.
References and Website Links in this Module


Website Links as References:


